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A Feasible Two-Step Estimator for Seasonal Cointegration†

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Abstract

This paper considers a feasible two-step estimator for seasonal cointegration as the extension of Brüggemann and Lütkepohl (2005). It is shown that the reduced-rank maximum likelihood(ML) estimator for seasonal cointegration can still produce occasional outliers as that for non-seasonal cointegration even though the sizes of them are not extreme as those in non-seasonal cointegration. The ML estimator(MLE) is compared with the two-step estimator in a small Monte Carlo simulation study and we find that the two-step estimator can be an attractive alternative to the MLE, especially, in a small sample.

Keywords: Reduced-rank estimation; error correction model; cointegrating vector.

1. Introduction

In cointegration analysis, the reduced-rank(RR) maximum likelihood(ML) approach has been the prevalent method for estimating the cointegration parameters in vector error correction models; see Johansen (1996) for the ML approach in non-seasonal cointegration and Lee (1992), Ahn and Reinsel (1994) and Johansen and Schaumburg (1999) for that in seasonal cointegration. Its popularity is occurred by its sound theoretical basis, computational simplicity and superior performance relative to some other estimators (Brüggemann and Lütkepohl, 2005: henceforth, BL). However, potentially poor small-sample performances of the ML estimator(MLE) were pointed out by several earlier works, especially, in non-seasonal cointegration analysis.

Among other, Phillips (1994) showed that finite-sample moments of the MLE do not exist and Gonzalo (1994) and Hansen *et al.* (1998) found that the small-sample properties of the MLE are not well approximated by its asymptotic distribution and in particular, that the MLE produces occasional outliers which are far away from the true parameter values. In this respect, BL considered a simple feasible two-step (or generalized least squares) estimator which does not produce the kind of outlying estimates observed for the MLE.

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In seasonal cointegration analysis, the two-step estimator has not attracted much attention from applied researchers in the past because it is at best used as an initial estimator for RR ML estimation by Ahn and Reinsel (1994). In this paper, we consider a feasible two-step estimator for seasonal cointegration as the extension of BL. We compare this with the MLE for seasonal cointegration and find through Monte Carlo simulations that it can also be an attractive alternative to the MLE in that it does not produce the outlying estimates.

The paper is structured as follows. In section 2, the model is presented and estimation procedures for seasonal cointegration are described. In section 3, Monte Carlo simulations are conducted for comparison of the MLE and the two-step estimator. Conclusions are drawn in section 4.

2. The Model and Estimations of Seasonal Cointegration

2.1. The model

We consider a vector autoregressive (VAR) model for an n-dimensional process X_t satisfying

$$\Pi(L)X_t = \left(I_n - \sum_{j=1}^{p} \Pi_j L^j \right) X_t = \Phi D_t + \varepsilon_t, \tag{2.1}$$

where ε_t are *iid.* $N_n(0, \Omega)$, D_t is a deterministic term that may contain a constant, a linear term or seasonal dummies and I_n denotes an $n \times n$ identity matrix. We assume that the initial values $X_0, \ldots, X_{-p+1}$ are fixed and that the roots of the determinant $|\Pi(z)| = 0$ are on or outside the unit circle.

For brevity, it is assumed that X_t are observed on a quarterly basis and $\Phi D_t = 0$. Models with deterministic terms, $\Phi D_t \neq 0$, can be easily implemented as in Johansen and Schaumburg (1999) and Cubadda (2001). Then, as in Ahn and Reinsel (1994), if the series are cointegrated of order (1, 1) at frequencies 0, π, $\pi/2$ and $3\pi/2$, model (2.1) can be rewritten in the following error correction model(ECM):

$$\begin{aligned} \Pi^*(L)Z_t &= A_1B_1U_{t-1} + A_2B_2V_{t-1} + (A_3B_4 + A_4B_3)W_{t-1} \\ &\quad + (A_4B_4 - A_3B_3)W_{t-2} + \varepsilon_t, \end{aligned} \tag{2.2}$$

