J. Appl. Math. & Informatics Vol. **26**(2008), No. 3 - 4, pp. 519 - 530
Website: http://www.kcam.biz

IDENTIFICATION OF THERMODYNAMIC PARAMETERS OF ARCTIC SEA ICE AND NUMERICAL SIMULATION

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ABSTRACT. This paper studies the multi-domain coupled system of one dimensional Arctic temperature field and establishes identification model about the thermodynamic parameters of sea ice (heat storage capacity, density and conductivity) by the so-called output least-square estimate according to the temperature data acquired by a monitor buoy installed in the Arctic ocean. By the optimal control theory, the existence and dependability of weak solution and the identifiability of identification model have been given. Moreover, necessary optimality condition is proposed. Furthermore, the optimal algorithm for the identification model is constructed. By using the optimal thermodynamic parameters of Arctic sea ice, the numerical simulation is implemented, and the numerical results of temperature distribution of Arctic sea ice are demonstrated.

AMS Mathematics Subject Classification : 49J20, 35K90, 93B30, 68W99.
Key words and phrases : The multi-domain coupled system, parameter identification, optimal algorithm, numerical simulation, Arctic sea ice.

1. Introduction

Sea ice which possesses seven percent of the ocean is an essential part of Arctic Circle. It affects the dynamics and thermodynamics between the atmosphere and the ocean obviously, so it has important influence on global climate change. In contrast with sea water and land, sea ice has its own characteristics: its albedo is at least 60% higher than that of land surface, as greatly reduces the absorption to the solar radiation of the sea; sea ice melting can absorb mass heat, so sea water can be diluted; the cover of sea ice can weaken the heat exchange between the ocean and the atmosphere, prevent the evaporation of sea water, and suppress sensible heat flux conduction from sea water to the atmosphere. Under the same atmospheric environmental condition, the heat flux in sea ice is one order of magnitude lower than that in neighboring sea water.

Received March 9, 2007. Revised March 23, 2008. *Corresponding author.
This work is supported by National Science Foundation of China (No.10471014, No.10671126, No.40233032, No.2003DEB5J057).

To be important, Arctic plays an extremely important role in global climate change as a result of its unique natural condition and geographical position. Since weather and hydrologic condition in different sea fields are variable, the thermodynamic parameters of snow, sea ice, and sea water are also different. In addition, these thermodynamic parameters play a key role in many fields, such as oil industry, seafaring, fishery etc. Especially, the quantitative analysis of the thermodynamic parameters is very important. M. Winton [2], C.M. Bitz [3] established the multi-layer thermodynamic model of sea ice. J. Su, H.D. Wu, S. Bai [4], [5] analyzed the sea ice thickness and heat budget at the sea ice surface according to the energy conversation law. Z.H. Lin, Q. Le, X.Q. Wang [6], B. Cheng [7] studied the ice temperature, the ice range and definite conditions via differential equation and partial differential equation. Zh.J. Li, X.L. Dong, Zh.H. Zhang [8] described the description of data collection of Arctic sea ice physical properties in the second Chinese Arctic Expedition. Q.F. Wang, D.X. Feng [9] made an qualitative analysis to the parameter identification problem of nonlinear abstract parabolic distributed parameter systems with one parameter via variational method. H. Gao [10] discussed the system governed by a semilinear parabolic equation and gave the necessary conditions for optimal control.

Based on the temperature of atmosphere, snow, sea ice and sea water acquired by a monitor buoy installed in the Arctic ocean from August, 2003 to April, 2004, we study the multi-domain coupled system of Arctic temperature field. By the optimal control theory, we describe the uniqueness and continuity of weak solution of this system. Moreover, we establish the distributed parameter identification model about the thermodynamic parameters of sea ice(heat storage capacity, density and conductivity) and discuss the identifiability of this model and propose necessary optimality condition. On the other hand, we construct the optimal algorithm of identification model and obtain the optimal thermodynamic parameters of sea ice from the ice floe. Furthermore, we realize the numerical simulation, and show the numerical results of temperature distribution of Arctic sea ice in this paper.

2. The model of one dimensional Arctic temperature field

From the temperature of Arctic snow, sea ice and sea water, we find that the gradient change along vertical direction is far greater than that along horizontal direction, and the heat exchange exists mainly in vertical direction and can be ignored in horizontal direction. Hence, the temperature distribution can be described by one dimensional temperature field equations.

Set any point of interface between snow and sea ice be coordinate origin, and axis z represent the vertical direction.

Let $l_1,\ l_2,\ l_3$ be the thickness of snow, sea ice and sea water(m) respectively. Set $D_1 = [-l_1,\ 0]$, $D_2 = [0,\ l_2]$, $D_3 = [l_2,\ l_2 + l_3]$ be the domain representing snow, sea ice and sea water respectively. $I = [0,\ t_f] \subset R$ be the time domain of the coupled system. Moreover, we set $D = D_1 \cup D_2 \cup D_3 \subset R$, $Q = D \times I \subset R^2$, $Q_j = D_j \times I \subset R^2$, $j = 1, 2, 3$.

