Feedback-Based Iterative Learning Control for MIMO LTI Systems

Tae-Yong Doh and Jung Rae Ryoo

Abstract: This paper proposes a necessary and sufficient condition of convergence in the $\mathcal{L}_2$-norm sense for a feedback-based iterative learning control (ILC) system including a multi-input multi-output (MIMO) linear time-invariant (LTI) plant. It is shown that the convergence conditions for a nominal plant and an uncertain plant are equal to the nominal performance condition and the robust performance condition in the feedback control theory, respectively. Moreover, no additional effort is required to design an iterative learning controller because the performance weighting matrix is used as an iterative learning controller. By proving that the least upper bound of the $\mathcal{L}_2$-norm of the remaining tracking error is less than that of the initial tracking error, this paper shows that the iterative learning controller combined with the feedback controller is more effective to reduce the tracking error than only the feedback controller. The validity of the proposed method is verified through computer simulations.

Keywords: Convergence, iterative learning control (ILC), $\mathcal{L}_2$-norm, least upper bound, multi-input multi-output (MIMO) linear time-invariant (LTI) systems, nominal performance, robust performance, structured singular value.

1. INTRODUCTION

Iterative learning control (ILC) has been used in control systems that perform repeated tasks [1,2]. Just as humans learn skills by trial and error, the ILC system learns the dynamics of the system by repeated trials. Studies on ILC systems, therefore, have been mainly focused on the convergence of proposed learning algorithms. Because ILC is an open-loop control scheme, it is applied to real systems along with feedback control for enhancing robustness against unrepeatable disturbances and for reducing the tracking error in the early stage of learning [3-8]. It is desirable that the least information about the plant is required in design of an iterative learning controller. However, at least a nominal plant model is needed because the convergence of the ILC system is guaranteed based on the mathematical model of the plant. The ILC system is designed to guarantee a convergence condition expressed in terms of the nominal plant model. As a result, the ILC system applied to a real plant may not converge because modeling errors are unavoidable to some extent. To solve the problem, some robust approaches to feedback-based ILC have been proposed. Moon *et al.* suggested a robust approach to ILC design for a unity feedback control system [4]. Doh *et al.* presented a sufficient condition for robust stability and robust convergence for uncertain linear systems and a method to design a feedback controller and an iterative learning controller simultaneously [5]. Hu *et al.* derived a necessary and sufficient condition of convergence for linear time-invariant systems with multiplicative uncertainties and time delays and showed that the condition is of the same form with the robust performance condition [9]. Tayebi and Zaremba proposed a result similar to that of Hu *et al.* [10].

Most studies on robust ILC with current feedback have focused on single-input and single-output (SISO) systems with multiplicative uncertainties. In this paper, motivated by the results in [9-11], we consider a feedback-based ILC scheme for multi-input and multi-output (MIMO) plants. We first obtain a necessary and sufficient condition of convergence in the $\mathcal{L}_2$-norm sense for nominal plants, which is equal to the nominal performance condition in the feedback control theory. And then a modified convergence condition considering the plant uncertainty is derived, which is described only by the nominal parts and is exactly the same as the robust performance condition expressed by the structured singular value (μ) [12-14]. The iterative input updating rule is established by

Manuscript received August 15, 2006; revised July 24, 2007; accepted November 16, 2007. Recommended by Editor Tae Woong Yoon.

Tae-Yong Doh is with the Department of Control and Instrumentation Engineering, Hanbat National University, San 16-1, Duckmyung-dong, Yuseong-gu, Daejeon 305-719, Korea (e-mail: dolerite@hanbat.ac.kr).

Jung Rae Ryoo is with the Department of Control and Instrumentation Engineering, Seoul National University of Technology, 172 Gongneung2-dong, Nowon-gu, Seoul 139-743, Korea (e-mail: jrryoo@snut.ac.kr).

using the weighting performance matrix as an iterative learning controller. Owing to these results, there is no need to design an iterative learning controller if a feedback controller is designed to satisfy the robust performance condition. Moreover, using the similar approach in [10], we demonstrate that the least upper bound of the $\mathcal{L}_2$-norm of the remaining tracking error is less than that of the initial tracking error in case of MIMO plants. Finally, a simulation study on a two-mass/spring/damper system is performed to verify the effectiveness of the proposed method.

Throughout this paper, signals in the time domain are denoted by lower-case letters and their capitals denote their own Laplace transforms, for example, $\mathcal{L}\{f(t)\} = F(s)$. If there is no other definition, capitals such as $G(s)$ or G stand for transfer matrices. The subspace of real rational matrices in $\mathcal{H}_2$ (respectively, $\mathcal{H}_\infty$) is denoted by $\mathcal{RH}_2$ (respectively, $\mathcal{RH}_\infty$). $\sigma(\cdot)$ and $\bar{\sigma}(\cdot)$ signify the singular value and the largest singular value, respectively. The Laplace variable s and the angular frequency ω will be omitted when these do not lead to any confusion. A lower linear fractional transformation (LFT) on Δ with $M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$ is $\mathcal{F}_l(M,\Delta) := M_{11} + M_{12}\Delta(I - M_{22}\Delta)^{-1}M_{21}$ [14]. In a similar manner, an upper LFT on Δ with M is $\mathcal{F}_u(M,\Delta) := M_{22} + M_{21}\Delta(I - M_{11}\Delta)^{-1}M_{12}$.

