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Blu-ray 디스크 드라이브 시스템 트래킹 서보시스템에 대한 견실비약성 H^{∞} 상태궤환 제어기 설계

(Robust and Non-fragile H^{∞} Controller Design for Tracking Servo of Blu-ray disc Drive System)

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요 약

본 논문에서는 blu-ray 디스크 드라이버의 트래킹 서보시스템에 대하여 플랜트와 제어기의 불확실성을 보상하는 견실비약성 H^{∞} 상태궤환 제어기 설계방법을 제안한다. 플랜트와 제어기의 불확실성을 매개변수화 선형행렬부등식(PLMI: parameterized linear matrix inequality)을 이용하여 구조화된 불확실성의 형태로 표현하며, Lyapunov 함수를 이용하여 구조적인 제어기의 이득섭동을 고려한 견실비약성 H^{∞} 상태궤환 제어기가 존재할 충분조건 및 제어기 설계방법을 PLMI의 형태로 제안한다. 또한, 완화기법(relaxation technique)을 통하여 PLMI를 유한개의 LMI의 형태로 변환하여 견실하고 최적화된 제어기 이득과 제어기 섭동 범위를 계산하고, 모의실험을 통해서 제시된 제어기의 타당성 및 견실성(robustness)과 비약성(non-fragility)을 검증한다.

Abstract

In this paper, we describe the synthesis of robust and non-fragile H^{∞} state feedback controllers for linear systems with affine parameter uncertain tracking servo system of blu-ray disc drive, as well as static state feedback controller with polytopic uncertainty. Similarity any other control system, the objective of the track-following system design for optical disc drives is to construct the system with better performance and robustness against modeling uncertainties and various disturbances. Also, the obtained condition can be rewritten as parameterized linear matrix inequalities (PLMIs), that is, LMIs whose coefficients are functions of a parameter confined to a compact set. We show that the resulting controller guarantees the asymptotic stability and disturbance attenuation of the closed loop system in spite of controller gain variations within a resulted polytopic region.

Keywords: Parameterized LMI, Robust, Non-fragile, Tracking, Blu-ray disc drive system.

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Ⅰ. Introduction

Blu-ray disc drive is a new developed type of compact disc(CD) and digital versatile disc(DVD). DVD drives use red laser, but blu-ray disc drives use blue laser. So, blu-ray disc drives use large storage than DVD. blu-ray disc drive is widely used optical storage medium for storage large amount of digital audio and video date. Blu-ray drive needed high speed and high accuracy, so blu-ray drive tracking is difficult problem. The optical spot must follow the track within $0.32\mu m$ reside tracking error in the face of disturbance.

Most plants in the industry have severe non-linearity and uncertainties. Thus, they post additional difficulties to the control theory of general non-linear systems and the design of their controllers. It is generally known that feedback systems designed for robustness with respect to plant parameters, or for optimization of a single performance measure, may require very accurate controllers[1]. However, in practice, controller do have a certain degree of variations due to finite word length and round-off errors in any digital systems, the imprecision inherent in analog systems and need for additional tuning of parameters in the final controller implements. Therefore, it is necessary that any controller should be able to tolerate some uncertainty in the controller as well as in the plant[1~9].

Cho et al.[10] proposed a robust and non-fragile H^{∞} controller design method for uncertain systems. Also the sufficient condition of controller existence, the design method of robust and non-fragile H^{∞} static state feedback controller, and the region of controllers which satisfies non-fragility are presented. The sufficient condition is presented using PLMIs, that is, LMIs whose coefficients are functions of a parameter confined to a compact set. However, in contrast to LMIs, PLMI feasibility problems involve infinitely many LMIs hence are inherently difficult to solve numerically. Therefore PLMIs are transformed into finitely many LMI problems using relaxation techniques[11~12].

In this paper is organized as follows. The modeling of tracking servo system is described in Section Ⅱ, the definition of PLMI and basic lemma and control structure are described in Section Ⅲ. Finally, Section Ⅳ and Ⅴ discusses simulation results for optical drive tracking servo problem.

Ⅱ. Tracking Modeling

Requirements of acceleration capability and operating range of Fig. 1. dictate the bandwidth and physical size of an actuator. With a single-stage actuator, it is almost impossible to meet both requirements in optical disc drives. In most cases, therefore, a compound actuator composed of a high-bandwidth fine actuator mounted on top of a large coarse actuator is used in optical disc drive systems. The coarse actuator provides a large operating range at the sacrifice of bandwidth. In contrast, the fine actuator, with a much smaller structure and a limited range, is capable of following high-frequency commands. Because the role of coarse actuator in track-following mode is simply to move the fine actuator slowly over the operating range, tracking performance is almost entirely dependent on how accurately the fine actuator is controlled.

Therefore, the literature usually considers only the fine actuator in the design of the track-following controller.

Fig. 2. shows a schematic view of an optical disc drive mechanism. A voice-coil motor and stepping