where $Z_t = (1-L^4)X_t$, $U_t = (1+L)(1+L^2)X_t$, $V_t = (1-L)(1+L^2)X_t$, $W_t = (1-L^2)X_t$, $\Pi^*(L)$ is a matrix polynomial, A_j and B_j are $n \times r_j$ and $r_j \times n$ matrices, respectively, with rank equal to r_j for $j = 1, \ldots, 4$ and $r_3 = r_4$. For a unique parameterization, we also need to normalize the B_j's such that $B_1 = [I_{r_1}, B_{10}]$, $B_2 = [I_{r_2}, B_{20}]$, $B_3 = [I_{r_3}, B_{30}]$ and $B_4 = [O_{r_3}, B_{40}]$ where O_{r_j} is a $r_j \times r_j$ matrix of zeros and B_{j0} is a matrix of unknown parameters. Note that r_1, r_2 and r_3 (r_4) denote the cointegrating(CI) rank at frequencies 0, π and $\pi/2$ $(3\pi/2)$, respectively and B_1U_t, B_2V_t, $(B_3 + B_4L)W_t$ and $(B_4 - B_3L)W_t$ are stationary processes, *i.e.*, existing CI relationships (vectors).

2.2. Reduced-rank maximum likelihood estimation

For given CI ranks r_j's, several types of RR ML estimation procedures have been considered by Lee (1992), Ahn and Reinsel (1994), Johansen and Schaumburg (1999), Cubadda (2001) and Cubadda and Omtzigt (2005), among others. Here, we adopt the ML procedure using the complex ECM of Cubadda (2001) since it can be easily implemented without using the iterative and switching procedures of the other earlier works.

Model (2.1) can be rewritten as the complex ECM:

$$X_t^{(0)} = \alpha_1\beta_1^* X_{t-1}^{(1)} + \alpha_2\beta_2^* X_{t-1}^{(2)} + \alpha_3\beta_3^* X_{t-1}^{(3)} + \Gamma(L)X_{t-1}^{(0)} + \varepsilon_t, \qquad (2.3)$$

where $X_t^{(0)} = (1-L)(1+L)(1+iL)X_t$, $X_t^{(1)} = (1+L)(1+iL)X_t/\{2(1+i)\}$, $X_t^{(2)} = (1-L)(1+iL)X_t/\{2(1-i)\}$, $X_t^{(3)} = (1-L^2)X_t/(2i)$, α_m and β_m are complex-valued $n\times r_m$ matrices with rank equal to r_m for $m = 1,2,3$, β_m^* denotes the conjugate transpose of β_m and $\Gamma(L)$ is a matrix polynomial. ECM (2.3) is related to (2.2) through the relationships:

$$B_1 = \text{real}(\tilde\beta_1^*), \quad B_2 = \text{real}(\tilde\beta_2^*), \quad B_3 = \text{real}(\tilde\beta_3^*), \quad B_4 = \text{imag}(\tilde\beta_3^*),$$
$$A_1 = \frac{\text{real}(\tilde\alpha_1)}{4}, \quad A_2 = -\frac{\text{real}(\tilde\alpha_2)}{4}, \quad A_3 = \frac{\text{real}(\tilde\alpha_3)}{2}, \quad A_4 = \frac{\text{imag}(\tilde\alpha_3)}{2},$$

where $\tilde\beta_m = \beta_m(\beta_m^{(1)})^{-1}$, $\tilde\alpha_m = \alpha_m\beta_m^{(1)*}$, $\beta_m^{(1)}$ denotes the first r_m rows of β_m for $m = 1,2,3$ and real(x) and imag(x) denote the real and the imaginary part of x, respectively. Note that

$$\tilde\beta_m = \left[I_{r_m},\ \beta_m^{(2)}\left(\beta_m^{(1)}\right)^{-1}\right],$$

where $\beta_m^{(2)}$ denotes the last $n - r_m$ rows of β_m.