2. MAIN RESULTS

Consider the feedback control system in Fig. 1. In this figure, $y_d(t) \in \mathbb{R}^p$ is the desired trajectory, $y(t) \in \mathbb{R}^p$ is the plant output, and $u(t) \in \mathbb{R}^q$ is the feedback control input. $C(s)$ is the feedback controller that stabilizes the feedback control system. $G(s)$ is the plant with q inputs and p outputs.

To apply ILC technique to the feedback system, we consider an ILC system shown in Fig. 2 with the additional input $v_k(t)$ of which iterative updating rule is given by

$$V_{k+1}(s) = W_p(s)(V_k(s) + E_k(s)) \tag{1}$$

with $V_1(s) = 0$ where the tracking error $E_k(s)$ is given by

$$E_k(s) = Y_d(s) - Y_k(s) \tag{2}$$

and $W_p(s)$ is a stable performance weighting matrix with the following type:

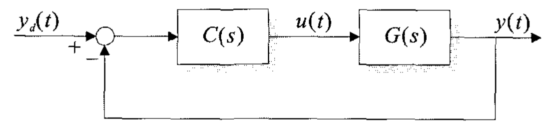


Fig. 1. Feedback control system.

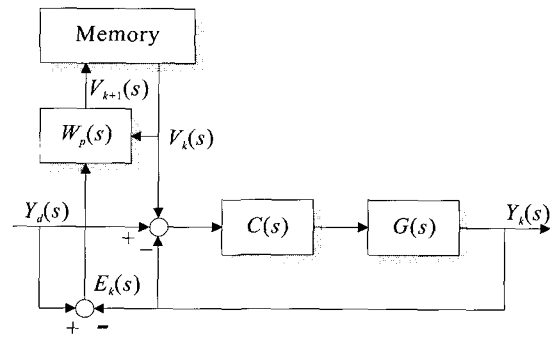


Fig. 2. Feedback-based ILC system.

$$W_p(s) = \begin{bmatrix} w_1(s) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & w_p(s) \end{bmatrix}. \tag{3}$$

Here, the subscript k means the number of iterations. (1) is similar to the laws proposed by Kavli [15] and Roh *et al.* [16] and generates a modified desired trajectory by learning. In the formulation of the ILC problem, the following assumptions are made:

Assumption 1: The desired trajectory $y_d(t)$ is bounded within the tracking interval, i.e., $Y_d(s) \in \mathcal{RH}_2$.

Assumption 2: Without loss of generality, the zero-input response $Y^0(s)$ of the plant $G(s)$ is invariant with respect to iteration and equal to zero, i.e., $Y^0(s) = Y_k^0(s) = 0$ for all $k \in \mathbb{N}$ where $Y_k^0(s)$ is the zero-input response of the plant at the k th iteration.

The convergence condition in the $\mathcal{L}_2$-norm sense is summarized as the following theorem.

Theorem 1: For the ILC system shown in Fig. 2, the proposed learning control algorithm (1) converges uniformly to

$$v_\infty(t) = \lim_{k\to\infty} v_k(t) = \mathcal{L}^{-1}\left\{\left(I - W_pS\right)^{-1}W_pSY_d\right\} \tag{4}$$

as $k \to \infty$, in the $\mathcal{L}_2$-norm sense if and only if

$$\| W_p(s)S(s) \|_\infty < 1, \tag{5}$$

where $S(s) = (I + G(s)C(s))^{-1}$ is the sensitivity matrix of the feedback control system revealed in Fig. 1. As the iteration tends to infinity, the remaining error is given by

$$
\begin{aligned}
e_\infty(t) &= \lim_{k\to\infty} e_k(t) \\
&= \mathcal{L}^{-1}\left\{(I - T(I - W_p S)^{-1})Y_d\right\} \\
&= \mathcal{L}^{-1}\left\{(I - W_p)S(I - W_p S)^{-1}Y_d\right\},
\end{aligned} \tag{6}
$$

where $T(s) = \left(I + G(s)C(s)\right)^{-1}G(s)C(s)$ is the complementary sensitivity matrix of the feedback control system shown in Fig. 1.

Proof: ($\Rightarrow$) In Fig. 2, the tracking output at the kth iteration is

$$Y_k = \left(I + GC\right)^{-1}GC\left(V_k + Y_d\right) = T\left(V_k + Y_d\right). \tag{7}$$

Using (1), (2), (7), and the relationship between S and T, i.e., $S + T = I$, the updated learning input at the $(k+1)$th iteration is given by

$$V_{k+1} = W_p S V_k + W_p S Y_d. \tag{8}$$

Similarly,

$$V_k = W_p S V_{k-1} + W_p S Y_d. \tag{9}$$

From (8) and (9), we get

$$V_{k+1} - V_k = W_p S\left(V_k - V_{k-1}\right). \tag{10}$$

By Parseval's theorem, (10) becomes

$$
\begin{aligned}
\| v_{k+1}(t) - v_k(t) \|_2 &= \| V_{k+1} - V_k \|_2 \\
&\le \| W_p S \|_\infty \| V_k - V_{k-1} \|_2 .
\end{aligned} \tag{11}
$$

Consequently, it is clear that if (5) is satisfied, the updated learning input $v_k(t)$ converges to $v_\infty(t) = \mathcal{L}^{-1}\{V_\infty(s)\}$ in the $\mathcal{L}_2$-norm sense as k approaches infinity. The fixed value $V_\infty(s)$ in (4) is obtained from (8) by substituting $V_k(s)$ and $V_{k+1}(s)$ with $V_\infty(s)$. With the help of (1) and (4), the remaining error $E_\infty(s)$ is also obtained as (6).

