

# On Color Cluster Analysis with Three-dimensional Fuzzy Color Ball

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## Abstract

The focus of this paper is on devising an efficient clustering task for arbitrary color data. In order to tackle this problem, the inherent uncertainty and vagueness of color are represented by a fuzzy color model. By taking a fuzzy approach to color representation, the proposed model makes a soft decision for the vague regions between neighboring colors. A definition on a three-dimensional fuzzy color ball is introduced, and the degree of membership of color is computed by employing a distance measure between a fuzzy color and color data. With the fuzzy color model, a novel fuzzy clustering algorithm for efficient partition of color data is developed.

**Key words** : Fuzzy color, fuzzy clustering, color space, color clustering

## 1. Introduction

Color is one of the most important features in our lives. Even though color can be considered as a simple and intuitive object, it is not an easy task to effectively describe color in our language [1][2][3]. This paper focuses on the color clustering problem. For a given set of color data and number of clusters, the color set is partitioned into homogeneous color sub-partitions. This kind of color clustering task can be widely used in a variety of applications, such as color image segmentation. The difficulties of this research include the lack of a correct color model that can describe the uncertain characteristics of color and the lack of an efficient clustering algorithm that can deal with vague color data.

All the studies related to color modeling, including RGB(Red, Green, and Blue), HSV(Hue, Saturation, and Value), and CIELAB(Commission Internationale del'Eclairage LAB) color spaces, have only handled the crisp representation of color data. However, color has inherently uncertain and vague characteristics. For overlapping areas between two major colors, absolute color classification is not possible because it depends on both visual decisions by the observer and the surrounding color effects. This phenomena is called "simultaneous contrast" [2][4]. Even though Vertan recently proposed a fuzzy approach to color modeling, his model has limitations [5]. First, there is no consideration of the relative strengths between colors (simultaneous contrast). Thus, the membership degree of a given color to a certain color has always the same value regardless of the given color distribution. Second, the sizes

of areas of all colors in a color space were fixed as the same value, which does not correspond to human color perception. According to human perception, for example, a green color has a larger area than red color [4]. Consequently we have tried to develop a new color model that can represent the uncertainty and vagueness of color. The proposed fuzzy color model is represented by a three-dimensional fuzzy color ball on CIELAB color coordinates. We have also proposed color distance measures that compute the distance between color elements, and the distance between a color element and a fuzzy color. With the distance measures, a computation method for color membership values was defined.

For effective partitioning of the color data set, fuzzy cluster analysis techniques are less prone to local minima than crisp clustering algorithms, and allow us to manage uncertainty on measures [6]. As mentioned before, color measurements are usually affected by uncertainty and subjectivity, and can be interpreted as degrees of membership to major colors. However, most of the previous fuzzy clustering algorithms were designed for crisp pattern data; thus we need to develop a new algorithm that can handle the fuzzy color data. This explains why a novel fuzzy clustering algorithm is required in order to deal with the vague mapping of color data to clusters, and compute degrees of membership that specify to what extent color data belong to fuzzy clusters. Hence we need to develop a fuzzy color model that can explain the uncertainty of color and can compute the relative difference between colors. With the proposed color model, we developed an effective fuzzy clustering algorithm for color data. By minimizing the

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evaluation function, we could obtain near-optimal convergence in an iterative color clustering.

## 2. Three-dimensional Fuzzy Color Ball

We should establish an approach to model three components, pattern representation, a proximity measure, and a clustering algorithm, to solve the color clustering problem. We create and use a new fuzzy color model as the pattern representation of color data, and define a color distance measure to solve the inter-color pattern proximity. Finally, we apply the fuzzy color model to the development of a new fuzzy clustering algorithm, where as cluster centroid is represented in the form of fuzzy color.

### 2.1 Description of Fuzzy Color Ball

The proposed fuzzy color is described with a three-dimensional spherical representation. Color, in general, has its own volumetric representation. The magnitude of color volume is determined on the basis of its own perception boundary, which is known as a JND (Just Noticeable Difference) [3][4]. This value is computed by changing the wavelength of color until there is a discernable difference. For the volumetric representation, we created a ball model due to its simplicity and ease of manipulation. When we look at a color or colored object, such as a red object, some pairs of red colors are difficult to distinguish. However, beyond a certain boundary, we can easily distinguish color pairs. The radius of the color ball takes the JND of each color. For a pair of two colors within the JND value, we assumed they are not easily distinguishable and we consider those two colors to be equal. Colors that do not belong to the same JND volume can be distinguished.