The MLE can be computed with the squared partial canonical correlations(SPCCs) between $X_t^{(0)}$ and $X_{t-1}^{(m)}$ for $m = 1,2,3$, adjusted for the other regressors from ECM (2.3). More specifically, from the regression of $X_t^{(0)}$ and $X_{t-1}^{(m)}$ on the other regressors, we obtain residuals $R_t^{(0)}$ and $R_t^{(m)}$, respectively. We then obtain the SPCCs by solving the eigenvalue problem

$$\left|\lambda S_{m,m} - S_{m,0}S_{0,0}^{-1}S_{0,m}\right| = 0,$$

where $S_{i,j} = \sum_{t=1}^T R_t^{(i)}R_t^{(j)*}$ for $i,j = 0, m$ and $m = 1,2,3$. For the ordered eigenvalues (SPCCs) $\hat\lambda_{1,m} > \cdots > \hat\lambda_{n,m}$ and corresponding matrix of eigenvectors $\hat V_m = (\hat v_{1,m},\ldots,\hat v_{n,m})$, normalized such that $\hat V_m^* S_{m,m}\hat V_m = I_n$, unnormalized estimators of β_m and α_m are given by $\hat\beta_m = (\hat v_{1,m},\ldots,\hat v_{r_m,m})$ and $\hat\alpha_m = S_{0,m}\hat\beta_m$ where r_m is the given CI rank. Therefore, post-multiplying by the inverse of the first r_m rows of $\hat\beta_m$ and conjugate-transposing the resulting matrix give the following normalized MLEs for CI vectors, B_{j0}'s, from the relationships between ECMs (2.2) and (2.3):

$$\hat B_{m0} = \text{real}\left(\left(\hat\beta_j^{(1)*}\right)^{-1}\hat\beta_j^{(2)*}\right), \quad \text{for } m = 1,2,3$$

and

$$\hat B_{40} = \text{imag}\left(\left(\hat\beta_3^{(1)*}\right)^{-1}\hat\beta_3^{(2)*}\right), \qquad (2.4)$$

where $\hat{\beta}_m^{(1)}$ and $\hat{\beta}_m^{(2)}$ denotes the first r_m and last $n-r_m$ rows of $\hat{\beta}_m$, respectively. Since we focus on the estimation of the CI vectors which are non-stationary parameters in model (2.2), details about estimation of A_j's, which are stationary parameters, are omitted here and hereafter.

In non-seasonal cointegration analysis, the MLE for CI vectors has no finite-sample moments (see, Phillips, 1994). This property is a consequence of the fact that it is effectively obtained by a ratio of two estimators which are needed for the normalization. In order to avoid a ratio of this type, BL suggest a feasible two-step (or generalized least squares) estimator. In seasonal cointegration, the MLE for seasonal CI vectors is still obtained by the ratio of two estimators using equation (2.4). Therefore, it is interesting to consider an extension of the feasible two-step estimator to seasonal cointegration, which is described in the next section.

2.3. A feasible two-step estimation

Ahn and Reinsel (1994) proposed an estimator for seasonal cointegration which can be viewed as a feasible two-step estimator and used it as an initial estimator for RR MLE. Define

$$C_j = \begin{cases} A_jB_j = [A_j,\ A_jB_{j0}], & \text{for } j = 1,2, \\ A_3B_4 + A_4B_3 = [A_4,\ A_3B_{40} + A_4B_{30}], & \text{for } j = 3, \\ A_4B_4 - A_3B_3 = [-A_3,\ -A_3B_{30} + A_4B_{40}], & \text{for } j = 4 \end{cases}$$

and let $\hat{\Pi}^*(L)$ and $\hat{C}_j$ be the ordinary least squares(OLS) estimator of $\Pi^*(L)$ and C_j, respectively, in model (2.2). Then, we can obtain $\hat{A}_1 = \hat{C}_{11}$, $\hat{A}_2 = \hat{C}_{12}$, $\hat{A}_3 = -\hat{C}_{14}$ and $\hat{A}_4 = \hat{C}_{13}$ as the OLS estimators for A_1, A_2, A_3 and A_4, respectively, where $\hat{C}_{1j}$ is the matrix with the first r_j columns of $\hat{C}_j$ for $j = 1, 2, 3, 4$.