($\Leftarrow$) The necessity can be proved using contradiction. Let ω_0 be such that

$$\| W_p S \|_\infty = \bar{\sigma}\left\{W_p(j\omega_0)S(j\omega_0)\right\} \ge 1. \tag{12}$$

Denote the singular value decomposition of $W_p(j\omega_0)S(j\omega_0)$ as

$$
\begin{aligned}
& W_p(j\omega_0)S(j\omega_0) \\
&= \bar{\sigma}\left\{W_p(j\omega_0)S(j\omega_0)\right\} g_1(j\omega_0)h_1^*(j\omega_0) \\
&\quad + \sum_{i=2}^{r} \sigma_i\left\{W_p(j\omega_0)S(j\omega_0)\right\} g_i(j\omega_0)h_i^*(j\omega_0),
\end{aligned}
$$

where r is the rank of $W_p(j\omega_0)S(j\omega_0)$ and g_i, h_i have unit length. Suppose the $g_1(j\omega_0)$ is written as

$$g_1(j\omega_0) = \left[\alpha_1 e^{j\theta_1}\ \alpha_2 e^{j\theta_2} \cdots \alpha_p e^{j\theta_p}\right]^T, \tag{13}$$

where $\alpha_i \in \mathbb{R}$ and $\theta_i \in (-\pi, 0]$. Let $0 \le \beta_i \le \infty$ be such that

$$\theta_i = \angle\left(\frac{\beta_i - j\omega_0}{\beta_i + j\omega_0}\right) \tag{14}$$

with $\beta_i = \infty$ if $\theta_i = 0$ and let $V_2(s)$ be given by

$$V_2(s) = \begin{bmatrix} \alpha_1 \dfrac{\beta_1 - s}{\beta_1 + s} \\ \alpha_2 \dfrac{\beta_2 - s}{\beta_2 + s} \\ \vdots \\ \alpha_p \dfrac{\beta_p - s}{\beta_p + s} \end{bmatrix} \hat{V}_2(s) \tag{15}$$

with 1 replacing $\frac{\beta_i - s}{\beta_i + s}$ if $\theta_i = 0$. A scalar function $\hat{V}_2(s)$ is chosen so that

$$|\hat{V}_2(j\omega)| = \begin{cases} c & \text{if } |\omega - \omega_0| < \varepsilon \text{ or } |\omega + \omega_0| < \varepsilon \\ 0 & \text{otherwise,} \end{cases} \tag{16}$$

where $0 < \varepsilon \ll 1$ and $c = c_0\sqrt{\pi/2\varepsilon}$. Then

$$
\begin{aligned}
\| V_2(s) \|_2^2 &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{Trace}[V_2^*(j\omega)V_2(j\omega)]d\omega \\
&= c_0^2 .
\end{aligned} \tag{17}
$$

In case of $\| V_3(s) - V_2(s) \|_2$,

$$
\begin{aligned}
& \| V_3(s) - V_2(s) \|_2^2 \\
&= \| W_p(s)S(s)(V_2(s) - V_1(s)) \|_2^2 \\
&= \frac{c_0^2}{2\pi}[\bar{\sigma}\left\{W_p(-j\omega_0)S(-j\omega_0)\right\}^2 \pi \\
&\quad + \bar{\sigma}\left\{W_p(j\omega_0)S(j\omega_0)\right\}^2 \pi] \\
&= \bar{\sigma}\left\{W_p(j\omega_0)S(j\omega_0)\right\}^2 \cdot c_0^2 \\
&= \| W_p(s)S(s) \|_\infty^2 \cdot c_0^2 \\
&\ge c_0^2 = \| V_2(s) - V_1(s) \|_2^2 ,
\end{aligned} \tag{18}
$$

because $\| W_p S \|_\infty$ is larger than 1 and $V_1(s) = 0$. Similarly, it can be shown that

$$\| V_{k+1}(s) - V_k(s) \|_2 \geq \| V_k(s) - V_{k-1}(s) \|_2 \quad (19)$$

for all $k \in \mathbb{N}$. Hence, the proof is completed. □

Remark 1: The convergence condition (5) is well known as the nominal performance condition in the feedback control theory [13]. In other words, the weighting matrix $W_p(s)$ to specify the performance of the feedback system plays a role of iterative learning controller and also determines the convergence of the ILC system.

Because the convergence condition shown in Theorem 1 includes the plant uncertainty, it is not proper to apply this condition to real plants. To examine the convergence of the ILC system with plant uncertainty, therefore, a modified convergence condition is required, which is expressed with only the known parts of the system.

Now, let the plant $G(s)$ be an element of the set of uncertain plants

$$\mathcal{G} = \{\mathcal{F}_u(G_o(s), \Delta(s)) : \Delta(s) \in \mathcal{M}(\underline{\Delta})\}, \quad (20)$$

where $G_o(s)$ is a plant free from the uncertainty, an $(n+p)\times(n+q)$ transfer matrix, and partitioned as

$$G_o(s) = \begin{bmatrix} G_{o_{11}}(s) & G_{o_{12}}(s) \\ G_{o_{21}}(s) & G_{o_{22}}(s) \end{bmatrix}, \quad (21)$$