To describe the fuzzy color ball, two numerical values should be specified: the center and JND value. The center value is the center point of the three-dimensional fuzzy color ball. The JND is the distance from the center to a volume boundary of a given color ball. Here we must distinguish two objects: the color element and the fuzzy color. Color element, denoted by  $x$ , is a point on CIELAB color coordinates, which is represented by  $(x_L, x_a, x_b)$  tuple value. Fuzzy color, denoted by  $\tilde{c}_i \in \tilde{C}$ , is a ball with a three dimensional volume representation on CIELAB space. Taking CIELAB as basic color coordinates, the proposed model has uniform color scaling, and provides a human perception-based color distance metric [7].

For a given color element  $x$ , the membership  $\mu_{\tilde{c}_i}(x)$  of  $x$  to fuzzy color  $\tilde{c}_i$  is obtained by the distance computation. Suppose that there are fuzzy color  $\tilde{c}_i$  and color element  $x$ . The fuzzy color has its own center value  $center_i$  and JND value  $jnd_i$ , which build three-dimensional spherical representation. That is,  $\tilde{c}_i = \langle center_i, jnd_i \rangle$ . If a color  $x$

is within JND distance, it strongly belongs to that color and has a membership degree 1.0. If color  $x$  is out of the JND range, the membership degree is computed relatively by comparison with neighbor colors. The left and right shapes of the fuzzy membership function are determined based on the distance from neighbor colors.

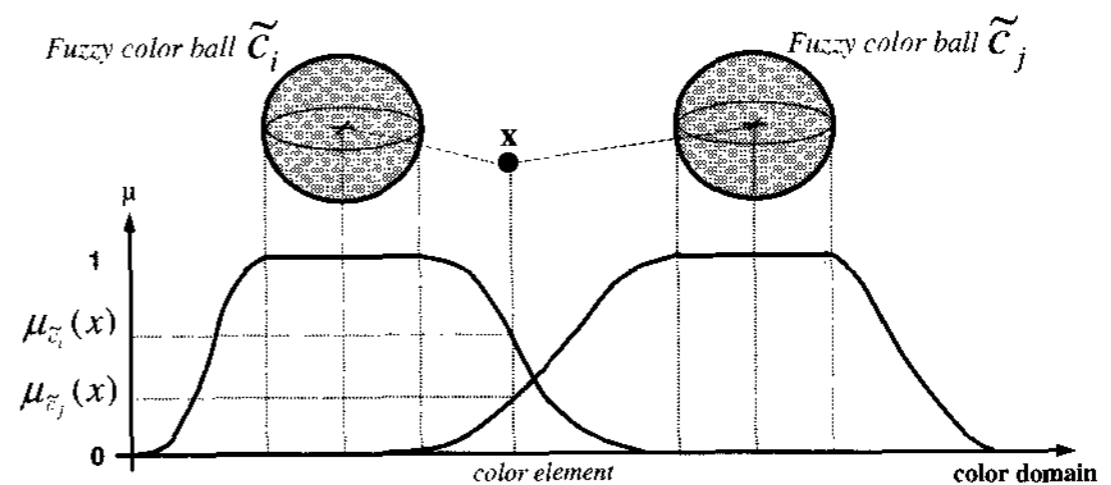


Figure 1: Membership computation between fuzzy colors

Figure 1 depicts the membership computation situation between two fuzzy colors. If a color element  $x$  strongly belongs to a given color  $\tilde{c}_i$ , it is classified to that color with a membership degree of 1.0. If the color  $x$  is within another color  $\tilde{c}_j$ , then color  $x$  has no relation to the fuzzy color  $\tilde{c}_i$ , and thus the membership degree to  $\tilde{c}_i$  is 0.0. Except for the above two cases, color  $x$  is located in the middle of fuzzy colors. Therefore, for example, the color  $x$  in Figure 1 has membership degrees  $\mu_{\tilde{c}_i}(x)$  and  $\mu_{\tilde{c}_j}(x)$ . The  $\mu_{\tilde{c}_i}(x)$  gets a higher value than  $\mu_{\tilde{c}_j}(x)$  because  $x$  is closer to fuzzy color  $\tilde{c}_i$  than  $\tilde{c}_j$ . The degree of membership  $\mu_{\tilde{c}_i}(x)$  can be determined by both distribution of neighbor fuzzy colors and the relative distances from  $x$  to each fuzzy color.