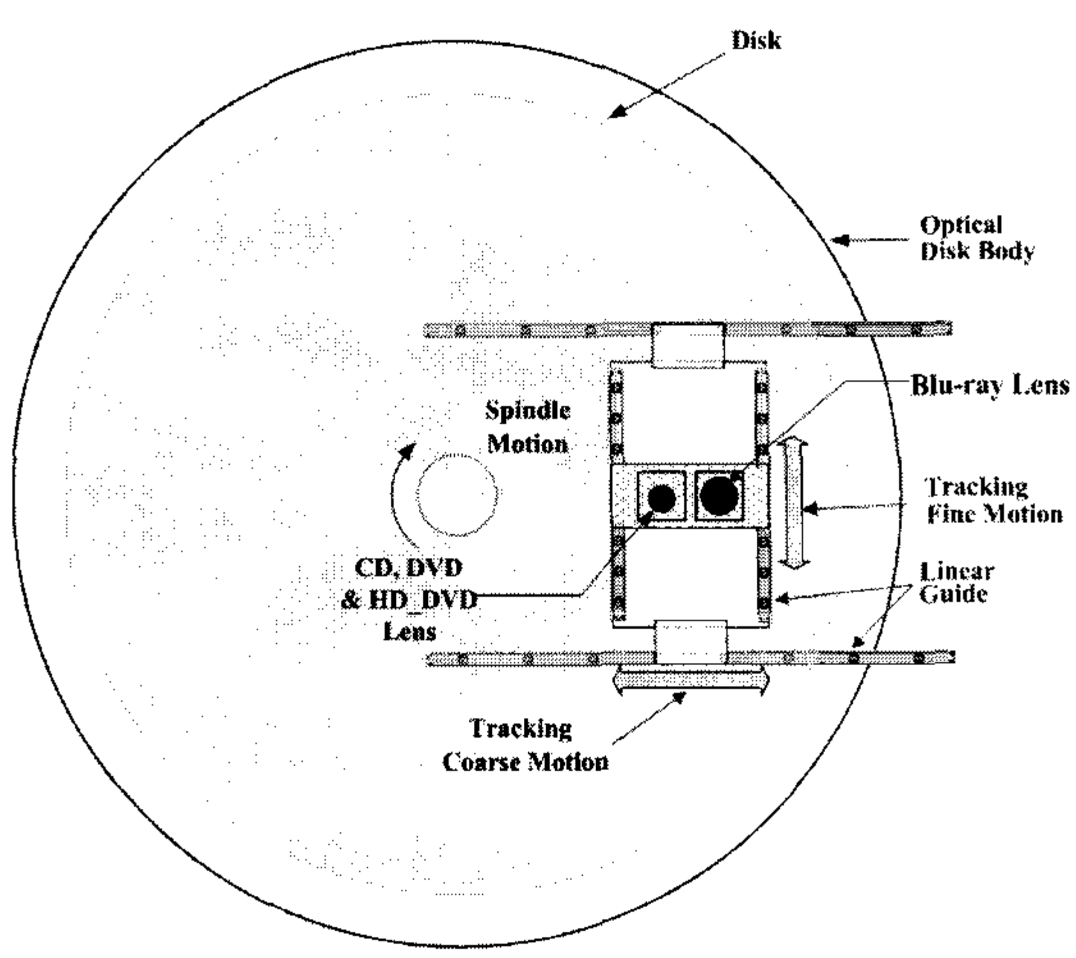

그림 1. 광학 디스크 드라이브 구조
Fig. 1. Optical disc drive structure.

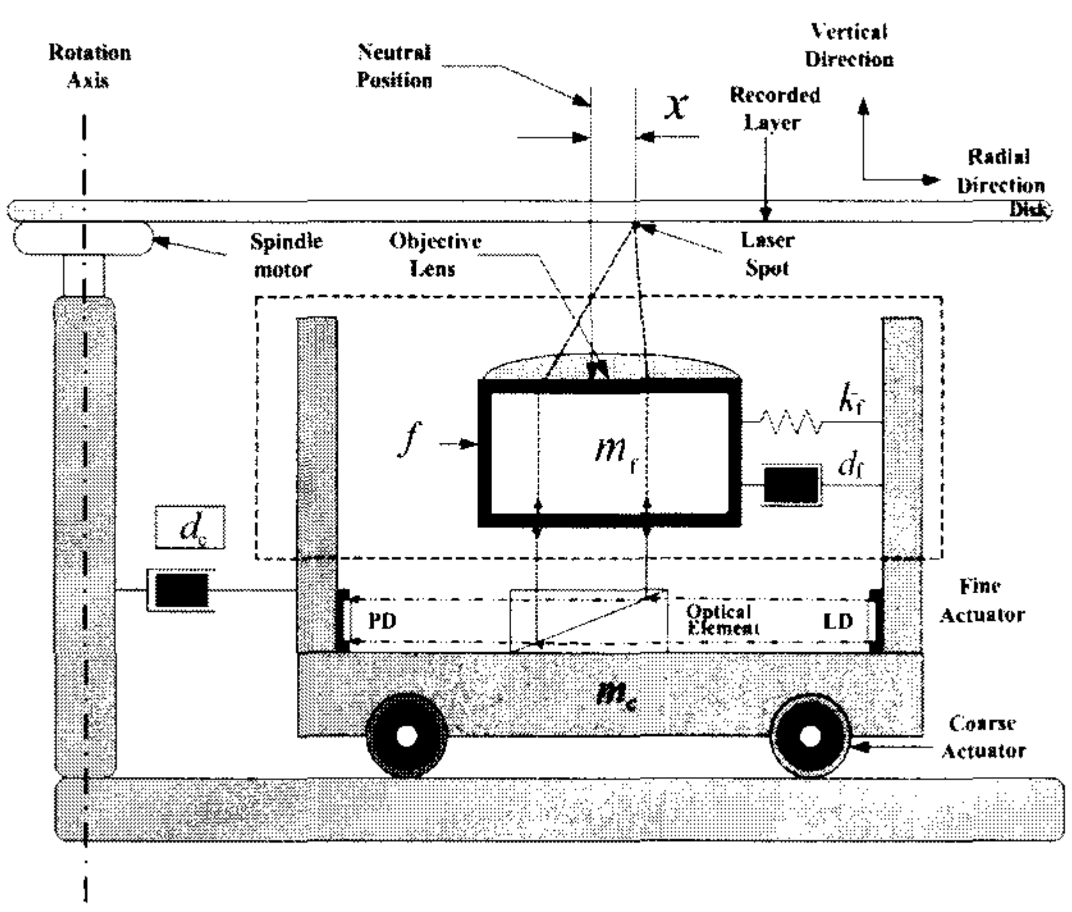

그림 2. 광학 디스크 드라이브 구조
Fig. 2. Optical disc drive structure.

motor are utilized as the fine actuator and coarse actuator. A linear system model is usually used to describe the track-following fine actuator. In this pater, the tracking actuator is modeled as a second-order nominal plant as follow:

$$P_0(s) = \frac{K_{act}\omega_0^2}{s^2 + 2\zeta_0\omega_0 s + \omega_0^2} \quad (1)$$

The second order nominal tracking actuator (1) is not included such as high resonance frequency, nonlinear characteristic, interference of focusing and tracking, and temperature characteristics. So the actual tracking actuator (2)

$$P_0(s) = \frac{[K_{act}^-, K_{act}^+][\omega_0^-, \omega_0^+]^2}{s^2 + 2\zeta_0[\omega_0^-, \omega_0^+]s + [\omega_0^-, \omega_0^+]^2} \quad (2)$$