$\Delta(s)$ is a model uncertainty, and $\mathcal{M}(\underline{\Delta})$ and $\underline{\Delta}$ are defined as

$$\mathcal{M}(\underline{\Delta}) := \{\Delta(\cdot) \in \mathcal{RH}_\infty : \Delta(s_0) \in \underline{\Delta} \text{ for all } s_0 \in \overline{\mathbb{C}}_+\}, \quad (22)$$

$$\underline{\Delta} = \{\text{diag}\left(\delta_1 I_{r_1}, \cdots, \delta_S I_{r_S}, \Delta_1, \cdots, \Delta_F\right) : \delta_i \in \mathbb{C}, \Delta_j \in \mathbb{C}^{m_j \times m_j}\} \quad (23)$$

with $\sum_{i=1}^{S} r_i + \sum_{j=1}^{F} m_j = n$ [13], respectively. For $M \in \mathbb{C}^{n\times n}$, the structured singular value $\mu_{\underline{\Delta}}(M)$ is defined as

$$\mu_{\underline{\Delta}}(M) = \frac{1}{\min\left\{\overline{\sigma}(\Delta) : \Delta \in \underline{\Delta}, \det(I - M\Delta) = 0\right\}}$$

unless no $\Delta \in \underline{\Delta}$ makes $I - M\Delta$ singular, in which case $\mu_{\underline{\Delta}}(M) := 0$ [12,13].

The following lemma, which is widely known as the robust performance test in the feedback control theory, will be used to derive the modified convergence condition.

Lemma 1 [13]: Let $\beta > 0$. Suppose that $P(s)$ is a stable, real-rational, proper transfer matrix with $n+q$ inputs and $n+p$ outputs and is partitioned as

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix}. \quad (24)$$

For all $\Delta(s) \in \mathcal{M}(\underline{\Delta})$ with $\|\Delta\|_\infty < 1/\beta$, $\mathcal{F}_l(P, \Delta)$ is well-posed, internally stable, and $\|\mathcal{F}_l(P, \Delta)\|_\infty \leq \beta$ if and only if

$$\sup_{\omega\in\mathbb{R}} \mu_{\underline{\Delta}_P}(P(j\omega)) \leq \beta \quad (25)$$

where $\underline{\Delta}_P := \left\{\text{diag}(\Delta, \Delta_f) : \Delta_f \in \mathbb{C}^{q\times p}\right\}$.

The uncertainty-free convergence condition is summarized as Theorem 2.

Theorem 2: Let $0 < \rho < 1$. For the ILC system shown in Fig. 2, the proposed learning control algorithm (1) converges uniformly to (6) as $k \to \infty$, in the $\mathcal{L}_2$-norm sense for all $\Delta(s) \in \mathcal{M}(\underline{\Delta})$ with $\|\Delta\|_\infty < 1/\rho$ if and only if

$$\sup_{\omega\in\mathbb{R}} \mu_{\underline{\Delta}_P}\{\mathcal{F}_l(M(j\omega), C(j\omega))\} \leq \rho < 1 \quad (26)$$

where

$$M = \begin{bmatrix} G_{o_{11}} & 0 & G_{o_{12}} \\ -W_p G_{o_{21}} & W_p & -W_p G_{o_{22}} \\ -G_{o_{21}} & I & -G_{o_{22}} \end{bmatrix}. \quad (27)$$

As the iteration tends to infinity, the remaining error converges to (6).

Proof: ($\Rightarrow$) Using the similar approach in the proof of Theorem 1, the relationship between consecutive updated learning inputs is given by

$$V_{k+1} - V_k = W_p(I + \mathcal{F}_u(G_o, \Delta)C)^{-1}(V_k - V_{k-1}), \quad (28)$$

which leads to

$$\|V_{k+1} - V_k\|_2 \leq \|W_p(I + \mathcal{F}_u(G_o, \Delta)C)^{-1}\|_\infty \|V_k - V_{k-1}\|_2 . \quad (29)$$

Under Assumptions 1 and 2, if

$$\|W_p\left(I + \mathcal{F}_u(G_o, \Delta)C\right)^{-1}\|_\infty < 1 \quad (30)$$

is satisfied, the updated learning input converges to $v_\infty(t)$ given in (4) in the $\mathcal{L}_2$-norm sense as $k \to \infty$.

$W_p\left(I + \mathcal{F}_u(G_o, \Delta)C\right)^{-1}$ is equal to the transfer matrix from w_2 to z_2 in the block diagram shown in Fig. 3 and can be rearranged as an LFT with $z_2 = \mathcal{F}_u(\mathcal{F}_l(M, C), \Delta)w_2$. Hence, the condition (30) can be represented as $\|\mathcal{F}_u(\mathcal{F}_l(M, C), \Delta)\|_\infty < 1$. Finally, according to Lemma 1, (30) is ensured if (26)

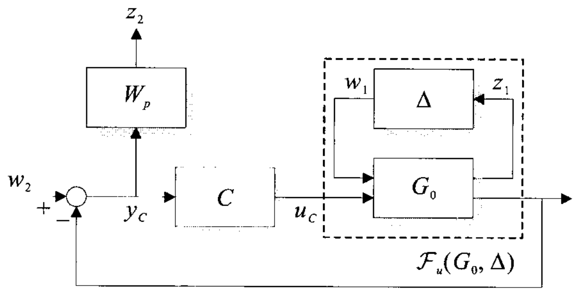


Fig. 3. Block Diagram of $W_p\left(I+\mathcal{F}_u(G_o,\Delta)C\right)^{-1}$.

is met for all $\Delta(s)\in\mathcal{M}(\underline{\Delta})$ with $\|\Delta\|_\infty<1/\rho$.