#### 2.1.1 Distance Measures Between Colors

We defined inter-color distances: distance between color elements and distance between a fuzzy color and a color element. Let  $x$  and  $y$  be two color elements on CIELAB color space, then the distance between  $x$  and  $y$ , denoted by  $\rho(x, y)$ , is defined as

$$\rho(x, y) = \sqrt{(x_L - y_L)^2 + (x_a - y_a)^2 + (x_b - y_b)^2} \quad (1)$$

where  $x = (x_L, x_a, x_b)$  and  $y = (y_L, y_a, y_b)$ . The above definition is easily obtained from CIELAB color difference formula.

One of the major concerns is to compute the distance between a fuzzy color and an arbitrary color element. With the above distance measure between color elements, we can define the distance between a color element and a fuzzy color. Let  $x$  and  $\tilde{c}_i$  be a color element and a fuzzy color on CIELAB color space respectively. Then the distance between  $x$  and  $\tilde{c}_i$ , denoted by  $\delta(x, \tilde{c}_i)$ , is defined as

$$\delta(x, \tilde{c}_i) = \|\rho(x, center_i)\| - jnd_i \quad (2)$$

where  $x = (x_L, x_a, x_b)$ , and  $center_i$  and  $jnd_i$  are the center and JND value of fuzzy color  $\tilde{c}_i = \langle (center_L^i, center_a^i, center_b^i), jnd_i \rangle$ .

As can be seen in the above definition, the distance measure considers not only the center point but also the JND value. Some colors have larger JND values, and others have smaller ones on a color space.

### 2.1.2 Definition of the Fuzzy Color Ball

With the distance measures defined in the above section, we can establish a formal fuzzy color model. Let fuzzy color  $\tilde{c}_i$  in a universe of discourse  $\tilde{C}$  be as a three dimensional fuzzy ball set  $\tilde{c}_i = \langle center_i, jnd_i \rangle \in \tilde{C}$  with a membership function such that

$$\mu_{\tilde{c}_i}(x) = \begin{cases} 1.0 & \text{if } \delta(x, \tilde{c}_i) \leq jnd_i \\ 0.0 & \text{if } \delta(x, \tilde{c}_j) \leq jnd_j \quad (i \neq j, \tilde{c}_j \in \tilde{C}) \\ (\sum_{j=1}^{|\tilde{C}|} \frac{\delta(x, \tilde{c}_i)}{\delta(x, \tilde{c}_j)})^{-1} & \text{otherwise} \end{cases} \quad (3)$$

where  $\delta$ -function means the distance between a fuzzy color and a color element.

If a color  $x$  strongly belongs to a given fuzzy color  $\tilde{c}_i$ , it is classified to that color with a membership degree 1.0. Conversely, if the color  $x$  is within an other fuzzy color  $\tilde{c}_j$ , then it means  $x$  has no relation to the fuzzy color  $\tilde{c}_i$ , thus the membership degree is 0.0. Except for the above two cases, the color  $x$  is located between fuzzy colors like in Figure 1. We compute the relative distance from fuzzy colors and determine the membership value. The sum of membership values to the whole fuzzy color families must be equal to 1.0.

Let us compute the fuzzy color distance with numerical examples. There are three fuzzy colors including  $\tilde{c}_{red}$ ,  $\tilde{c}_{green}$ , and  $\tilde{c}_{blue}$ , and one color element  $x$ . We calculate the distance between fuzzy color  $\tilde{c}_{red}$  and a given color element  $x$ . Let's compute the membership degree of  $x$  to fuzzy color  $\tilde{c}_{red}$ . Suppose that color element  $x$  doesn't belong to any fuzzy colors at all, and the distances from each fuzzy color are given like this: 20 units to  $\tilde{c}_{red}$ , 30 units to  $\tilde{c}_{green}$ , and 10 units to  $\tilde{c}_{blue}$ . In this case, we should first compute the relative strengths between fuzzy colors, and based on the strength we compute the relative color membership values. Thus, the final membership degrees of color  $x$  to fuzzy colors are as follows:

$$\begin{aligned} \mu_{\tilde{c}_{red}}(x) &= (20/20 + 20/30 + 20/10)^{-1} = 0.27 \\ \mu_{\tilde{c}_{green}}(x) &= (30/20 + 30/30 + 30/10)^{-1} = 0.18 \\ \mu_{\tilde{c}_{blue}}(x) &= (10/20 + 10/30 + 10/10)^{-1} = 0.55 \end{aligned}$$

From this result, we can say that color element  $x$  is close to fuzzy color  $\tilde{c}_{red}$  with a membership 0.27.

## 3. Proposed Fuzzy Clustering Algorithm

The given color element data are  $x_1, \dots, x_n \in X$  where each  $x_j$  ( $j \in 1, \dots, n$ ) is a color element represented as

three-dimensional  $L^*a^*b^*$  value on CIELAB color space. The algorithm objective is to cluster a collection of given color elements into  $p$  homogeneous groups represented as fuzzy sets ( $\tilde{F}_i, i = 1, \dots, p$ ).

The collection of the fuzzy cluster sets is denoted by  $\tilde{F} = \{\tilde{F}_1, \tilde{F}_2, \dots, \tilde{F}_p\}$ . Each fuzzy cluster is represented by its centroid denoted by fuzzy color  $\tilde{c}_i$ . The pattern matrix  $M$ , which is handled by clustering algorithm, is represented as an  $p \times n$  pattern matrix. An element  $e_{ij}$  in matrix  $M$  means the degree of membership of a color element  $x_j$  to a fuzzy cluster  $\tilde{F}_i$ . Equation 4 shows the evaluation function  $J(\tilde{F})$  of clustering results, which extends the typical evaluation function of FCM in order to apply the fuzzy color model to the color clustering problem [8]. The objective is to minimize the evaluation function for a given pattern matrix.

$$J(\tilde{F}) = \sum_{i=1}^p \sum_{j=1}^n [\mu_{\tilde{F}_i}(x_j)]^m \delta(x_j, \tilde{c}_i) \quad (4)$$

where  $\mu_{\tilde{F}_i}(x_j)$  is the membership degree of color element  $x_j$  to a fuzzy cluster set  $\tilde{F}_i$ , and  $\delta(x_j, \tilde{c}_i)$  is the proposed color distance between the color element  $x_j$  and the centroid  $\tilde{c}_i$  of the fuzzy cluster set  $\tilde{F}_i$ . The parameter  $m$  controls the fuzziness of membership of each color elements. The goal is to iteratively improve a sequence of sets of fuzzy clusters  $\tilde{F}(1), \tilde{F}(2), \dots, \tilde{F}(t)$  ( $t$  means the iteration step) until no further improvement in  $J(\tilde{F})$  is possible.

### 3.1 Initialization of Clustering

Before we describe the detailed clustering algorithm, we discuss how to obtain the initial partition of the color cluster set. In clustering algorithm, initialization plays an important role. The clustering algorithm might terminate at a different clustering result according to the selected initial partition. There is no general agreement on a good initialization scheme. In this paper, we propose a novel initial selection method based on the notion of fuzzy color because it is simple and intuitive.

The basic concept of initialization is to seek dominant colors from a given color pattern set. The dominant color is defined as the color that occupies a large portion of the given color set rather than others. Starting a clustering algorithm with the initials from the dominant colors is more helpful to avoid falling into local minima rather than the random initialization. To accomplish this, a notion of reference fuzzy color, denoted by  $\tilde{C}_R$ , is employed to cover all the dominant colors in a color space. Ten reference fuzzy colors are adopted from the hue families of the Munsell color system: Red, Yellow, Green, Blue, Purple, Yellow-Red, Green-Yellow, Blue-Green, Purple-Blue, and Red-Purple. The Munsell color system has been used for

color reference guides for color selection and communication. Furthermore, this color arrangement takes an effect in both separating each of colors and managing the simultaneous contrast phenomena [2]. For the color element  $x_j \in X$ , we compute the matching score to the ten reference fuzzy colors, and increase the occurrence count of the reference fuzzy color that has the highest matching score. The input of initial color selection system is color elements in pattern space, and the output is a set of dominant fuzzy colors that have larger matching scores. The first  $p$  fuzzy colors of the list are chosen as the initial fuzzy cluster centroid where  $p$  is the number of groups to be clustered.