The tracking servo system (2) is represented by a linear time-varying delayed system with affine parameter uncertainties by transforming

$$\begin{aligned} \dot{x}(t) &= A(t,\xi)x(t) + A_d(t,\xi)x(t-\tau(t)) \\ &+ B_1(t,\xi)\omega(t) + B_2(t,\xi)u(t),\ z(t) = C(t,\xi)x(t) \end{aligned} \quad (3)$$

where, $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the control input, $w(t) \in R^L$ is the disturbance input, and $z(t) \in R^p$ is the controlled output. The system matrices $A(t,\xi)$, $B_d(t,\xi)$, $B_1(t,\xi)$, $B_2(t,\xi)$ and $C(t,\xi)$ are supposed to have appropriate dimension and the following time-varying structured uncertainties:

$$\begin{aligned} A(t,\xi) &= A_0 + \textstyle\sum_{i=1}^{L}\xi_i(t)A_i, \\ A_d(t,\xi) &= A_{d0} + \textstyle\sum_{i=1}^{L}\xi_i(t)A_{di}, \\ B_1(t,\xi) &= B_{10} + \textstyle\sum_{i=1}^{L}\xi_i(t)B_{1i}, \\ B_2(t,\xi) &= B_{20} + \textstyle\sum_{i=1}^{L}\xi_i(t)B_{2i}, \\ C(t,\xi) &= C_0 + \textstyle\sum_{i=1}^{L}\xi_i(t)C_i. \end{aligned} \quad (4)$$

To obtain a less conservative result, the uncertain system matrices are expressed as structured form. also, the time-delay is time-varying and satisfies

$$0 \le \tau(t) \le h \quad \dot{\tau}(t) \le d < 1 \quad (5)$$

Although one finds the robust H^∞ state feedback controller $u(t) = Kx(t)$, the actual controller with additive perturbations implemented is assumed as

$$u(t) = [K_0 + \Delta K(t)]x(t) = K(t,\xi)x(t) \quad (6)$$

where, $K(t,\xi)$ is the region of controller variations, and K_j is the vertices of polytope. And the region of controller variations is rewritten as

$$\begin{aligned} K(t,\xi) &= K_0 + \Sigma_{j=1}^{L}\xi_j(t)\tilde{K}_j(t),\ \tilde{K}_j = K_j - K_0, \\ &\xi_j(t) \ge 0,\ \Sigma_{j=1}^{L}\xi_j(t) = 1,\ j = 1,2,\cdots,L. \end{aligned} \quad (7)$$

Here, the value of $\tilde{K}_j$ indicates the measure of non-fragility against controller gain variations. System (3) without time-delay is transformed to the closed loop system of affine form as

$$\dot{x}(t) = [A(t,\xi) + B_2(t,\xi)\{K_0 + \Sigma_{j=1}^{L}\xi_j(t)\tilde{K}_j\}]x(t) \quad (8)$$

III. Controler Design

We consider parameterized LMIs (PLMIs), that is, LMIs depending on a parameter θ evolving in a compact set. The parameter θ can designate parameter uncertainties or system operations but virtually appears. In this case, a particular emphasis is placed on PLMIs of the form

$$M_0(z)+\sum_{i=1}^{L}\theta_i M_i(z)+\sum_{1\le i\le j\le L}\theta_i\theta_j M_{ij}(z)<0 \quad (9)$$

where, z is the decision variable, $M_i(z)$, $M_{ij}(z)$ are affine symmetric matrix-valued functions of z, and θ is a parameter confined to either the polytope

$$\theta\in\Gamma:=\{\theta=(\theta_1,\theta_2,\cdots,\theta_L): \sum_{i=1}^{L}\theta_i=1, \theta_i\ge 0, i=1,2,\cdots,L\}, \quad (10)$$

or the parameter hyper-rectangle

$$\theta\in\Gamma:=[\alpha \ \ \beta]; \ \ \alpha,\beta\in\Re^L, \ \ \alpha\ge 0,\ \beta>0,\ \alpha_i\ge 0,\ \beta_i>0,\ \ i=1,2,\cdots,L \quad (11)$$

where, α_i and β_i are elements of vector α,β each other.

However, PLMI feasibility problems involve an infinite amount of LMIs according to the variations of parameters, hence are very difficult to solve numerically. Computational efforts for locating feasible points are expected to be much greater than those of LMIs. In this paper, we use relaxation techniques where PLMIs are replaced by a finite number of LMIs. Such approaches are potentially conservative but often provide practically exploitable solutions of difficult problems with a reasonable computational effort.

Lemma 1: The PLMI problem (9) and (10) has a solution z whenever the following quadratic conditions hold,

$$x^T M_0(z)x+\sum_{i=1}^{L}\theta_i x^T M_i(z)x + \sum_{1\le i\le j\le L}\max\left\{-x^T M_{ij}(z)x\cdot\left(\frac{\theta_i^2+\theta_j^2}{2}-\frac{\theta_i+\theta_j}{2}+0.125\right), x^T M_{ij}(z)x\cdot\frac{\theta_i^2+\theta_j^2}{2}\right\}<0, \quad \theta\in vert\Gamma. \quad (12)$$

The latter conditions are readily rewritten as LMIs and can be easily expressed as an LMI feasibility problem. The third term is a tight upper bound of $\theta_i\theta_j x^T M_{ij}(z)x$ with $\theta_i+\theta_j\le 1$. Therefore, if the set Γ is alternatively defined as

$$\theta\in\Gamma:=\{\theta=(\theta_1,\theta_2,\cdots,\theta_L): \sum_{i=1}^{L}\theta_i=\upsilon, \theta_i\ge 0, i=1,2,\cdots,L\}, \quad (13)$$

with $\upsilon>1$, one can use the change of variable $\overline{\theta_i}=\theta_i/\upsilon$ to recover the case $\overline{\theta_i}+\overline{\theta_j}\le 1$. Analogously, applying the change of variable $\theta_i+\theta_j\le 1$ to the constraint (11) yields the relation $\overline{\theta}\in[0\ 1]^L$.

Theorem 1: Consider the linear parameter uncertain system (3) without time-delay. If there exists the positive definite matrix Q, matrices Y_0, and $Y_j\,(j=1,2,....,L)$ such that

$$\begin{bmatrix}\Psi & B_1^T(t,\xi) & QC^T(t,\xi)\\ B_1(t,\xi) & -\rho I & 0\\ C(t,\xi)Q & 0 & -I\end{bmatrix}<0,$$

$$\Psi=QA^T(t,\xi)+A(t,\xi)Q+B_2(t,\xi)Y_0+Y_0^T B_2^T(t,\xi) +\sum_{j=1}^{L}\xi_j(t)\left[B_2(t,\xi)Y_j+Y_j^T B_2^T(t,\xi)\right]. \quad (14)$$

then the closed loop system (8) is asymptotically stable with disturbance attenuation γ and non-fragility. Here, some variables are defined as follows:

$$Q=P^{-1},\ \rho=\gamma^2,\ Y_0=K_0Q,\ \text{and}\ \ Y_j=\widetilde{K}_jQ \quad (15)$$

Proof: When Lyapunov derivative corresponding to the closed loop system with Lyapunov functional $V(x(t),t)=x^T(t)Px(t)$ is negative, suppose that the disturbance input is zero for all time. The closed loop system is asymptotically stable.