Using these estimators, calculate $\tilde{Z}_t$ and $\tilde{P}_t$ as follows:

$$\tilde{Z}_t = Z_t - \hat{A}_1U_{1t-1} - \hat{A}_2V_{1t-1} - \hat{A}_4W_{1t-1} + \hat{A}_3W_{1t-2} - \sum_{j=1}^{p-4}\hat{\Pi}_j^*Z_{t-j},$$

$$\tilde{P}_t = \left[\hat{A}_1 \otimes U'_{2t-1},\ \hat{A}_2 \otimes V'_{2t-1},\ \hat{A}_4 \otimes W'_{2t-1} - \hat{A}_3 \otimes W'_{2t-2},\ \hat{A}_3 \otimes W'_{2t-1} + \hat{A}_4 \otimes W'_{2t-2}\right],$$

where U_{1t}, V_{1t} and W_{1t} are the first r_1, r_2 and r_3 components of U_t, V_t and W_t, respectively, U_{2t}, V_{2t} and W_{2t} are the last $n-r_1$, $n-r_2$ and $n-r_3$ components of U_t, V_t and W_t, respectively and $\otimes$ denotes the Kronecker product. Then, the feasible two-step estimator for seasonal CI vectors can be given by

$$\hat{b} = \left(\sum_{t=1}^{T}\tilde{P}_t'\tilde{\Omega}^{-1}\tilde{P}_t\right)^{-1}\left(\sum_{t=1}^{T}\tilde{P}_t'\tilde{\Omega}^{-1}\tilde{Z}_t\right), \tag{2.5}$$

where $\tilde{\Omega}$ is the usual residual covariance matrix from the OLS procedure, $b = (b_1', b_2', b_3', b_4')'$ and $b_j = \text{vec}(B'_{j0})$ for $j = 1, \ldots, 4$, where vec$(\cdot)$ vectorizes a matrix columnwise from left to right. This two-step estimator is asymptotically equivalent to the RR MLE (see, Ahn and Reinsel, 1994). However, the finite-sample properties of them may be very different as we will see in the following Monte Carlo simulations.

3. Monte Carlo Experiments

Monte Carlo simulations are conducted to compare finite-sample properties of the feasible two-step estimator with those of the MLE in seasonal cointegration by using three data generating processes(DGPs).

In each DGP, we generate 10,000 replications of the sample series with $T = 30$, 50, 100 and 200 and apply the ML and two-step estimations to obtain estimates of the cointegration parameters by using a correctly specified model that uses the true values for seasonal CI ranks and VAR order, p. Initial values are set to zero and the first 50 observations are truncated in order to eliminate the impact of zero start-up values.

Note that, in earlier literature for (seasonal) cointegration, Monte Carlo simulations have been conducted under various types of covariance matrices of the error term in order to check if inference (estimation) procedures are sensitive to the degree and sign of correlation among innovations. However, the results under such simulations are not reported in the paper because it is observed that they did not make any changes on relative performances between the ML and the two-step estimators.

3.1. DGP I

For the first experiment, the DGP considered is identical to the one used in Cubadda (2001):

$$(1 + L^2)X_t = \begin{pmatrix} 0 & 0 \\ \gamma_1 & 0 \end{pmatrix} X_{t-1} + \begin{pmatrix} 0 & 0 \\ 0 & -\gamma_1 \end{pmatrix} X_{t-2} + \varepsilon_t. \tag{3.1}$$

This model is seasonally cointegrated at the roots $\pm i$ with a polynomial CI vector $\{(1,\, 0)' + (0,\, -1)'L\}$ and CI rank of one, if $-2 < \gamma_1 < 0$. As in Cubadda (2001) we choose $\gamma_1 = -0.2$ where the roots of the characteristic equation $|\Pi(z)| = 0$ are $\{\pm i,\, \pm 1.1180i\}$ and for the variance of $\varepsilon_t = (\varepsilon_{1,t},\, \varepsilon_{2,t})'$ we choose $\mathrm{Var}(\varepsilon_{1,t}) = \mathrm{Var}(\varepsilon_{2,t}) = 1$ and $\mathrm{Cov}(\varepsilon_{1,t},\, \varepsilon_{2,t}) = 0.5$. Model (3.1) has the following ECM:

$$(1 + L^2)X_t = A_4(B_3 + B_4 L)X_{t-1} + \varepsilon_t,$$

where $A_4 = (A_{14},\, A_{24})'$, $B_3 = (1,\, B_{30})'$ and $B_4 = (0,\, B_{40})'$ with $A_{14} = 0$, $A_{24} = \gamma_1 = -0.2$, $B_{30} = 0$ and $B_{40} = -1$. Note that the first elements in B_3 and B_4 are not parameters to be estimated but are normalizing constants.