For a given color element  $x_j$ , the matching score is computed by considering the membership degree  $\mu_{\tilde{r}_i}(x_j)$  to all reference fuzzy colors  $\tilde{r}_i \in \tilde{C}_R$ . The membership degree  $\mu_{\tilde{r}_i}(x_j)$  is obtained by calculating all distances from reference fuzzy colors. Each reference fuzzy color  $\tilde{r}_i$  has two additional attributes denoted by  $occur_i$  and  $candidate_i$ . The  $occur_i$  refers to how many color elements belong to the given reference fuzzy color  $\tilde{r}_i$ .

$$occur_i \leftarrow occur_i + 1 \text{ if } \mu_{\tilde{r}_i}(x_j) \geq \mu_{\tilde{r}_k}(x_j) \quad (5)$$

for  $\forall \tilde{r}_k \in \tilde{C}_R$ . The  $candidate_i$  is determined by the input color element that has the maximum membership degree to this reference fuzzy color, and it will be used as a center point of a newly generated initial centroid.

$$candidate_i \leftarrow x_j \text{ if } \mu_{\tilde{r}_i}(x_j) \geq \mu_{\tilde{r}_i}(x_k) \quad (6)$$

for  $\forall x_k \in X$ . The output set  $R$  of the initial color selection process can be described in equation 7.

$$R = \{ \dots, \{ \tilde{r}_i, occur_i, candidate_i \}, \dots \} \quad (7)$$

for  $i = 1, \dots, 10$ . From the output set  $R$ , we select the dominant fuzzy color set, denoted by  $D \subseteq R$ , which includes the first  $p$  reference fuzzy colors that have a larger occurrence count than other colors. Each  $candidate_i \in D$  is employed as the center point of the initial fuzzy color centroids when the reference color  $\tilde{r}_i$  is selected.

### 3.2 Fuzzy Clustering using Fuzzy Color

As mentioned in an earlier section, the objective is to obtain the optimal partition  $\tilde{F} = \{ \tilde{F}_1, \tilde{F}_2, \dots, \tilde{F}_p \}$  for given color elements  $x_1, \dots, x_n$  and the number of clusters  $p$  by minimizing the evaluation function  $J(\tilde{F})$ . We extend the conventional FCM procedure to exploit the notion of the fuzzy color centroid and cluster membership computation. The complete clustering algorithm includes the following steps:

Step 1) Given  $p$  and  $x_1, \dots, x_n \in X$ , run the proposed cluster initial selection method to obtain the dominant color set  $D$  that is used to create the initial centroids  $\tilde{c}_1, \dots, \tilde{c}_p$  of fuzzy clusters  $\tilde{F}_1, \dots, \tilde{F}_p$ .

Step 2) Create the initial color centroids,  $\tilde{c}_1(t), \dots, \tilde{c}_p(t)$  where  $t$  means the iteration step (initially  $t = 0$ ) with  $D$ . The newly created fuzzy color centroid  $\tilde{c}_i$  ( $i = 1, \dots, p$ ) is obtained as follows:

$$\begin{aligned} \tilde{c}_i &= \langle center_i, jnd_i \rangle \\ center_i &\leftarrow candidate_i \quad (candidate_i \in D) \\ jnd_i &\leftarrow jnd_{\tilde{r}_i} \quad (\tilde{r}_i \in D) \end{aligned}$$

where  $\tilde{r}_i$  and  $candidate_i$  are elements of the  $i$ th dominant fuzzy color, and  $jnd_{\tilde{r}_i}$  is the JND value of fuzzy color  $\tilde{r}_i$ .