($\Leftarrow$) Let the condition (26) not be satisfied, that is, let ω_0 be such that

$$\mu_{\underline{\Delta}_P}\{\mathcal{F}_l(M(j\omega_0),C(j\omega_0))\}<1 \tag{31}$$

for all $\Delta(s)\in\mathcal{M}(\underline{\Delta})$ with $\|\Delta\|_\infty<1/\rho$. Then, by Lemma 1, $\|\mathcal{F}_u(\mathcal{F}_l(M,C),\Delta)\|_\infty=\|W_p(I+\mathcal{F}_u(G_o,\Delta)C)^{-1}\|_\infty\geq 1$. From this result, as shown in the proof of Theorem 1, it is easily obtained that $\|V_{k+1}(s)-V_k(s)\|_2\geq\|V_k(s)-V_{k-1}(s)\|_2$ for all $k\in\mathbb{N}$. Therefore, (26) is the necessary condition for the convergence in the $\mathcal{L}_2$-norm sense.

Remark 2: The convergence condition obtained in Theorem 2 is equal to that of robust performance by structured singular value. Therefore, if the feedback control system satisfies the robust performance condition, the convergence of the feedback-based ILC system is automatically ensured, and vice versa. Accordingly, the ILC design problem is equivalent to finding a feedback controller $C(s)$ satisfying (26).

Remark 3: Using the so-called *D-K iteration*, we can obtain a controller $C(s)$ which not only stabilizes the overall feedback system but also satisfies the condition (26) [13]. The upper bound on the model uncertainty can be easily found using several methods in the books related with robust control [13,17].

Remark 4: Let the plant $G(s)$ be a SISO system with the multiplicative uncertainty, i.e.,

$$G(s)=(1+\Delta(s)W_2(s))G_n(s), \tag{32}$$

where $G_n(s)$ is the nominal plant, $W_2(s)$ is a known stable weighting function, and $\Delta(s)$ is an unknown stable transfer function satisfying $\|\Delta\|_\infty<1/\rho$. Then, as in [9] and [10], (26) boils down to the robust performance condition

$$\left\|\,|W_pS_n|+|W_2T_n|\,\right\|_\infty\leq\rho, \tag{33}$$

where $S_n=1/(1+CG_n)$ and $T_n=CG_n/(1+CG_n)$.

As Theorem 1, the tracking error converges uniformly to the remaining error given in (6) in the $\mathcal{L}_2$-norm sense as the iteration tends to infinity. However, no one can guarantee that the remaining error is less than the initial tracking error. Therefore, a condition is needed that the tracking error converges to a less value than the initial one, which is given in the following theorem.

Theorem 3: For the ILC system shown in Fig. 2, if there exists a $W_p(s)$ such that $\|I-W_p\|_\infty<1$ and if

$$\|W_p^*S\|_\infty<1, \tag{34}$$

where $W_p^*=W_p/(1-\|I-W_p\|_\infty)$, then

i) the tracking error is bounded for all $k\in\mathbb{N}$ and converges uniformly to the remaining error (6) as $k\to\infty$ in the $\mathcal{L}_2$-norm sense;
ii) the least upper bound of the $\mathcal{L}_2$-norm of the remaining error is less than the least upper bound of the $\mathcal{L}_2$-norm of the initial tracking error, i.e., $\|e_\infty\|_2\leq\alpha_1$, $\|e_1\|_2\leq\alpha_2$, with $\alpha_1<\alpha_2$;
iii) the infinity norm of the remaining error is less than that of the initial tracking error, i.e., $\|E_\infty\|_\infty<\|E_1\|_\infty$.

Proof:

i) If $W_p=I$, (34) is equal to condition (5) and the proof of convergence of the tracking error is the same as that of Theorem 1. Consider $W_p\neq I$. Since $\|I-W_p\|_\infty<1$, (34) becomes

$$\|W_pS\|_\infty<1-\|I-W_p\|_\infty, \tag{35}$$

which implies that the convergence condition (5) is satisfied and then the tracking error is bounded for all iterations. By Theorem 1, the remaining error can be also given by (6).

ii) The initial tracking error is obtained by setting $V_1(s)=0$. Let α_1 and α_2 be the least upper bounds of the remaining error and the initial tracking error, respectively:

$$\|e_\infty\|_2=\|E_\infty\|_2 \leq\|(I-W_p)S(I-W_p)^{-1}\|_\infty\|y_d\|_2=\alpha_1, \tag{36}$$

$$\|e_1\|_2=\|E_1\|_2\leq\|S\|_\infty\|y_d\|_2=\alpha_2. \tag{37}$$

From the condition (35), we get the following inequality:

$$|I-W_p|<1-|W_pS|,\ \forall\omega. \tag{38}$$

We have

$$1 = |I + W_pS - W_pS| \le |W_pS| + |I - W_pS|, \ \forall\omega \quad (39)$$

and therefore

$$1 - |W_pS| \le |I - W_pS|, \ \forall\omega. \quad (40)$$

Finally, from (38) and (40), the following inequality

$$|I - W_p| < |I - W_pS|, \ \forall\omega \quad (41)$$

is obtained, which implies that

$$\|(I - W_p)S(I - W_pS)^{-1}\|_\infty < \|S\|_\infty . \quad (42)$$

Therefore, $\alpha_1 < \alpha_2$ is established.

iii) (41) also yields that

$$\|(I - W_p)S(I - W_pS)^{-1}Y_d\|_\infty < \|SY_d\|_\infty , \quad (43)$$

which means that $\|E_\infty\|_\infty < \|E_1\|_\infty$. □

Remark 5: Theorem 3 shows the boundedness of the tracking error. Under some conditions, the iterative learning controller diminishes the tracking error below, which only the feedback controller generates. In other words, by adding an iterative learning controller with a simple structure, we can reduce the tracking error more effectively than when only the feedback controller is used. If the plant with multiplicative uncertainty, i.e., $G = (1 + \Delta W_2)G_n$ is considered, the conditions for the boundedness of the tracking error are modified to be more or less conservative as shown in Theorem 2 of [10].