3.2. DGP II

For the second experiment, the DGP considered is the one used in Ahn and Reinsel (1994), that is

$$(1 - L^4)X_t = A_1 B_1 U_{t-1} + A_2 B_2 V_{t-1} + A_4 B_2 W_{t-1} - A_3 B_2 W_{t-2} + \varepsilon_t, \tag{3.2}$$

where $A_1 = (A_{11}, A_{21})' = (0.6, 0.6)'$, $A_2 = (A_{12}, A_{22})' = (-0.4, 0.6)'$, $A_3 = (A_{13}, A_{23})' = (0.6, -0.6)'$, $A_4 = (A_{14}, A_{24})' = (0.4, -0.8)'$, $B_1 = (1, B_{10})' = (1, -0.7)'$, $B_2 = (1, B_{20})' = (1, 0.4)'$, $\mathrm{Var}(\varepsilon_{1,t}) = \mathrm{Var}(\varepsilon_{2,t}) = 1$ and $\mathrm{Cov}(\varepsilon_{1,t}, \varepsilon_{2,t}) = 0.5$. The model is contemporaneously cointegrated at the roots $\pm i$ and -1 with a CI vector $B_2 = (1, 0.4)'$ and also (non-seasonally) cointegrated at the root 1 with a CI vector $B_1 = (1, -0.7)'$. We note that the first components in B_1 and B_2 are normalizing constants and the roots of characteristic equation are $\{\pm 1, \pm i, -1.336, 1.344, 0.117 \pm 1.494i\}$.

3.3. DGP III

The third DGP is an extension of DGP I to 3-dimensional process:

$$(1+L^2)X_t = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \gamma_2 & 0 & 0 \end{pmatrix} X_{t-1} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \gamma_2 & -\gamma_2 \end{pmatrix} X_{t-2} + \varepsilon_t, \tag{3.3}$$

where $\varepsilon_t \overset{iid}{\sim} N_3(0, \Omega)$. The parameters are set at the following values:

$$\gamma_2 = -0.2 \quad \text{and} \quad \Omega = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1 & 1 & \rho\sigma_2 \\ \rho\sigma_1\sigma_2 & \rho\sigma_2 & \sigma_2^2 \end{pmatrix},$$

where $(\sigma_1^2, \sigma_2^2) = (0.5,\ 2)$ and $\rho = 0.5$. Model (3.3) has the following ECM:

$$(1+L^2)X_t = A_4(B_3 + B_4 L)X_{t-1} + \varepsilon_t,$$

where $A_4 = (A_{14}, A_{24}, A_{34})'$, $B_3 = (1, B_{30,1}, B_{30,2})'$ and $B_4 = (0, B_{40,1}, B_{40,2})'$ with $A_{14} = 0$, $A_{24} = 0$, $A_{34} = \gamma_2 = -0.2$, $B_{30,1} = 0$, $B_{30,2} = 0$, $B_{40,1} = 1$ and $B_{40,2} = -1$. We note that X_t is seasonally cointegrated at the roots $\pm i$ with a polynomial CI vector $\{(1, 0, 0)' + (0, 1, -1)'L\}$ and CI rank of one.

3.4. Results of Monte Carlo simulations

We compare different aspects of estimation precision of the two estimators on the basis of various criteria including standard measures such as the mean bias and the mean squared error(MSE). The criteria include other characteristics of the empirical distribution of estimators: the median bias and the sample dispersion, which is measured by the interquartile range $\mathrm{IQR}_{50} = q_{75} - q_{25}$ (q_i is the i^{th} quantile of the empirical distribution). As pointed out by Gonzalo (1994), these are more fair and effective criteria for estimators that do not have the finite-sample.