Step 3) Update the membership degree of fuzzy cluster sets  $\tilde{F}_i(t+1)$  by the following procedure. For each color element  $x_j$ :

- if  $\delta(x_j, \tilde{c}_i) < jnd_{\tilde{c}_i}$ , then update the membership of  $x_j$  in  $\tilde{F}_i$  by  $\mu_{\tilde{F}_i}(x_j)(t+1) = 1.0$ .
- if  $\delta(x_j, \tilde{c}_k) < jnd_{\tilde{c}_k}$ , for  $k \neq i, \tilde{c}_k \in \tilde{C}$  then update the membership of  $x_j$  in  $\tilde{F}_i$  by  $\mu_{\tilde{F}_i}(x_j)(t+1) = 0.0$ .
- otherwise, update the membership of  $x_j$  by Eq. 3.

Step 4) Update the fuzzy color centroid  $\tilde{c}_i$  of each fuzzy cluster. The  $center_i$  is computed with the degree of membership of each color element, and the  $jnd_i$  is updated by  $jnd_{\tilde{r}_q}$  where  $\tilde{r}_q$  is the closest reference fuzzy color to the newly updated  $center_i(t+1)$ .

$$center_i(t+1) = \frac{\sum_{j=1}^n \mu_{\tilde{F}_i}(x_j)(t) \cdot x_j}{\sum_{j=1}^n \mu_{\tilde{F}_i}(x_j)(t)} \quad (8)$$

$$jnd_i(t+1) = jnd_{\tilde{r}_q} \text{ s.t. } \mu_{\tilde{r}_q}(center_i) \leq \mu_{\tilde{r}_k}(center_i) \quad (9)$$

where  $q \neq k, \forall \tilde{r}_k \in \tilde{C}_R$ , and  $\tilde{C}_R$  is the universe of discourse of the reference fuzzy colors.

Step 5) If  $|\tilde{F}_i(t+1) - \tilde{F}_i(t)| < \epsilon$  for all  $\tilde{F}_i \in \tilde{F}$ , where  $\epsilon$  is a termination threshold, then stop; otherwise,  $t \leftarrow t+1$  and go to step 3.

## 4. Clustering Examples

The original color image is mapped into feature space where each pixel corresponds to a color element  $x_j$  in color space and each segmented region to a cluster set  $\tilde{F}_i$ . As a simple illustration of the proposed procedure, we consider a color data set consisting of 15 elements from CIELAB color space in Table 1. We suppose the number of clusters is given as three ( $p = 3$ ).

Table 1: Sample color elements and their CIELAB values

Color element	$L^*$	$a^*$	$b^*$
$x_1$	53.24	76.16	68.65
$x_2$	87.74	-89.94	85.77
$x_3$	32.30	76.16	-102.67
$x_4$	26.96	52.84	-31.34
$x_5$	88.35	-82.17	54.90
$x_6$	94.14	-26.40	97.21
$x_7$	39.60	79.11	-92.32
$x_8$	64.99	43.94	74.62
$x_9$	47.70	-14.82	54.75
$x_{10}$	60.32	93.81	-55.62
$x_{11}$	91.11	-52.39	-8.74
$x_{12}$	54.62	79.75	13.21
$x_{13}$	89.54	-74.71	88.00
$x_{14}$	11.21	42.84	-57.79
$x_{15}$	44.32	-29.56	-4.91

According to the proposed initialization method, the degree of membership of each data to the ten reference colors is calculated in order to get the occurrence and candidate value of each reference color  $\tilde{r}_i$ . Table 2 shows the degree of membership of each data to the reference colors. The ten reference colors from the Munsell hue families are originally described in a Munsell notation. Thus, we convert the Munsell notation to  $L^*a^*b^*$  space in order to obtain the center and JND value using a conversion software [9].

Table 3: The output set  $R$  of the color decision process

$\tilde{r}_i$	Center( $L^*a^*b^*$ )			JND	Occur	Candi
$\tilde{r}_1$	51.5	99.7	50.8	17.6	2	$x_1$
$\tilde{r}_2$	81.3	41.5	125.81	12.5	1	$x_8$
$\tilde{r}_3$	91.0	-5.1	141.2	10.5	0	$x_6$
$\tilde{r}_4$	81.3	-43.7	127.0	10.5	5	$x_6$
$\tilde{r}_5$	51.5	-153.7	23.9	14.3	0	$x_5$
$\tilde{r}_6$	51.5	-117.1	-20.3	14.3	0	$x_{11}$
$\tilde{r}_7$	51.5	-48.2	-61.8	20.1	2	$x_{15}$
$\tilde{r}_8$	20.5	55.4	-102.8	13.1	2	$x_3$
$\tilde{r}_9$	30.7	65.6	-52.4	13.1	2	$x_4$
$\tilde{r}_{10}$	51.5	113.8	-18.1	15.6	1	$x_{10}$