3. SIMULATION RESULTS

Consider a two-mass/spring/damper system as given in Fig. 4 [13]. The dynamical system can be described by the following differential equations:

$$\dot{x}(t) = Ax(t) + Bu(t),$$
$$y(t) = Cx(t)$$

with $x(t) = [x_1(t)\ x_2(t)\ x_3(t)\ x_4(t)]^T$,

$$u(t) = \left[u_1(t)\ u_2(t)\right]^T, \ y(t) = \left[y_1(t)\ y_2(t)\right]^T,$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_1}{m_1} & \frac{k_1}{m_1} & -\frac{b_1}{m_1} & \frac{b_1}{m_1} \\ \frac{k_1}{m_2} & -\frac{k_1+k_2}{m_2} & \frac{b_1}{m_2} & -\frac{b_1+b_2}{m_2} \end{bmatrix},$$

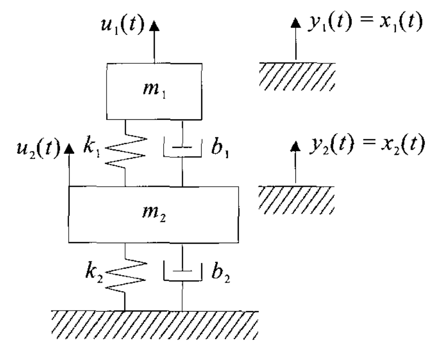


Fig. 4. A two-mass/spring/damper system.

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{m_1} & 0 \\ 0 & \frac{1}{m_2} \end{bmatrix}, \ C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

Suppose that the transfer matrix from (u_1, u_2) to (x_1, x_2) is the nominal plant $G_n(s)$ and suppose that parameters $k_1 = 1$, $k_2 = 4$, $b_1 = 0.2$, $b_2 = 0.1$, $m_1 = 1$, and $m_2 = 2$ with appropriate units. The plant $G(s)$ is described in the following multiplicative uncertain form:

$$G(s) = G_n(s)(I + \Delta(s)W_u(s)), \quad (44)$$

where $W_u(s)$ is given as

$$W_u(s) = \begin{bmatrix} \frac{s+5}{s+50} & 0 \\ 0 & \frac{s+5}{s+50} \end{bmatrix}. \quad (45)$$

A method to find $G_o(s)$ from $G(s)$ is introduced in [5]. To improve the tracking performance of $y_1(t) = x_1(t)$ and $y_2(t) = x_2(t)$ in a frequency range $0 \le \omega \le 2$, the performance weighting matrix $W_p(s)$ is given as

$$W_p(s) = \begin{bmatrix} \frac{1}{s/2+1} & 0 \\ 0 & \frac{1}{s/2+1} \end{bmatrix}. \quad (46)$$

Using the μ-Analysis and Synthesis Toolbox of Matlab [18], the controller $C(s) = \begin{bmatrix} C_{11}(s) & C_{12}(s) \\ C_{21}(s) & C_{22}(s) \end{bmatrix}$ satisfying the robust performance condition that can be obtained where

$$C_{11}(s)=\frac{24358.02(s+50)(s+5.51)(s^2+1.62s+1.03)(s^2+3.69s+10.54)}{(s+3.05e4)(s+5.47)(s+2)(s^2+3.58s+10.37)(s^2+9.27s+28.71)}$$

$$C_{12}(s)=\frac{8385.68(s+50)(s+4.94)(s+2.80)(s-0.80)(s^2+2.37s+8.57)}{(s+3.05e4)(s+5.47)(s+2)(s^2+3.58s+10.37)(s^2+9.27s+28.71)}$$

$$C_{21}(s)=\frac{9177.31(s+50)(s+4.99)(s+2.31)(s-0.25)(s^2+2.10s+6.85)}{(s+3.05e4)(s+5.47)(s+2)(s^2+3.58s+10.37)(s^2+9.27s+28.71)}$$

$$C_{22}(s)=\frac{3161.02(s+50)(s+13.45)(s^2+0.40s+1.56)(s^2+8.98s+32.65)}{(s+3.05e4)(s+5.47)(s+2)(s^2+3.58s+10.37)(s^2+9.27s+28.71)}.$$

The largest structured singular value with this controller is

$$\sup_{\omega\in\mathbb{R}} \mu_{\Delta_P}\{\mathcal{F}_l(M(j\omega),C(j\omega))\}=0.9013$$

as shown in Fig. 5. In simulations, let the desired trajectory $y_d(t)=\left[\hat{y}_d(t)\;\hat{y}_d(t)\right]^T, 0\le t\le 15$ be the following signal shown in Fig. 6:

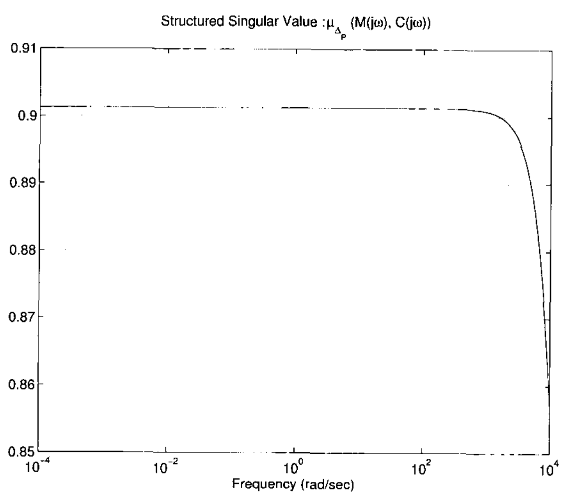


Fig. 5. $\mu_{\Delta_P}\{\mathcal{F}_l(M(j\omega),C(j\omega))\}$ versus frequency.