Under zero initial condition, let us introduce

$$J=\int_0^\infty[z^T(t)z(t)-\gamma^2 w^T(t)w(t)]dt \quad (16)$$

Then performance measure (16) for any nonzero

$$w(t) \in L_2[0\ \infty),$$

$$J = \int_0^{\infty}[z^T(t)z(t) - \gamma^2 w^T(t)w(t) + \frac{d}{dt}\{x^T(t)Px(t)\}]dt$$
$$-x^T(\infty)Px(\infty) \qquad (17)$$

then robust H^{∞} condition

$$\begin{bmatrix} x^T(t) & w^T(t) \end{bmatrix} \begin{bmatrix} \Xi & PB_1(t,\xi) \\ B_1^T(t,\xi)P & -\gamma^2 I \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \end{bmatrix} < 0,$$

$$\Xi = A^T(t,\xi)P + PA(t,\xi) + C^T(t,\xi)C(t,\xi)$$
$$+PB_2(t,\xi)K(t,\xi) + K(t,\xi)^T B_2^T(t,\xi)P \qquad (18)$$

implies $\|z(t)\|_2 \le \gamma \|w(t)\|_2$ for all nonzero disturbances. Also, the inequality (18) can be transformed to (14) using Schur complements and change variables in (15). ■

The proposed sufficient condition of existence for robust and non-fragile H^{∞} static feedback controller (8) is presented using PLMIs. That is LMIs whose coefficients are functions of a parameter confined to a compact set. However, in contrast to LMIs, PLMI feasibility problems involve an infinite number of LMIs, hence are transformed into finitely many LMI problems using relaxation techniques.

Theorem 2: The linear parameter uncertain system (3) is asymptotically stable with disturbance attenuation γ and non-fragility whenever there exist matrices Y_0, $Y_j(j=1,2,....,L)$, positive definite matrix Q, and positive constant ρ such that

$$x^T M_0(z)x + \sum_{i=1}^{L}\xi_i x^T M_i(z)x + \sum_{j=1}^{L}\xi_j x^T N_j(z)x$$
$$+ \sum_{1\le i\le j\le L}\max\left\{ -x^T M_{ij}(z)x \cdot \left(\frac{\xi_i^2+\xi_j^2}{2} - \frac{\xi_i+\xi_j}{2} + 0.125 \right), \right.$$
$$\left. x^T M_{ij}(z)x \cdot \frac{\xi_i^2+\xi_j^2}{2} \right\} < 0$$

$$\forall \|x\|=1,\ (\xi_i,\ \xi_j) \in vert\ \Gamma \qquad (19)$$

holds for z, $M_j(z)$, $N_j(z)$, and $M_{ij}(z)$ defined below:

$$M_0(z) = \begin{bmatrix} QA_0^T + A_0Q + B_{20}Y_0 + Y_0^T B_{20}^T & B_{10}^T & QC_0^T \\ B_{10} & -\rho I & 0 \\ C_0Q & 0 & -I \end{bmatrix},$$

$$M_i(z) = \begin{bmatrix} QA_i^T + A_iQ + B_{2i}Y_j + Y_j^T B_{2i}^T & B_{1i}^T & QC_i^T \\ B_{1i} & 0 & 0 \\ C_iQ & 0 & 0 \end{bmatrix},$$

$$N_j(z) = \begin{bmatrix} B_{20}Y_j + Y_j^T B_{20}^T & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$M_{ij}(z) = \begin{bmatrix} B_{2i}Y_j + Y_j^T B_{2i}^T & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \qquad (20)$$

Proof: Using the modified PLMI form and applying lemma 1, the proof is easily obtained. ■

Remark 1: The inequality (14) is converted to a finite number of LMI problems in terms of Q, ρ, Y_0, and $Y_j(j=1,2,....,L)$ using the relaxation technique of lemma 1. Therefore, the proposed robust and non-fragile H^{∞} state feedback controller K_0 and the region of controllers that satisfy non-fragility can be calculated from $\widetilde{K_j} = Y_j Q^{-1}(j=1,2,\cdots,L)$ after determining the LMI solutions from (18). In addition, the value of disturbance attenuation γ can be obtained by $\gamma = \sqrt{\rho}$ in (20).

Because the controller implementation using IIR filter Fig. 4 is subject to imprecision inherent in analog-digital

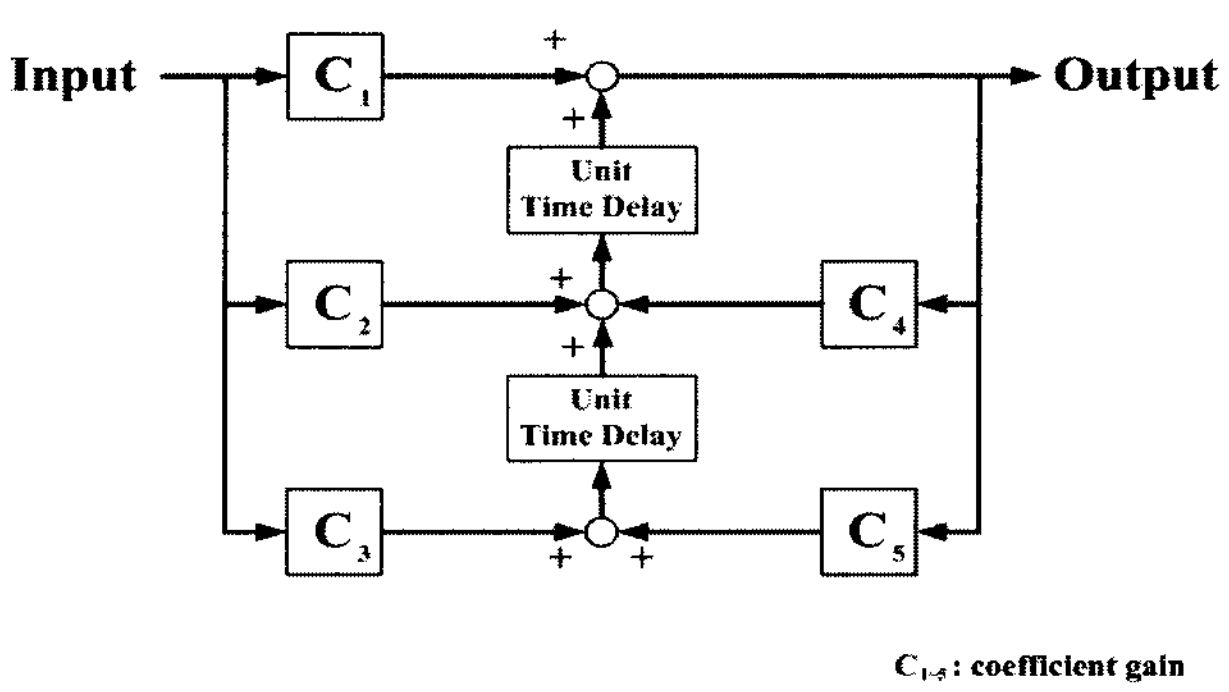

그림 4. IIR 필터를 이용한 컨트롤러 구현
Fig. 4. Controller implementation using IIR filter.