The results for DGP I are summarized in Table 3.1. We observe that extremely outlying MLEs do not exist in terms of MSE, as in non-seasonal cointegration by BL. However, the two-step estimator is superior to MLE in terms of MSE and IQR_{50} for all considered sample sizes. Especially, this phenomenon is more distinguished in terms of MSE for $T = 30$ and 50. In terms of bias in mean or in median, superiority between two

Table 3.1: Comparison of performances of the ML and two-step estimators for CI vectors in DGP I (10,000 replications)

		$\hat{B}_{30}$		$\hat{B}_{40}$	
T		MLE	Two-Step	MLE	Two-Step
30	Bias in mean	0.0338	0.0462	0.0162	−0.0114
	Bias in median	0.0340	0.0445	0.0060	−0.0053
	MSE	0.7483	0.0792	0.2001	0.0806
	IQR_{50}	0.2815	0.2638	0.2753	0.2662
50	Bias in mean	0.0167	0.0276	0.0160	−0.0062
	Bias in median	0.0181	0.0257	0.0044	−0.0045
	MSE	0.0800	0.0364	0.5269	0.0361
	IQR_{50}	0.1943	0.1870	0.1943	0.1880
100	Bias in mean	0.0042	0.0109	0.0085	−0.0018
	Bias in median	0.0057	0.0109	0.0039	−0.0009
	MSE	0.0144	0.0111	0.0123	0.0110
	IQR_{50}	0.1083	0.1062	0.1069	0.1056
200	Bias in mean	0.0022	0.0044	0.0027	−0.0004
	Bias in median	0.0023	0.0037	0.0015	0.0002
	MSE	0.0030	0.0030	0.0030	0.0030
	IQR_{50}	0.0587	0.0584	0.0583	0.0585

estimators depends on $\hat{B}_{30}$ and $\hat{B}_{40}$ and it is remarkable that the MLE shows outlying estimates in sample size $T = 200$ where it has bias in mean (bias in median) about 7.62 times (7 times) larger than the corresponding two-step estimator.

In Table 3.2, we observe similar results to those of DGP I. The newly observed points are that, for sample sizes, $T = 30$, 50 and 100, the two-step estimator dominates the MLE independently of $\hat{B}_{j0}$ for $j = 1, \ldots, 4$, in terms of almost all considered performance criteria and the MLE shows extremely outlying estimates comparatively to the corresponding two-step estimator, especially, in sample size $T = 30$. In $T = 200$, the two-step estimator shows an outlying estimate which is about 44 times larger than the MLE in terms of bias in median. It is interesting that we can not observe extremely outlying MLE for (non-seasonal) CI vectors at the unit root 1, as in BL.

In Table 3.3, we can also observe similar results to those of DGPs I and II but the remarkable point is that, in terms of biases in mean and median, the MLE dominates the two-step estimator in cases of sample sizes $T = 50$, 100 and 200, independently of $\hat{B}_{i0,j}$ for $i = 3,\ 4$ and $j = 1, 2$.

In conclusion, the results indicate that the RR MLE for seasonal cointegration may still produce rather outlying estimates of the cointegration parameters, especially, in terms of MSE and IQR_{50}, even if the outlying sizes are not large as those in non-seasonal cointegration analysis.

Table 3.2: Comparison of performances ($\times 10^{-2}$) of the ML and two-step estimators for CI vectors in DGP II (10,000 replications)

		$\hat{B}_{10}$		$\hat{B}_{20}$	
T		MLE	Two-Step	MLE	Two-Step
30	Bias in mean	6.0283	0.0034	−0.5386	−0.0421
	Bias in median	0.4094	−0.0099	−0.1119	−0.0156
	MSE	2418.7145	0.1019	0.8796	0.0269
	IQR_{50}	5.3373	2.0580	2.8202	1.1001
50	Bias in mean	0.3607	0.0138	−0.0848	−0.0146
	Bias in median	0.0991	0.0110	−0.0312	−0.0040
	MSE	0.1152	0.0275	0.0224	0.0076
	IQR_{50}	2.3134	1.2823	1.2814	0.6787
100	Bias in mean	0.0897	0.0123	−0.0198	−0.0078
	Bias in median	0.0295	0.0019	−0.0055	−0.0069
	MSE	0.0121	0.0063	0.0032	0.0018
	IQR_{50}	0.9821	0.6942	0.5318	0.3671
200	Bias in mean	0.0160	0.0022	−0.0053	−0.0032
	Bias in median	0.0117	0.0020	−0.0030	−0.0021
	MSE	0.0022	0.0016	0.0006	0.0005
	IQR_{50}	0.4518	0.3785	0.2401	0.1943