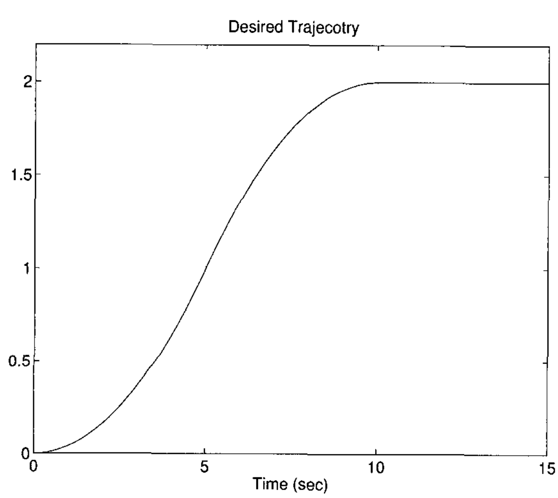


Fig. 6. Desired trajectory.

$$\hat{y}_d(t)=\begin{cases} t^2/25, & 0\le t<5 \\ -(t-10)^2/25+2, & 5\le t<10 \\ 2, & 10\le t\le 15. \end{cases} \quad (47)$$

We performed a simulation using the obtained controller and the proposed iterative updating rule. The learning control input was initialized as $v_1(t)=0$, thus the tracking error at the first iteration was only the result of the feedback control. Fig. 7 indicates the desired trajectory, tracking outputs, and tracking errors $e_1(t)=\hat{y}_d(t)-y_1(t)$, $e_2(t)=\hat{y}_d(t)-y_2(t)$, at $k=1$, $k=2$, $k=10$, and $k=20$. Fig. 8 presents the root mean square (rms) values of the tracking errors versus the number of iterations, where the performance evolution by the added iterative learning controller is clearly seen. More quantitatively, the rms values of $e_1(t)$ and $e_2(t)$ decrease steadily to approach around 0.037 and 0.31, 5.2 % and 26.5 % of the initial rms values, 0.70 and 1.17,

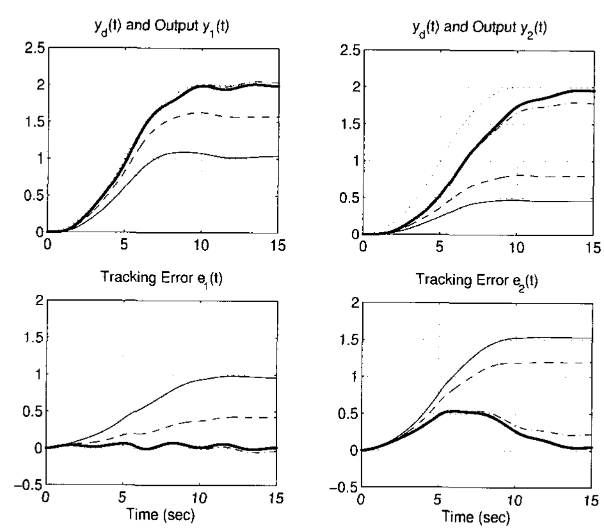


Fig. 7. Desired trajectory (dotted line), tracking outputs, and tracking errors at $k=1$ (solid line), $k=2$ (dashed line), $k=10$ (dashed and dotted line), and $k=20$ (bold line).

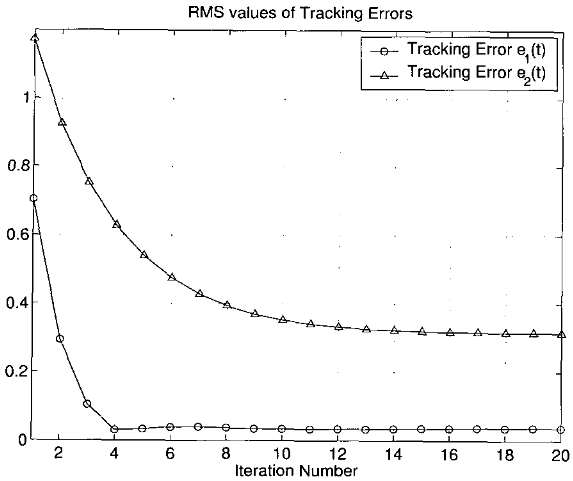


Fig. 8. Root mean square values of tracking errors versus the number of iterations.

respectively, which verifies the benefit of iterative learning control.

Remark 6: By iteratively performing *D-K iteration* or selecting a new performance weighting matrix $W_p(s)$, we can obtain a lower ρ in the condition (26) and a controller with higher gain. Then, it is possible to improve the initial tracking performance when only the feedback controller is applied.