By using the energy conversation law and the theory of Fourier heat exchange, the temperature field equations and definite conditions of Arctic snow, sea ice and sea water can be given as:

$$\begin{cases} (c\rho)(z,t)\dfrac{\partial T(z,t)}{\partial t} = \dfrac{\partial}{\partial z}\left(K(z,t)\dfrac{\partial T(z,t)}{\partial z}\right) + g(z,t), & (z,t) \in Q \\ T(z,0) = T_0(z), & z \in D \\ \dfrac{\partial T(z,t)}{\partial z}|_{z=-l_1} = h_0(t), & t \in I \\ T(z,t)|_{z=l_2+l_3} = T_3(t), & t \in I \end{cases} \quad (1)$$

where $h_0(t) = (h(T(z,t)-T_c)/K_s)|_{z=-l_1}$. h is heat exchange parameter between snow and atmosphere. T_c is air temperature.

Let $S_i(z)$ be the salinity in $z \in D_2$, which can be expressed approximately as follows:

$$S_i(z) = \begin{cases} 14.24 - 19.39z, & 0 < z < 0.57m; \\ 3.2, & 0.57m \leq z \leq l_2. \end{cases}$$

Let c_s, ρ_s, K_s, c_i, ρ_i, K_i, c_w, ρ_w and K_w be average heat storage capacity, density and heat conductivity of snow, sea ice and sea water respectively. So, in equation (1), we have

$$(c\rho)(z,t) = \begin{cases} c_s\rho_s, & (z,t) \in Q_1 \\ c_i\rho_i, & (z,t) \in Q_2 \\ c_w\rho_w, & (z,t) \in Q_3 \end{cases} \quad (2)$$

and

$$K(z,t) = \begin{cases} K_s, & (z,t) \in Q_1 \\ K_i, & (z,t) \in Q_2 \\ K_w, & (z,t) \in Q_3 \end{cases} \quad (3)$$

Set c_s, ρ_s, K_s, c_w, ρ_w and K_w be constants, and $q = (c_i, \rho_i, K_i)$ be the identification parameters vector. We have $q \in R^3_+$ obviously.

Let $q_- \in R^3_+$ and $q_+ \in R^3_+$ be lower and upper bounds of $q \in R^3_+$ because of engineering meaning. Thus, the admissible set of q should be

$$Q_{ad} = \{q : q_- \leq q \leq q_+\} \subset R^3. \quad (4)$$

where the symbol '$\leq$' is for every elements of the vectors.

In equation (1), the heat source term $g(z,t)$ can be described by :

$$g(z,t) = \begin{cases} g_1(z,t), & (z,t) \in Q_1 \\ g_2(z,t), & (z,t) \in Q_2 \\ 0, & (z,t) \in Q_3 \end{cases} \quad (5)$$

where

$$g_j(z,t) = \gamma_j(1-\alpha_j)r_j I_{oj} exp(-r_j|z|), \qquad j = 1,2 \quad (6)$$

γ_j, α_j, I_{oj} and r_j are the solar shortwave radiation, sea ice albedo, transmission and extinction coefficient of sea ice respectively.

From above, the multi-domain coupled system (TCS) of Arctic temperature field can be defined by equation (1).

3. The properties of multi-domain coupled system

Based on the physical properties of Arctic snow, sea ice and sea water, we introduce the assumption:

$A1$: In the coupled system TCS, functions $g : D \times I \to R$, $T_0 : D \to R$, $h_0 : I \to R$, $T_3 : I \to R$ should be bounded, continuously differentiable. The derivatives in terms of all the variables are also bounded.

As for the system TCS, according to the assumption $A1$, we set $u(z,t) = T(z,t) - (z - l_2 - l_3)h_0(t) - T_3(t)$. Then the system TCS has been changed:

$$\begin{cases} \frac{\partial u}{\partial t} - \alpha(z,t)\frac{\partial}{\partial z}\left(K(z,t)\frac{\partial u}{\partial z}\right) = p(z,t;q), \\ u(z,0) = u_0(z), \\ \left.\frac{\partial u}{\partial z}\right|_{z=-l_1} = 0, \\ u|_{z=l_2+l_3} = 0, \end{cases}$$

where

$$\alpha(z,t) = ((c\rho)(z,t))^{-1},$$

$$p(z,t;q) = ((c\rho)(z,t))^{-1}\Big(h_0(t)\frac{\partial K(z,t)}{\partial z} + g(z,t) - (c\rho)(z,t)(z - l_2 - l_3)h_0'(t) - (c\rho)(z,t)T_3'(t)\Big),$$

$$u_0(z) = T_0(z) - (z - l_2 - l_3)h_0(0) - T_3(0).$$

We write this system as $TTCS$. We introduce $H(D)$, real Hilbert space of $D \subset R$. We set $V = \left\{v : v \in H(D),\ \frac{\partial v}{\partial z}|_{z=-l_1} = v|_{z=l_2+l_3} = 0\right\}$, and then $V \subset H(D)$. Let $(\cdot,\cdot)_H$ and $<\cdot,\cdot>_V$ be the inner product of H and V, $|\cdot|_H$ and $\|\cdot\|_V$ be the norm of H and V, H^* and V^* be the dual space, and $<\cdot,\cdot>_{V^*,V}$ denote the duality pairing respectively. Then (V, H, V^*) is *Gelfand* triple space with $V \hookrightarrow H = H^* \hookrightarrow V^*$, which means that the embedding $V \hookrightarrow H$ is continuous and V is dense in H. Similarly, the embedding $H^* \hookrightarrow V^*$ is also continuous and H^* is dense in V^*.