and digital-analog conversion, finite word length, finite resolution measuring instruments and round-off errors in numerical computations, as well as a useful design procedure should generate a controller which also has sufficient space for readjustment of its coefficients. The inequality (19) provides a sufficient condition for the existence of the robust controller under additive control gain perturbations of the form (7).

Remark 2: The proposed robust H^∞ controller is not fragile under additive control gain perturbations and less conservative than controller design algorithms regarding control gain perturbations as system uncertainties. Because control gain perturbations should be independent of system uncertainties, the proposed sufficient condition is less conservative than a conventional robust H^∞ controller design algorithm for a linear uncertain system.

In optical disc drives, tracking error inherently contains a significant sinusoidal disturbance as the disc rotates at a constant angular velocity. Because the sinusoidal disturbance of the disc rotational frequency has a great influence on the performance of the track-following system, the proposed control problem deals with finding a feedback controller that guarantees the internal stability of a closed-loop system and suppresses the infinity norm of the transfer functions between disturbances and controlled outputs less than a given bound.

Corollary 1: Consider the linear system with affine parameter uncertainties in (3) and the time-varying delay (5). If there exist three positive-definite matrices P, X_1, and X_2 such that

$$\begin{bmatrix} \Pi & \tilde{h}A_d(t,\xi)P & \tilde{d}_h A_d(t,\xi)P & B_1(t,\xi)P & C^T(t,\xi) \\ \tilde{h}PA_d^T(t,\xi) & -X_1 & 0 & 0 & 0 \\ \tilde{d}_h PA_d^T(t,\xi) & 0 & -X_2 & 0 & 0 \\ PB_1^T(t,\xi) & 0 & 0 & -\rho I & 0 \\ C(t,\xi) & 0 & 0 & 0 & -I \end{bmatrix} < 0,$$

$$\begin{aligned} \Pi = {} & A^T(t,\xi)P + PA(t,\xi) + A_d^T(t,\xi)P + A_d^T(t,\xi)P \\ & + K^T(t,\xi)B_2^T(t,\xi)P + PB_2(t,\xi)K(t,\xi) \end{aligned} \quad (21)$$

then the closed-loop system is asymptotically delay-dependent stable with disturbance attenuation γ and non-fragility, where $\tilde{h} = \sqrt{h^2+1}$ and $\tilde{d_h} = h/(1-d)$.

proof: In Appendix I. And using the modified PLMI form and applying lemma 1, the PLMI (21) are transformed into the LMI problems. ∎

To evaluate the robust and non-fragile H^∞ controller design method presented in this paper, we apply to the tracking servo system of Blu-ray optical disc drive system. Consider a linear system (3) with affine parameter uncertainties satisfying

$$\begin{aligned} A(t,\xi) &= A_0 + \xi_1(t)\cdot A_1 + \xi_2(t)\cdot A_2, \\ B_2(t,\xi) &= B_{20} + \xi_1(t)\cdot B_{21} + \xi_2(t)\cdot B_{22}, \end{aligned} \quad (22)$$

and parameters $\xi_1(t)$ and $\xi_2(t)$ satisfying

$$\xi \in \Gamma := \left\{ \xi = (\xi_1,\ \xi_2) : \textstyle\sum_{i=1}^{2} \xi_i(t) = 1,\ \xi_i(t) \ge 0. \right\} \quad (23)$$

System matrices in (22) are represented in Appendix II.

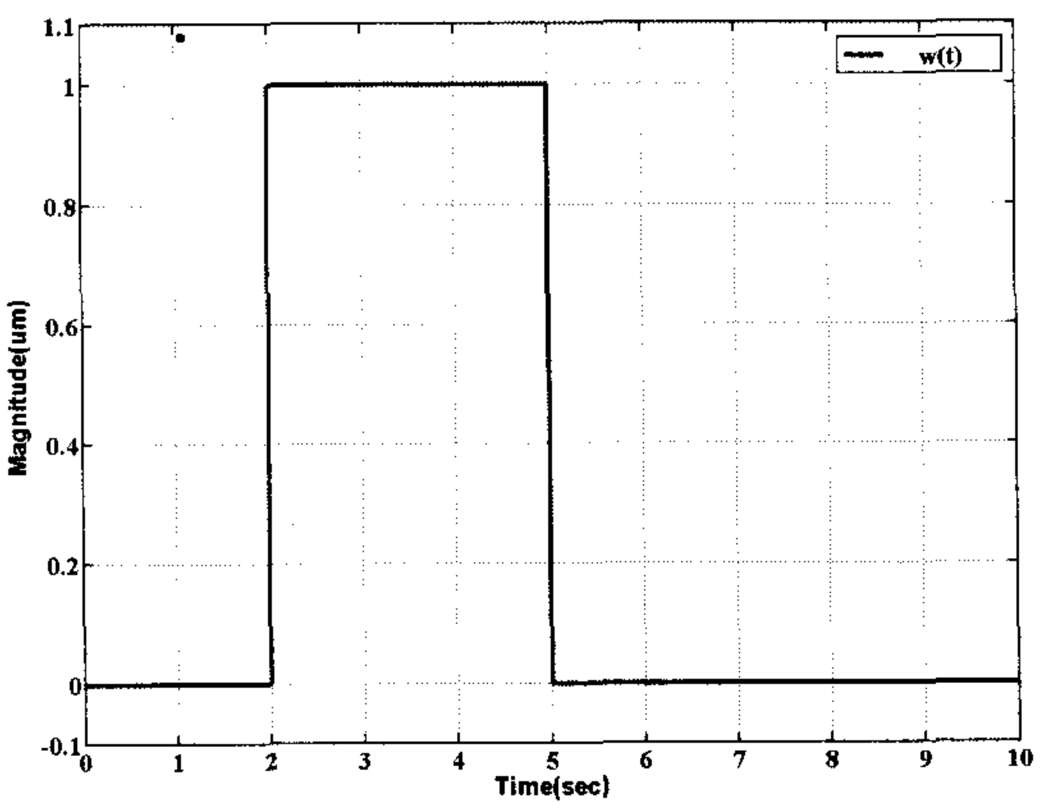

그림 5. 외란 $w(t)$에 대한 시뮬레이션

Fig. 5. Simulation result of disturbance $w(t)$.