		$\hat{B}_{30}$		$\hat{B}_{40}$	
T		MLE	Two-Step	MLE	Two-Step
30	Bias in mean	−0.0704	0.0262	0.1655	0.0086
	Bias in median	0.0051	0.0292	0.0682	0.0009
	MSE	0.0804	0.0155	0.0857	0.0162
	IQR_{50}	2.8906	1.1622	2.9445	1.1508
50	Bias in mean	−0.0318	0.0048	0.0661	0.0061
	Bias in median	−0.0187	0.0107	0.0362	0.0063
	MSE	0.0171	0.0054	0.0171	0.0057
	IQR_{50}	1.3753	0.7334	1.3986	0.7570
100	Bias in mean	−0.0059	0.0038	0.0145	0.0026
	Bias in median	−0.0061	0.0023	0.0036	0.0042
	MSE	0.0030	0.0015	0.0029	0.0016
	IQR_{50}	0.5968	0.4139	0.6077	0.4126
200	Bias in mean	−0.0030	0.0041	0.0044	0.0049
	Bias in median	−0.0023	0.0024	0.0001	0.0044
	MSE	0.0006	0.0004	0.0006	0.0004
	IQR_{50}	0.2785	0.2234	0.2786	0.2251

Table 3.3: Comparison of performances of the ML and two-step estimators for CI vectors in DGP III (10,000 replications)

		$\hat{B}_{30,1}$		$\hat{B}_{30,2}$	
T		MLE	Two-Step	MLE	Two-Step
30	Bias in mean	0.1193	0.1423	0.1226	0.1071
	Bias in median	0.1233	0.1511	0.1095	0.1030
	MSE	2.2950	0.3249	1.4098	0.1681
	IQR_{50}	0.8544	0.6179	0.6746	0.4455
50	Bias in mean	0.0682	0.1226	0.0782	0.0815
	Bias in median	0.0800	0.1207	0.0693	0.0776
	MSE	1.9564	0.2285	1.5576	0.1246
	IQR_{50}	0.7804	0.5411	0.6410	0.4022
100	Bias in mean	0.0198	0.0776	0.0233	0.0459
	Bias in median	0.0258	0.0702	0.0254	0.0446
	MSE	0.6707	0.1161	0.5232	0.0688
	IQR_{50}	0.5160	0.3909	0.4258	0.3001
200	Bias in mean	−0.0081	0.0290	0.0078	0.0237
	Bias in median	0.0013	0.0247	0.0106	0.0228
	MSE	0.0813	0.0399	0.0583	0.0258
	IQR_{50}	0.2634	0.2344	0.2149	0.1853

		$\hat{B}_{40,1}$		$\hat{B}_{40,2}$	
T		MLE	Two-Step	MLE	Two-Step
30	Bias in mean	0.6100	0.6726	−0.5279	−0.5954
	Bias in median	0.6147	0.6815	−0.5458	−0.6113
	MSE	2.9158	0.7799	2.0315	0.5423
	IQR_{50}	0.9131	0.6636	0.7430	0.5264
50	Bias in mean	0.4023	0.5329	−0.3365	−0.4774
	Bias in median	0.3934	0.5276	−0.3380	−0.4786
	MSE	3.5984	0.5399	1.9904	0.3832
	IQR_{50}	0.8091	0.6161	0.6744	0.4992
100	Bias in mean	0.0974	0.3248	−0.0769	−0.2907
	Bias in median	0.1192	0.3051	−0.1009	−0.2723
	MSE	0.5786	0.2342	0.4503	0.1694
	IQR_{50}	0.5060	0.4497	0.4143	0.3775
200	Bias in mean	−0.0033	0.1416	0.0055	−0.1268
	Bias in median	0.0207	0.1241	−0.0188	−0.1122
	MSE	0.1269	0.0664	0.0857	0.0478
	IQR_{50}	0.2553	0.2657	0.2133	0.2216

4. Conclusions

This paper considers a feasible two-step estimator for seasonal cointegration. It is shown that the RR MLE for seasonal cointegration can produce occasional outliers, similarly to that for non-seasonal cointegration. Through a small Monte Carlo simulation, it is found that the two-step estimation can be an attractive alternative to the ML estimation, especially, in a small sample.

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