By using principle of virtual work, we define

$$a(t,q;u,v) = \int_D K(z,t)\frac{\partial u}{\partial z}\frac{\partial(\alpha(z,t)v)}{\partial z}dz, \qquad \forall\, u,v \in V,$$

with the properties below:

Property $B1$: $a(t,q;u,v)$ is a bilinear functional on $V \times V$, and there exists a const $M > 0$, such that the inequality $|a(t,q;u,v)| \le M\|u\|_V\|v\|_V$ holds.

Property $B2$: As for $a(t,q;u,v)$, there exist $\beta_0 > 0$ and $\lambda_0 \in R$ such that the inequality $a(t,q;v,v) + \lambda_0|v|_H^2 \ge \beta_0\|v\|_V^2$ holds for any $v \in V$.

From Property $B1$, Property $B2$ and [13], we have,

$$\exists\ A(t,q) \in B(V,V^*), \qquad s.t. \quad a(t,q;u,v) = < A(t,q)u, v >_{V^*,V},$$

where $B(V,V^*)$ denotes all the bounded linear operators mapped from V into V^*.

For $\forall\ q \in Q_{ad}$, $\int_D p(z,t;q)vdz$ is bounded. Hence, according to Riesz representation theorem, there exists $f(z,t;q) \in V^*$, for $\forall\ v \in V$, such that

$$\int_D p(z,t;q)vdz = < f(z,t;q), v >_{V^*,V}\ .$$

So, the system $TTCS$ can be changed to abstract parabolic equations below:

$$\begin{cases} \dfrac{\partial u(z,t;q)}{\partial t} + A(t,q)u = f(z,t;q), & t \in I \\ u(z,0;q) = u_0(z). \end{cases}$$

We denote this system as APE.

According to [9], we give the definition of weak solution to the system APE.

Definition 1. *Function $u = u(z,t;q)$ is called as a weak solution to the system APE if for any $v \in V$, u satisfies the following:*

$$< \frac{\partial u(z,\cdot;q)}{\partial t}, v >_{V^*,V} + a(\cdot,q;u(z,\cdot;q),v) = (f(z,\cdot;q),v)_H.$$

Moreover, we can give the definition of weak solution to the system TCS similarly.

By [9] and [13] and analysis above, we can get the theorem of weak solution to the system APE as follows.

Theorem 1. *For $\forall\ q \in Q_{ad}$, the system APE admits a unique weak solution $u(z,t;q)$, and $u(z,t;q)$ is continuous with respect to parameter $q \in Q_{ad}$.*

Similarly, using Theorem 1, we have,

Corollary 1. *For $\forall\ q\ \in Q_{ad}$, the system TCS admits a unique weak solution $T(z,t;q)$, and $T(z,t;q)$ is continuous with respect to parameter $q \in Q_{ad}$.*

Let $S \subset L^2(I;H(D)) \cap C(I;H(D))$ be the set of weak solutions to the system TCS, i.e.,

$S = \{T(z,t;q) \in L^2(I;H(D)) \cap C(I;H(D)) \mid T(z,t;q)$ is the weak solution to the system TCS with respect to q $\in Q_{ad}\}$.

Proposition 1. *Let S be defined by the equality above. Then S is a compact set in $L^2(I;H(D)) \cap C(I;H(D))$.*

Proof. From Corollary 1, the map $q \to T(z,t;q) : Q_{ad} \to S \subset L^2(I;H(D)) \cap C(I;H(D))$ is continuous and $Q_{ad} \subset R^3_+$ is bounded and closed. Hence, S is a compact set in $L^2(I;H(D)) \cap C(I;H(D))$. □

4. Identification of multi-domain coupled system

For the sake of researching the characteristic and correlativity of Arctic snow, sea ice, sea water and surface atmosphere, monitoring the sea ice freezing and melting process and evaluating its influence on global weather, satellite tracker-localizer was installed in the Arctic ocean which can monitor the data including wind speed, wind direction, air temperature, air pressure, sea ice and sea water temperature at test points from August, 2003 to April, 2004. Moreover, the thickness of snow, sea ice and salinity had also been provided.