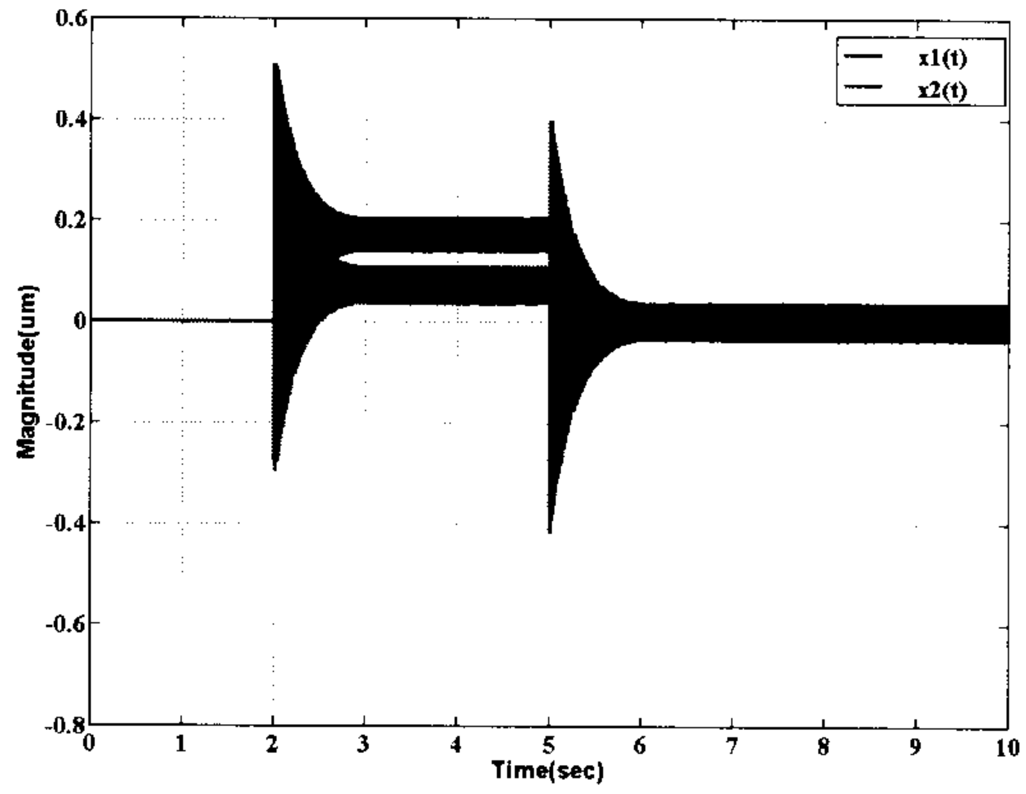

그림 6. 상태 $x(t)$에 대한 시뮬레이션
Fig. 6. Simulation result of state $x(t)$.

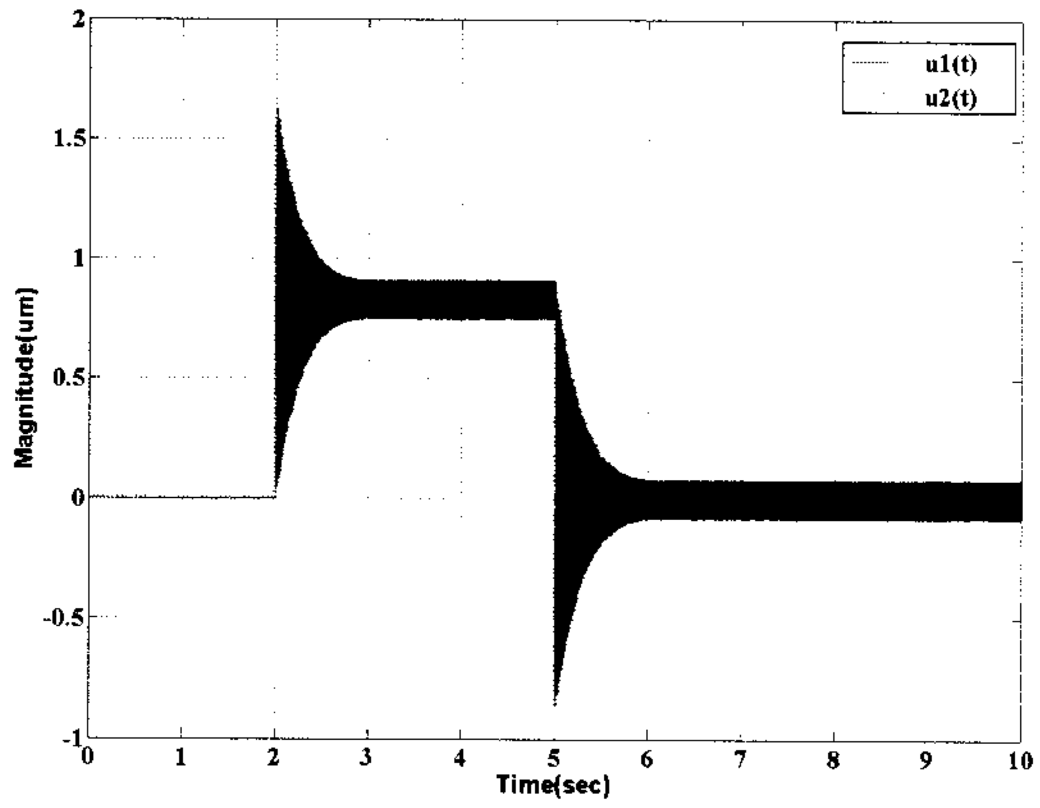

그림 7. 컨트롤러 입력 $u(t)$에 대한 시뮬레이션
Fig. 7. Simulation result of control input $u(t)$.