Table 3 shows the count of occurrence and the candidate value of each reference color that are calculated by the equations 5 and 6. This table corresponds to the output set  $R$  of the initial color selection process. From the output set  $R$ , the dominant color set  $D$  is extracted by sorting  $R$  by the largest count of occurrence. In this example, a given number of clusters is three, thus the rows of  $\{\tilde{r}_4, \tilde{r}_1, \tilde{r}_7\}$  become the elements of  $D$ . Even though the  $\{\tilde{r}_8, \tilde{r}_9\}$  have the same occurrences as the  $\{\tilde{r}_1, \tilde{r}_7\}$  do, the  $\{\tilde{r}_1, \tilde{r}_7\}$  get the right for  $D$  since they precede others in the list. Table 4 shows the initial cluster centroids represented by the fuzzy color. We can see, for example,  $x_6$  is selected as the center of the initial cluster centroid because it is the closest color to the reference color  $\tilde{r}_4$ . For three fuzzy cluster sets  $\tilde{F}_1, \tilde{F}_2$ , and  $\tilde{F}_3$ , three fuzzy color centroids have their own centers and JNDs.

Table 4: Initial fuzzy color centroids for three clusters

Cluster ( $\tilde{F}$ )	Fuzzy color centroid( $\tilde{c}$ )				
	Candidate	$L^*$	$a^*$	$b^*$	JND
$\tilde{F}_1$	$x_6$	94.14	-26.4	97.21	10.57
$\tilde{F}_2$	$x_1$	53.24	76.16	68.65	17.61
$\tilde{F}_3$	$x_{15}$	44.32	-29.56	-4.91	20.13

Given this initialization, the clustering algorithm iteratively computes the membership degree of each color element to the fuzzy cluster sets, and updates the fuzzy color centroids of each cluster. Table 5 shows a series of iteration steps until the algorithm reaches at convergence. The termination threshold was  $|\tilde{F}_i(t+1) - \tilde{F}_i(t)| < \epsilon = 0.001$ . The clustering algorithm stopped at step 15. According to the final fuzzy membership of each color elements, the color elements  $\{x_2, x_5, x_6, x_{11}, x_{13}\}$  belong to the first fuzzy cluster  $\tilde{F}_1$ ,  $\{x_1, x_8, x_9, x_{12}, x_{15}\}$  belong to  $\tilde{F}_2$ , and  $\{x_3, x_4, x_7, x_{10}, x_{14}\}$  belong to  $\tilde{F}_3$ . In particular, color elements  $\{x_3, x_4, x_5, x_7, x_9, x_{10}\}$  is considered to be vague at the first step since the maximum membership degrees of those elements are around 0.5. However, as the iteration step increases, they turn out to be elements of clusters  $\tilde{F}_3, \tilde{F}_3, \tilde{F}_1, \tilde{F}_3, \tilde{F}_2, \tilde{F}_3$  respectively at the final step.

## 5. Concluding Remarks

In this paper we discussed the color clustering problem. To successfully partition color pattern data, we first proposed a new fuzzy color model that can describe the vagueness underlying natural colors. The fuzzy color model is based on CIELAB color space which gives uniform color scaling. We defined the concept of fuzzy color and a new distance measure between fuzzy color and a color element. Each fuzzy color has a tuple of its center and JND value. With the fuzzy color distance, we proposed a color membership computation method to a specific fuzzy color and its desirable properties. In order to effectively deal with color data in clustering, we adopted a fuzzy cluster analysis. We developed a new fuzzy clustering algorithm with the proposed fuzzy color model. The key concept was to exploit the fuzzy color centroids. Each fuzzy color centroid can help to calculate the membership degree of each color data.

As further work, we would like to study the following issues. We should conduct an analysis on the proposed fuzzy color model and its properties. The color's asymmetric characteristics and the reference color selection policy should be considered. Furthermore, clustering issues include automatic determination of the number of clusters and a centroid initialization technique.