Let $T_d(z_k, t_j)$, $j = 1, 2, \cdots, l_t$; $k = 1, 2, \cdots, l_z$ be the test temperature at $z_k \in D$ and $t_j \in I$. The temperature distribution function $T_d(z,t)$, $(z,t) \in Q$ can be fitted by the test data collection $\{T_d(z_k, t_j)\}$. According to the continuity of temperature change, the function $T_d(z,t) : Q \to R$ is continuous and the coupled system TCS admits a unique weak solution $T(z,t;q)$ with respect to parameter $q \in Q_{ad}$. In order to estimate the error between $T(z,t;q)$ and $T_d(z,t)$, by the so-called output least-square estimate, the cost function is given by:

$$J(q) = \|T(z,t;q) - T_d(z,t)\|^2_{C(Q,R)}. \tag{7}$$

Then, identification model of the coupled system TCS can be expressed as:

$$\begin{aligned} ITCSP : \min \quad & J(q) \\ \text{s.t.} \quad & T(z,t;q) \in S \\ & q \in Q_{ad} \end{aligned}$$

Because the map $q \to T(z,t;q) : Q_{ad} \to S$ is continuous, $J(q)$ defined by (7) is continuous with respect to parameter $q \in Q_{ad}$. We note that $Q_{ad} \subset R^3_+$ is nonempty, bounded and closed, therefore, there exists at least one optimal parameter $q^* \in Q_{ad}$, such that

$$J(q^*) \leq J(q), \qquad \forall\, q \in Q_{ad}.$$

So, we can get the following theorem:

Theorem 2. *Assume that A1 holds. Let Q_{ad} be given by (4) and let $T(z,t;q)$ be a weak solution of the system TCS with respect to parameter q. If the cost function is given by (7), there exists at least one optimal parameter $q^* \in Q_{ad}$ such that $\|T(z,t;q) - T_d(z,t)\|^2_{C(Q,R)}$ attains its minimum $\|T(z,t;q^*) - T_d(z,t)\|^2_{C(Q,R)}$.*

We now consider necessary optimality condition. As we all know, one classical method to obtain the necessary condition for q^* is to calculate the first variation of $J(q)$ around q^*. If $T(z,t;q)$ is Gâteaux differentiable at $q^* \in Q_{ad}$ and $T'(z,t;q^*)$ is its Gâteaux derivative at $q = q^*$, then $J(q)$ is Gâteaux differentiable at $q = q^*$, and the necessary condition for the optimal parameter q^* is characterized by the following variational inequality:

$$J'(q^*)(q - q^*) \geq 0, \qquad \forall\, q \in Q_{ad},$$

where $J'(q^*)$ denotes the Gâteaux derivative of $J(q)$ at $q = q^*$. In [9], as is proved, $T(z,t;q)$ is Gâteaux differentiable at $q^* \in Q_{ad}$. Therefore, our necessary optimality condition is feasible.

In order to be convenient for calculating, the cost function (7) can be modified

$$J_d(q) = \sum_{k=1}^{l_z}\sum_{j=1}^{l_t}(T(z_k,t_j;q) - T_d(z_k,t_j))^2, \tag{8}$$

Thus the practical identification problem which we write as model $ITCSP_d$ can be converted to:

$$\begin{aligned} ITCSP_d: \min \quad & J_d(q) \\ \text{s.t.} \quad & T(z,t;q) \in S \\ & q \in Q_{ad} \end{aligned}$$

Similarly, using Theorem 2, we have,

Corollary 2. *Assume that A1 holds. Let Q_{ad} be given by (4) and let $T(z,t;q)$ be a weak solution of the system TCS with respect to parameter q. If the cost function is given by (8), there exists at least one optimal parameter $q^* \in Q_{ad}$, such that $J_d(q)$ attains its minimum $J_d(q^*)$.*

5. Optimal algorithm and numerical simulation

5.1 Optimal algorithm

In this section, we propose the concrete optimal algorithm according to classical Hooke-Jeeves direct search algorithm. Considering magnitude of these parameters, we set different precision and step-size to every search direction, different from classical Hooke-Jeeves direct search algorithm. Our algorithm is as follows, where n is the number of parameters:

Algorithm1 :

Step 1. Select starting points $q^{(1)} \in Q_{ad}$, lower and upper bounds of the admissible set q_-, q_+, n directions $e_1, e_2, \cdots, e_n$, starting step-size $d_1, d_2, \cdots, d_n$, acceleration factor $\alpha > 0$, precision $\epsilon_1, \epsilon_2, \cdots, \epsilon_n > 0$ and maximum iteration number k_{max}.

Step 2. Set $v^{(1)} = q^{(1)}$, $k = j = 1$.

Step 3. If $d_j > \epsilon_j$ and $(q_-)_j \le q_j^{(k)} \le (q_+)_j$, solve the system TCS with respect to parameter $v^{(j)}$ by using finite difference scheme, get numerical solution $T(z,t;v^{(j)})$, and compute $J_d(v^{(j)})$ by (8), and go to Step 4. Else, go to Step 5.

Step 4. Solve the system TCS with respect to parameter $v^{(j)} + d_j e_j$ in the same way, get $T(z,t;v^{(j)} + d_j e_j)$, and compute $J_d(v^{(j)} + d_j e_j)$ by (8). go to Step 5.