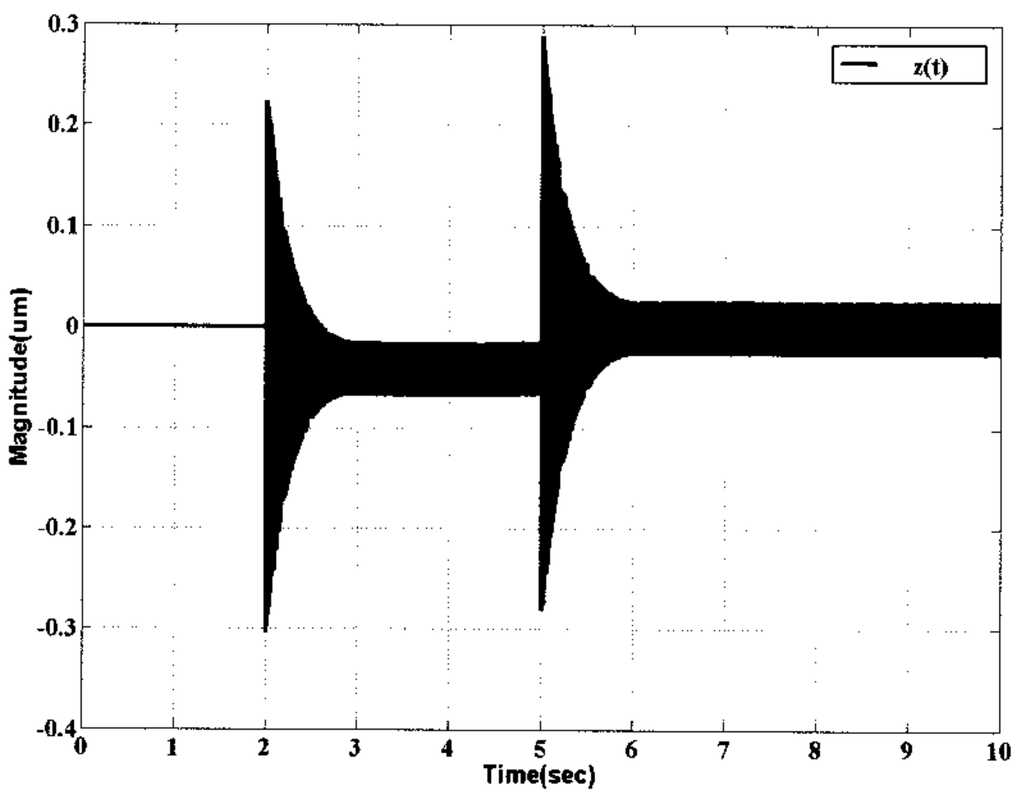

그림 8. 컨트롤러 출력 $z(t)$에 대한 시뮬레이션
Fig. 8. Simulation result of control output $z(t)$.

In Theorem 2, all solutions are obtained simultaneously as follows:

$$Q=\begin{bmatrix} 43.5917 & 33.1369 \\ 33.1369 & 42.8498 \end{bmatrix},$$

$$Y_0 = 10^3 \times [5.3933 \quad -6.0968],$$

$$Y_1 = [-66.0843 \quad -53.7548],$$

$$Y_2 = [-56.8235 \quad 46.4522] \qquad (24)$$

therefore, the robust and non-fragile H^{∞} state feedback gain, vertex of perturbation satisfying non-fragility, and the value of disturbance attenuation in a closed loop system are represented from the changes of variables (15) as follows:

$$K_0 = [562.6236 \quad -577.3742],$$

$$K_1 = [568.6157 \quad -583.2626],$$

$$K_2 = [557.4613 \quad -572.2980],$$

$$\gamma = 0.3162. \qquad (25)$$

This simulation shows that the tracking of blu-ray disc drive controller polytope guarantees the asymptotic stability. And the disturbance $w(t)$ is defined by (26).

$$w(k) = \begin{cases} 1, & \text{if } 2 \le t \le 5 \text{ sec}, \\ 0, & \text{otherwise.} \end{cases} \qquad (26)$$

Ⅳ. Conclusions

In this paper, we presented the robust and non-fragile H^{∞} controller design method for optical disc drive with affine parameter uncertainties and state feedback controller with polytopic uncertainties. Also, the robust and non-fragile tracking controller and the region of controllers which satisfies non-fragility were calculated at a time using PLMI approach.

In spite of the controller gain variations within the resulted polytopic region, the obtained robust and non-fragile H^{∞} controller guaranteed the asymptotic stability and disturbance attenuation of the closed loop system.

Appendix Ⅰ

$$x(t-\tau(t)) = x(t) - \int_{t-\tau(t)}^{t} \dot{t}(s)ds$$

$$= x(t) - \int_{t-\tau(t)}^{t} \{A(s,\xi)x(s)$$

$$+A_d(s,\xi)x(s-\tau(s)) + B_2(s,\xi)u(s)\}ds$$

$$= x(t) - \int_{t-\tau(t)}^{t} \{A(s,\xi) + B_2(s,\xi)K(t,\xi)\}x(s)ds$$

$$-\int_{t-\tau(t)}^{t} A_d(s,\xi)x(s-\tau(s))ds.$$

If we take a Lyapunov functional as

$$V(x,t) = x^T Px$$

$$+h\cdot \int_0^h \int_{t-\theta}^{t} x^T(s)A^T(s,\xi)X_1A(s,\xi)x(s)\,ds\,d\theta$$

$$+\frac{h}{(1-d)^2}\cdot \int_{\tau}^{h+\tau} \int_{t-\theta}^{\theta} x^T(s)A_d^T(s,\xi)X_2A_d(s,\xi)x(s)\,ds\,d\theta.$$