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Table 2: Degrees of membership to ten reference colors ( $\tilde{r}_i, i = 1, \dots, 10$ )

$\tilde{r}_i$	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	$x_8$	$x_9$	$x_{10}$	$x_{11}$	$x_{12}$	$x_{13}$	$x_{14}$	$x_{15}$
$\tilde{r}_1$ RED	1.00	0.06	0.00	0.00	0.07	0.06	0.06	0.20	0.11	0.08	0.07	0.29	0.06	0.06	0.08
$\tilde{r}_2$ Y-R	0.00	0.09	0.00	0.00	0.09	0.13	0.03	0.22	0.12	0.04	0.07	0.07	0.10	0.04	0.07
$\tilde{r}_3$ Yellow	0.00	0.13	0.00	0.00	0.11	0.21	0.03	0.12	0.12	0.04	0.07	0.05	0.14	0.03	0.07
$\tilde{r}_4$ G-Y	0.00	0.22	0.00	0.00	0.16	0.30	0.03	0.10	0.14	0.03	0.08	0.05	0.26	0.03	0.08
$\tilde{r}_5$ Green	0.00	0.14	0.00	0.00	0.16	0.06	0.03	0.05	0.08	0.03	0.10	0.03	0.11	0.03	0.09
$\tilde{r}_6$ B-G	0.00	0.11	0.00	0.00	0.15	0.06	0.04	0.05	0.09	0.04	0.17	0.04	0.10	0.04	0.13
$\tilde{r}_7$ Blue	0.00	0.08	0.00	0.00	0.11	0.05	0.06	0.06	0.10	0.06	0.22	0.06	0.08	0.09	0.24
$\tilde{r}_8$ P-B	0.00	0.04	1.00	0.00	0.05	0.04	0.38	0.05	0.06	0.13	0.07	0.07	0.05	0.20	0.08
$\tilde{r}_9$ Purple	0.00	0.05	0.00	1.00	0.06	0.05	0.24	0.07	0.09	0.27	0.08	0.12	0.05	0.39	0.10
$\tilde{r}_{10}$ R-P	0.00	0.05	0.00	0.00	0.06	0.05	0.11	0.09	0.08	0.27	0.07	0.23	0.05	0.09	0.07

Table 5: Iteration steps of the proposed clustering

$t$	$\tilde{F}_i$	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	$x_8$	$x_9$	$x_{10}$	$x_{11}$	$x_{12}$	$x_{13}$	$x_{14}$	$x_{15}$
1	$\tilde{F}_1$	0.00	0.70	0.16	0.11	0.53	1.00	0.16	0.06	0.31	0.15	0.09	0.07	0.80	0.11	0.00
	$\tilde{F}_2$	1.00	0.09	0.33	0.33	0.09	0.00	0.34	0.91	0.16	0.45	0.05	0.79	0.06	0.25	0.00
	$\tilde{F}_3$	0.00	0.22	0.51	0.56	0.38	0.00	0.50	0.03	0.52	0.40	0.86	0.14	0.14	0.63	1.00
2	$\tilde{F}_1$	0.03	0.91	0.11	0.04	0.88	0.93	0.10	0.10	0.43	0.08	0.23	0.00	0.97	0.06	0.02
	$\tilde{F}_2$	0.94	0.03	0.35	0.35	0.04	0.04	0.36	0.80	0.23	0.50	0.11	0.99	0.01	0.25	0.02
	$\tilde{F}_3$	0.04	0.05	0.55	0.60	0.08	0.04	0.54	0.10	0.33	0.42	0.66	0.01	0.02	0.69	0.96
⋮	⋮							⋮								
15	$\tilde{F}_1$	0.01	0.97	0.01	0.03	0.99	0.75	0.00	0.00	0.38	0.02	0.60	0.04	0.99	0.02	0.37
	$\tilde{F}_2$	0.97	0.02	0.02	0.13	0.01	0.21	0.01	0.99	0.56	0.06	0.25	0.80	0.01	0.05	0.39
	$\tilde{F}_3$	0.02	0.01	0.97	0.85	0.00	0.04	0.98	0.00	0.06	0.92	0.15	0.17	0.00	0.94	0.23

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