Step 5. If $J_d(v^{(j)} + d_j e_j) < J_d(v^{(j)})$, set $v^{(j+1)} = v^{(j)} + d_j e_j$, and go to Step 6. Else, go to Step 7.

Step 6. If $j < n$, set $j = j + 1$ and go to Step 3. Else, go to Step 9.

Step 7. Solve the system TCS with respect to parameter $v^{(j)} - d_j e_j$, get $T(z,t;v^{(j)} - d_j e_j)$, and compute $J_d(v^{(j)} - d_j e_j)$ by (8). go to Step 8.

Step 8. If $J_d(v^{(j)} - d_j e_j) < J_d(v^{(j)})$, set $v^{(j+1)} = v^{(j)} - d_j e_j$, and go to Step 6. Else, set $v^{(j+1)} = v^{(j)}$, and go to Step 6.

Step 9. If $J_d(v^{(n+1)}) < J_d(q^{(k)})$, set $q^{(k+1)} = v^{(n+1)}$, $v^{(1)} = q^{(k+1)} + \alpha(q^{(k+1)} - q^{(k)})$, $k = k+1$, $j = 1$, and go to Step 3. Else, go to Step 10.

Step 10. If $d_j \le \epsilon_j$, $j = 1, 2, \cdots, n$ or $k \ge k_{max}$, stop the programming, and select $q^* = q^{(k)}$. Else, go to Step 11.

Step 11. If $d_j > \epsilon_j$ and $(q_-)_j \le q_j^{(k)} \le (q_+)_j$, set $d_j = \frac{d_j}{2}$, $j = 1, 2, \cdots, n$. Set $v^{(1)} = q^{(k)}$, $q^{(k+1)} = q^{(k)}$, $k = k+1$, $j = 1$, and go to Step 3.

where the temperature $T(z, t; \cdot)$ is calculated by using the semi-implicit finite difference scheme, which is stable unconditionally.

5.2 Numerical simulation

According to the optimal algorithm above, we carry out the numerical simulation and calculate the temperature distribution of Arctic sea ice from the real data collection acquired by a monitor buoy installed in the Arctic ocean.

During the computation, we cite the following experimented formulas:

$$c_i\rho_i = c_{i0}\rho_{i0} + \lambda S_i(z)/(T(z,t) - 273.15)^2,$$

$$K_i = K_{i0} + \beta S_i(z)/(T(z,t) - 273.15),$$

where λ, $\beta > 0$.

We choose $n = 5$, and give the optimal parameters according to *Algorithm1*. i.e., $c_{i0} = 2100.44$ J/(kg·K), $\rho_{i0} = 923.442$ kg/m^3, $K_{i0} = 1.7993$ W/(m·K), $\lambda = 1.72007 * 10^7$ J·K/kg, $\beta = 0.124442$ W·m^2/kg.

During implement, the time ranges from November.1, 2003 to March.10, 2004. The average error between the calculated temperature $T(z, t; q^*)$ and the test temperature $T_d(z, t)$ for these days is defined by:

$$e = \sqrt{\sum_{k=1}^{l_z}\sum_{j=1}^{l_t}(T(z_k, t_j; q) - T_d(z_k, t_j))^2/(l_z l_t)}. \tag{9}$$

After calculation by (9), the average error is $e = 1.07^oC$.

And the absolute average error for these days is defined by:

$$e1 = \left|\frac{e}{\sum_{k=1}^{l_z}\sum_{j=1}^{l_t} T_d(z_k, t_j)/(l_z l_t)}\right|. \tag{10}$$

After calculation by (10), the absolute average error is $e1 = 14.2\%$.

In Fig. 1, Fig. 2, Fig. 3 and Fig. 4, the time ranges from November.17, 2003 to March.10, 2004. Let the horizontal direction be the time $t(/minute)$ and the vertical direction be the temperature$(/centigrade)$. Set $T(z, t; q)$ denote the calculated temperature and $Td(z, t)$ denote the test temperature respectively in the figures.

Fig. 1 shows the calculated temperature and test temperature at test points.