Then the first time-derivative term is obtained as

$$\dot{x}^T Px + x^T P\dot{x}$$

$$= x^T\{A^T(t,\xi)P + PA(t,\xi) + A_d^T(t,\xi)P + PA_d(t,\xi)$$

$$+K^T(t,\xi)B_2^T(t,\xi)P + PB_2(t,\xi)K(t,\xi)\}x$$

$$-2\cdot\left\|x^T PA_d(t,\xi)\int_{t-\tau(t)}^{t}\overline{A}(s,\xi)x(s)ds\right\|$$

$$-2\cdot\left\|x^T PA_d(t,\xi)\int_{t-\tau(t)}^{t}A_d(s,\xi)x(s-\tau(s))ds\right\|$$

$$\le x^T\Xi x + 2\left\|x^T PA_d(t,\xi)X_1^{-1/2}\right\|\cdot\left\|\int_{t-\tau(t)}^{t} X_1^{1/2}\overline{A}(s,\xi)x(s)ds\right\|$$

$$+2\left\|x^T PA_d(t,\xi)X_2^{-1/2}\right\|\cdot\left\|\int_{t-\tau(t)}^{t} X_2^{1/2}\cdot A_d(s,\xi)\cdot x(s-\tau(s))ds\right\|$$

$$\le x^T\Xi x + \left\|x^T PA_d(t,\xi)X_1^{-1/2}\right\|^2 + \left\|\int_{t-\tau(t)}^{t} X_1^{1/2}\overline{A}(s,\xi)x(s)ds\right\|^2$$

$$+\left\|x^T PA_d(t,\xi)X_2^{-1/2}\right\|^2 + \left\|\int_{t-\tau(t)}^{t} X_2^{1/2}\cdot A_d(s,\xi)\cdot x(s-\tau(s))ds\right\|^2$$

$$\le x^T\Xi x + \left\|x^T PA_d(t,\xi)X_1^{-1/2}\right\| + \tau(t)\int_{t-\tau(t)}^{t}\left\|X_1^{1/2}\overline{A}(s,\xi)x(s)\right\|^2 ds$$

$$+\left\|x^T PA_d(t,\xi)X_2^{-1/2}\right\|^2 + \tau(t)\int_{t-\tau(t)}^{t}\left\|X_2^{1/2}\cdot A_d(s,\xi)\cdot x(s-\tau(s))\right\|^2 ds$$

where,

$$\Xi = A^T(t,\xi)P + PA(t,\xi) + A_d^T(t,\xi)P$$

$$+PA_d(t,\xi) + K^T(t,\xi)B_2^T(t,\xi)P + PB_2(t,\xi)K(t,\xi)$$

and

$$\int_{t-\tau(t)}^{t}\left\|X_2^{1/2}\cdot A_d(s,\xi)\cdot x(s-\tau(s))\right\|^2 ds$$

$$= \int_{t-\tau(t)-\tau(t')}^{t-\tau(t)}\left\|X_2^{1/2}\left[\frac{1}{1-\dot{\tau}(\theta)}A_d(\theta)\right]_{\theta-\tau(\theta)=s} x(s)\right\|^2 ds$$

$$\le \frac{1}{1-d}\cdot\int_{t-\tau(t)-h}^{t-\tau(t)}\left\|X_2^{1/2}[A_d(\theta)]_{\theta-\tau(\theta)=s}x(s)\right\|^2 ds.$$

The second and third time-derivative terms are obtained as

$$\frac{d}{dt}\int_0^h\int_{t-\theta}^{t} x^T(s)A^T(s,\xi)X_1A(s,\xi)x(s)\,ds\,d\theta$$

$$= h\cdot x^T A^T(t,\xi)X_1A(t,\xi)x(t) - \int_{t-h}^{t} x^T(s)A^T(s,\xi)X_1A(s,\xi)ds$$

and

$$\frac{d}{ds}\int_{\tau}^{h+\tau}\int_{t-\theta}^{\theta} x^T(s)A_d^T(s,\xi)X_2A_d(s,\xi)x(s)\,ds\,d\theta$$

$$= h\cdot x^T A_d^T(t,\xi)X_2A_d(t,\xi)x - (1-\dot{\tau}(t)),$$

$$\cdot\int_{t-\tau(t)-h}^{t-\tau(t)} x^T(s)A_d^T(s,\xi)X_2A_d(s,\xi)x(s)\,ds\,d\theta$$

$$\le h\cdot x^T A_d^T(t,\xi)X_2A_d(t,\xi)x - (1-d),$$

$$\cdot\int_{t-\tau(t)-h}^{t-\tau(t)} x^T(s)A_d^T(s,\xi)X_2A_d(s,\xi)x(s)\,ds\,d\theta.$$

By using the above relations, we get

$$\frac{d}{dt}V(x,t)$$

$$\le x^T\begin{bmatrix}\Xi + (h^2+1)PA_d^T(t,\xi)X_1^{-1}A_d(t,\xi)P \\ +\left\{\dfrac{h^2}{(1-d)^2}+1\right\}PA_d^T(t,\xi)X_2^{-1}A_d(t,\xi)P\end{bmatrix}x$$

Appendix II

$$A_0 = 10^2\times\begin{bmatrix}-0.2239 & 4.7070\\ -4.7070 & -0.2474\end{bmatrix},$$

$$A_1 = \begin{bmatrix}-0.1940 & 5.3340\\ -4.0790 & -0.2144\end{bmatrix},$$

$$A_2 = \begin{bmatrix} -0.2537 & 4.0790 \\ -5.3340 & -0.2804 \end{bmatrix},$$

$$B_{20} = \text{driver IC gain} \times \begin{bmatrix} -0.3881 \\ -0.3881 \end{bmatrix},$$

$$B_{21} = \begin{bmatrix} 0.0268 \\ 0.0268 \end{bmatrix},\ B_{22} = -\begin{bmatrix} 0.0268 \\ 0.0268 \end{bmatrix},\ B_{10} = \begin{bmatrix} -1 \\ -0.9 \end{bmatrix},$$

$$C_0 = [-0.3881 \quad 0.3881],$$

$$C_1 = [0.0379 \quad 0.0379],\ C_2 = [-0.0379 \quad -0.0379].$$

Where, the matrix B_{20} amplified by driver IC gain 12 at quality factor $Q < 20\text{dB}$.

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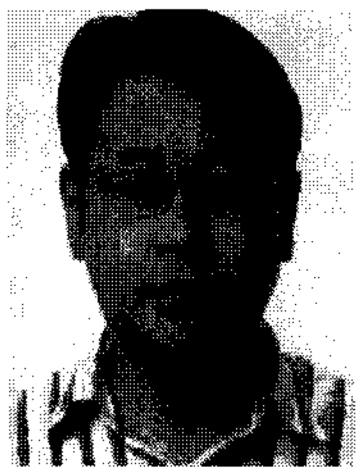

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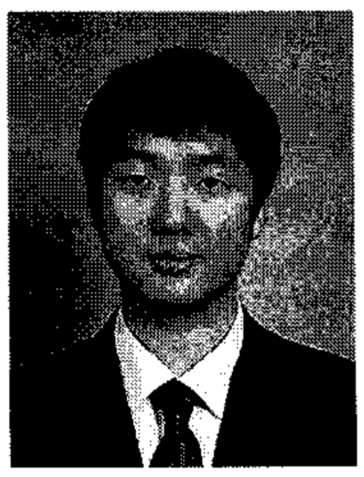

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