4. CONCLUDING REMARKS

In this paper, the convergence for a MIMO feedback-based ILC was considered. It was shown that convergence in the $\mathcal{L}_2$-norm sense is closely related with the nominal performance condition and the robust performance condition in the feedback control theory. Therefore, the design problem of the iterative learning controller for an uncertain MIMO system is equivalent to designing a feedback controller $C(s)$ or selecting a performance weighting function $W_p(s)$ to satisfy the robust performance condition. Moreover, it was shown that the least upper bound of $\mathcal{L}_2$-norm of the remaining tracking error is less than that of the initial error. From the result, it was verified that the iterative learning controller is useful in reducing the tracking error. Simulation on a 2-mass/spring/damper system was performed and its results were presented to validate the effectiveness of the proposed method.

REFERENCES

[1] S. Arimoto, S. Kawamura, S, and F. Miyazaki, "Betterment operation of robots by learning," *J. Robotic Systems*, vol. 1, no. 2, pp. 123-400, 1984.

[2] K. L. Moore, "Iterative learning control - an expository overview," *Applied and Computational Controls, Signal Processing and Circuits*, vol. 1, no. 1, pp. 425-488, 1998.

[3] T. Y. Kuc, J. S. Lee, and K. Nam, "An iterative learning control theory for a class of nonlinear dynamic systems," *Automatica*, vol. 28, no. 6, pp. 1215-1221, 1992.

[4] J.-H. Moon, T.-Y. Doh, and M. J. Chung, "A robust approach to iterative learning control design for uncertain systems," *Automatica*, vol. 34, no. 8, pp. 1001-1004, 1998.

[5] T.-Y. Doh, J.-H. Moon, K. B. Jin, and M. J. Chung, "Robust ILC with current feedback for uncertain linear systems," *Int. J. Systems Science*, vol. 30, no. 1, pp. 39-47, 1999.

[6] D. de Roover, O. H. Bosgra, and M. Steinbuch, "Internal-model-based design of repetitive and iterative learning controllers for linear multivariable systems," *Int. J. Control*, vol. 73, no. 10, pp. 914-929, 2000.

[7] M. Norrlöf and S. Gunnarsson, "Disturbance aspects of iterative learning control," *Engineering Application of Artificial Intelligence*, vol. 14, pp. 87-94, 2001.

[8] P. B. Goldsmith, "On the equivalence of causal LTI iterative learning control and feedback control," *Automatica*, vol. 38, pp. 703-708, 2002.

[9] Q. Hu, J.-X. Xu, and T. H. Lee, "Iterative learning control design for Smith predictor," *Systems and Control Letters*, vol. 44, pp. 201-210, 2001.

[10] A. Tayebi and M. B. Zaremba, "Robust iterative learning control design is straightforward for uncertain LTI systems satisfying the robust performance condition," *IEEE Trans. on Automatic Control*, vol. 48, no. 1, pp. 101-106, 2003.

[11] T.-Y. Doh, "Comments on "Robust iterative learning control design is straightforward for uncertain LTI systems satisfying the robust performance condition"," *IEEE Trans. on Automatic Control*, vol. 49, no. 4, pp. 629-630, 2004.

[12] J. C. Doyle, "Analysis of feedback systems with structured uncertainties," *IEE Proceedings, Part D*, vol. 133, pp. 45-56, 1982.

[13] K. Zhou and J. C. Doyle, *Essentials of Robust Control*, Prentice-Hall, Inc., 1998.

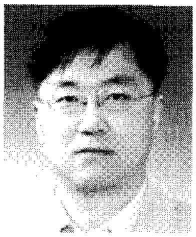
[14] J. C. Doyle, A. Packard, and K. Zhou, "Review of LFTs, LMIs and μ," *Proc. of the 30th IEEE Conf. Decision and Control*, pp. 1227-1232, 1991.

[15] T. Kavli, "Frequency domain synthesis of trajectory learning controller for robot manipulator," *J. Robotic Systems*, vol. 9, no. 5, pp. 663-680, 1992.

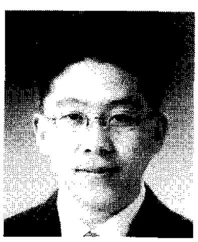
[16] C. L. Roh, M. N. Lee, and M. J. Chung, "ILC for nonminimum phase system," *Int. J. Systems Science*, vol. 27, no. 4, pp. 429-424, 1996.

[17] J. C. Doyle, B. A. Francis, and A. R. Tannenbaum, *Feedback Control Theory*, Maxwell Mcmillan, 1992.

[18] G. J. Balas, J. C. Doyle, K. Glover, A. Packard, and R. Smith, μ *-Analysis and Synthesis Toolbox for Use with Matlab*, Mathworks, 1998.



Tae-Yong Doh received the B.S. degree in Electronics Engineering from Kyungpook National University in 1992, and the M.S. and Ph.D. degrees in Electrical Engineering from the Korea Advanced Institute of Science and Technology in 1994 and 1999, respectively. In 2002, he joined the faculty of the Department of Control and Instrumentation Engineering, Hanbat National University, Korea, where he is currently an Associate Professor. From 1997 to 2001, he worked for Samsung Electronics Co., Ltd as a Senior Research Engineer. His research interests include iterative learning control, repetitive control, robust control, embedded control systems, and intelligent robotics.



Jung Rae Ryoo received the B.S., M.S., and Ph.D. degrees in Electrical Engineering from the Korea Advanced Institute of Science and Technology in 1996, 1998, and 2004, respectively. In 2005, he joined the faculty of the Department of Control and Instrumentation Engineering, Seoul National University of Technology, Korea, where he is an Assistant Professor. From 2004 to 2005, he worked for Samsung Electronics Co., Ltd as a Senior Engineer. His research interests include robust motion control, optical disk drive servo control, and DSP-based digital control systems.