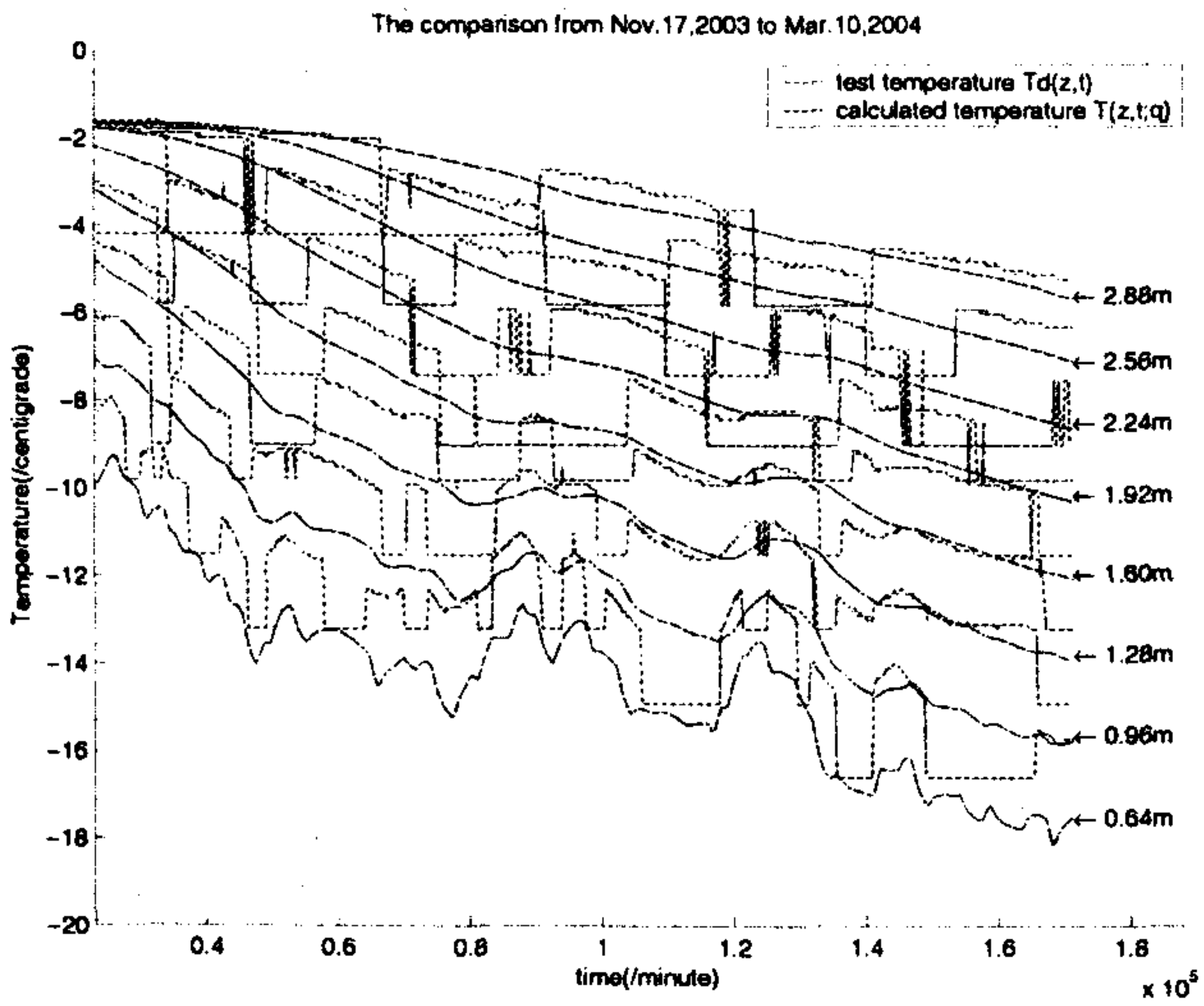

Fig. 1 The calculated temperature and the test temperature at test points

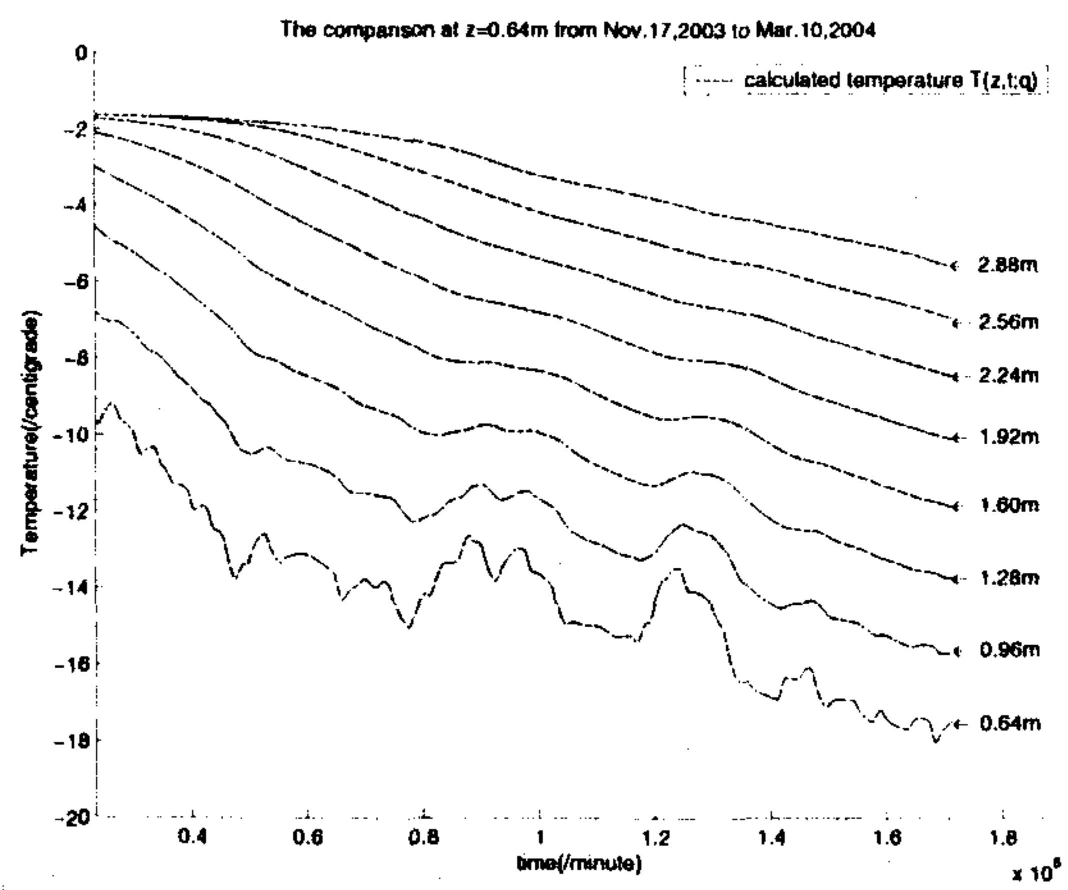

Fig. 2 The calculated temperature at test points

From Fig. 1 and Fig. 2, we find our numerical simulation is valid, because the calculated temperature curves correspond to geophysical laws. The deeper the sea ice is, and the least it is influenced by the air temperature change.

Fig. 3 and Fig. 4 show the comparisons between the calculated temperature and test temperature at location $z_k = 0.64$m and $z_k = 1.28$m respectively.

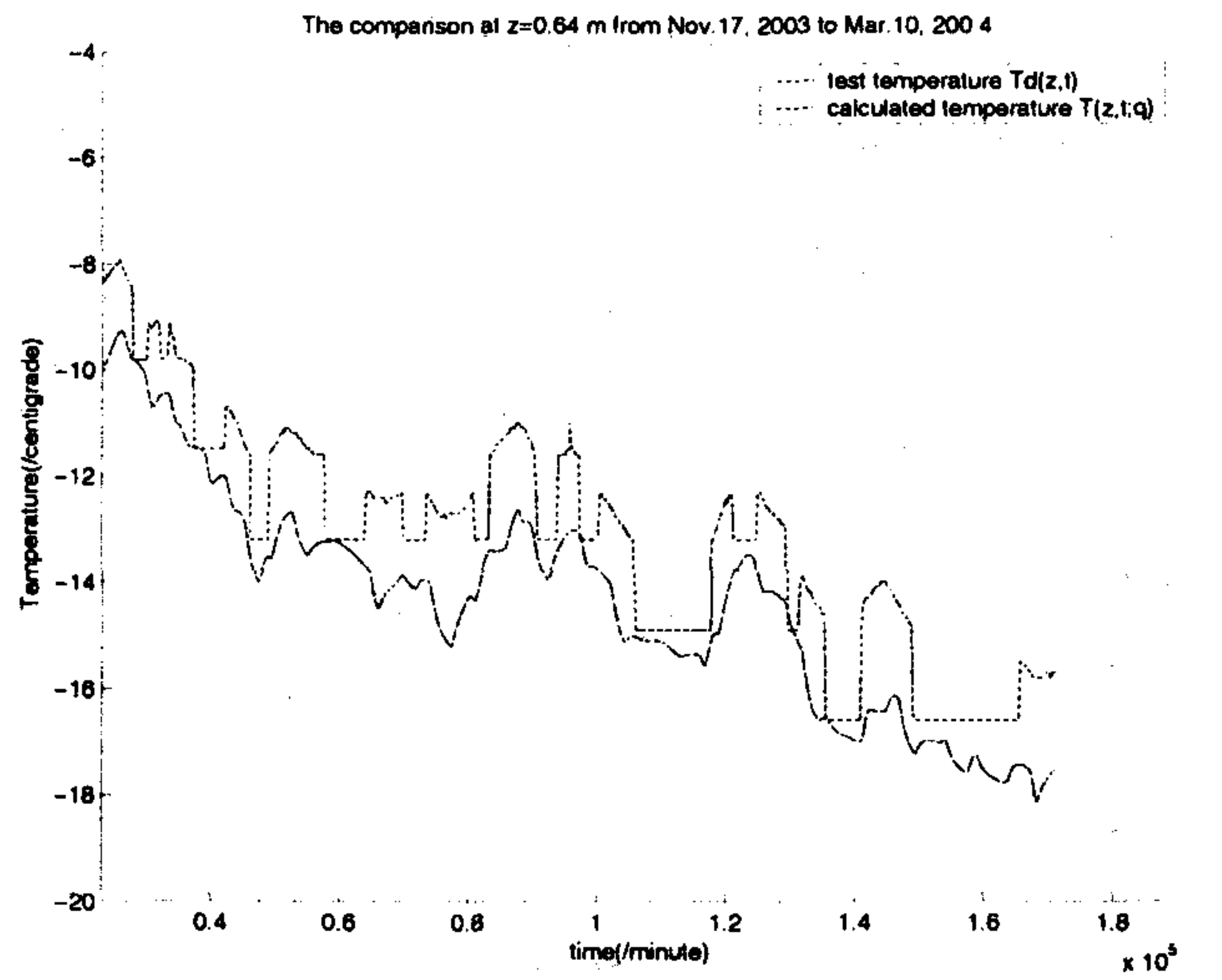

Fig. 3 The comparisons at location $z_k = 0.64\text{m}$

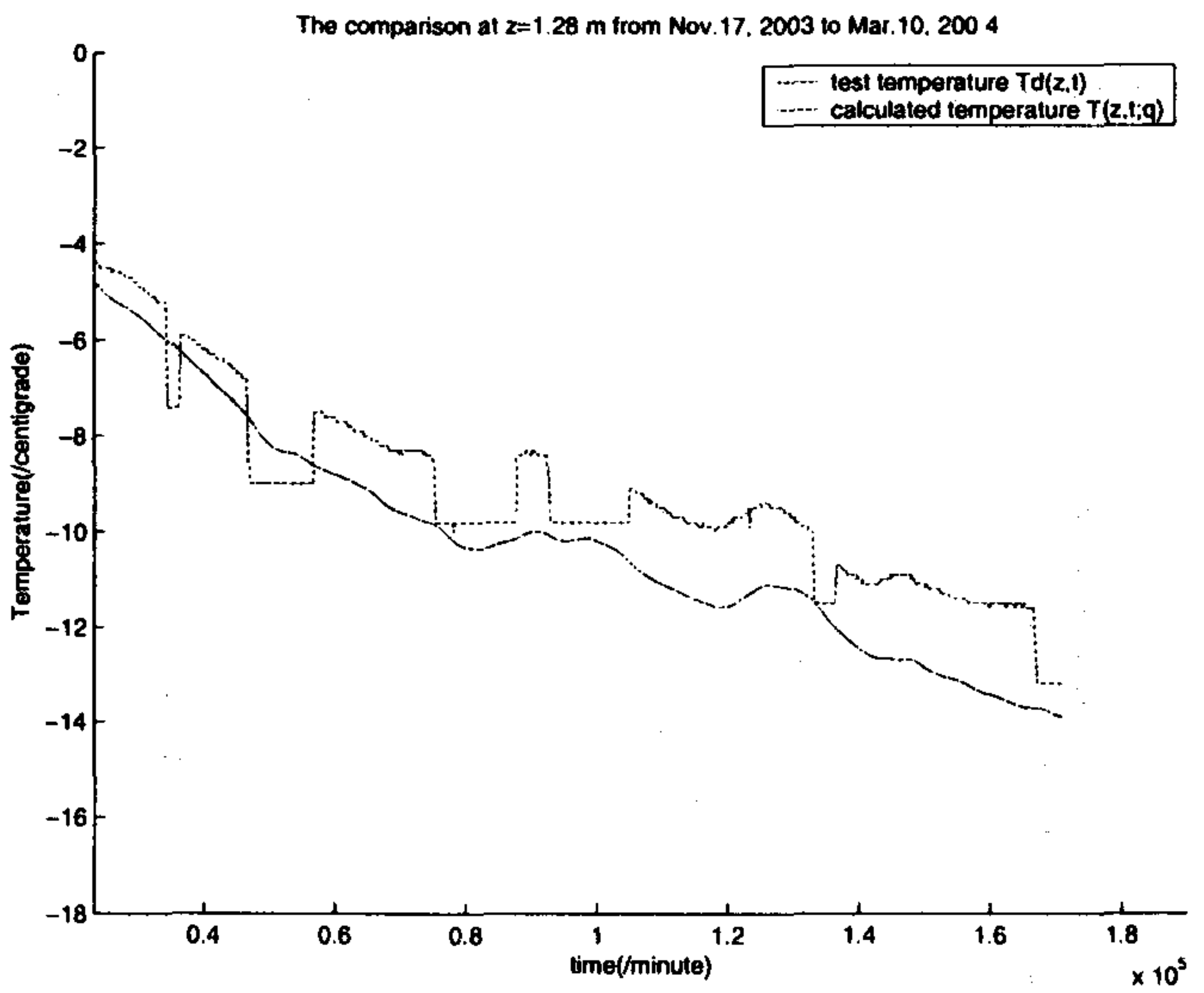

Fig. 4 The comparisons at location $z_k = 1.28\text{m}$

From Fig. 3 and Fig. 4, we find the error between the calculated temperature and test temperature is less than 1^oC at most of time points. Therefore, our algorithm and numerical simulation are valid.

6. Conclusions

This paper studies the multi-domain coupled system of one dimensional Arctic temperature field, and establishes identification model about the thermodynamic parameters of sea ice (heat storage capacity, density and conductivity). The uniqueness and continuity of the weak solution of this system are described.

Moreover, the identifiability of the identification model has been established and necessary optimality condition is proposed. On the other hand, the optimal algorithm is constructed. Furthermore, the numerical simulation is carried out, and the numerical results of temperature distribution of Arctic sea ice are shown in this paper.

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