제어기 영점의 영향을 감소시키는 종속형 저차 제어기의 설계

論 文
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Design of Low Order Cascade Controller to Reduce the Effects of Its Zeros

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Abstract - This paper represents a design method for PID or low-order controllers cascaded with a linear plant in the unit feedback system where it is required to meet the given time response specifications such as overshoot and settling time. This problem is difficult to solve because the zeros of the controller appear in the numerator of the overall system and thus those zeros may make the time response design difficult. In this paper, we propose a new approach based on the partial model matching and the so called K-polynomial. The partial matching problem is formulated to an optimization problem in which a quadratic function of coefficient errors between a target model and the resulting closed loop system is minimized. For the sake of satisfying the closed loop stability, a set of quadratic constraints associated with the cost function is introduced. As a result, the controller designed meets both time response requirements and the closed loop stability, if any. It is shown through several examples that the present method can be easily applied to these problems.

Key Words : Partial model matching, Cascade structure, Low order controller

1. INTRODUCTION

In [1], a "practical control design problem" is referred to a feedback design problem with the conditions; (i) a relatively simple, fixed-structure controller (e.g. PID and 1st order controllers), (ii) multiple performance specifications, and (iii) robust closed-loop stability and performance. It is well known that most practical control problems do not have analytical solutions. In the same category, the present paper deals with a problem of designing low-order cascade controller that satisfies mainly the time response specifications such as overshoot limit and settling time as well as the closed-loop stability for a given linear plant. When a controller is implemented in the cascade structure, we are faced with a difficult problem that is caused by the zeros of controller itself. The reason is that the zeros of the controller must appear in the numerator of the overall closed-loop transfer function. Those zeros may be a substantial obstacle when designing a good damping response.

Kitamori [2] has developed a simple solution for this problem, in which he has applied the observation fact that the step response is dominantly affected by the coefficients of low power in s of closed-loop transfer function. He also proposed a special form of polynomial (so called "Kitamori polynomial") that can be easily used when we choose a target polynomial with good damping.

The Kitamori approach begins from approximating the rational plant model into an all pole system (APS) and generating a target transfer function in the form of APS using the Kitamori polynomial. Subsequently, the PID gains are computed by equating so that the resulting closed-loop transfer function partially matches the target model in the sense of partial model matching [3,4].

However, the Kitamori solution does not guarantee the stability. Furthermore, it does not allow us to adjust the overshoot and settling time of the target model and cannot be applied to the case where the plant has integrators.

In this paper, we suggest a new method of designing PID or low order controllers in cascade structure. The present method introduces the so called "K- polynomial" instead of the Kitamori polynomial for the purpose of composing a target characteristic polynomial. To solve the stability problem, we use a sufficient condition by Lipatov and Sokolov [5]. As will be shown, this problem is formulated by a non-convex optimization problem subject to quadratic function inequalities. As a simple algorithm for solving the optimization problem, we can use the Gloptipoly software [6], which is open source freeware based on the linear matrix inequalities (LMI)

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solver SeDuMi [11]. We will demonstrate our approach through several examples and compare the results with those by Kitamori's method.

2. THE KITAMORI METHOD AND PROBLEM STATEMENT

In this section, we first give a brief summary of the Kitamori approach and point out some problems on it. The objectives of this paper will be defined.

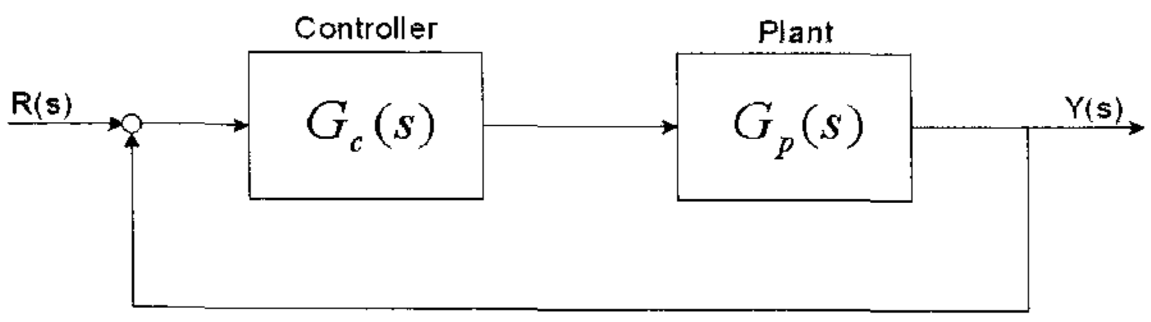

Fig. 1 Cascaded control system

Consider a unit feedback system shown in Fig. 1. We assume that the plant is a linear time invariant (LTI), single-input single-output (SISO) system with a delay free and PID controller is employed.

$$G_p(s)=\frac{N(s)}{D(s)}=\frac{b_0+b_1s+\cdots+b_ms^m}{a_0+a_1s+\cdots+s^n}\quad(m\le n-1)\tag{1}$$

$$G_c(s)=\frac{C(s)}{s}=\frac{c_0+c_1s+c_2s^2}{s}\tag{2}$$

Also, suppose that the design objective for PID controller is to meet the good time responses.

2.1 The Kitamori method

Now, the Kitamori method is summarized as follows. From (1), dividing $D(s)$ by $N(s)$, the plant can be expanded by

$$G_p'(s)=\frac{1}{D'(s)}=\frac{1}{d_0'+d_1's+\cdots}.\tag{3}$$

The reference model is expressed by

$$T^*(s)=\frac{1}{P(s)}=\frac{1}{\gamma_0+\gamma_1\sigma s+\gamma_2\sigma^2s^2+\cdots}\tag{4}$$

where γ_i's are called by the Kitamori parameters and σ is a time scaling parameter. Kitamori [2] has proposed a special values of γ_i's of which the corresponding $T^*(s)$ has relatively good damping property. The Kitamori parameters are

$$[\gamma_0\ \gamma_1\ \gamma_2\ \cdots]=[1\ 1\ 0.5\ 0.15\ 0.03\ \cdots].\tag{5}$$

The closed-loop transfer function can be written by

$$T(s)=\frac{1}{1+s(D'(s)/C(s))}.\tag{6}$$

According to the partial model matching, if we let (6) to be identical to (4), $C(s)$ in (2) becomes

$$C(s)=\frac{sD'(s)}{P(s)-1}.\tag{7}$$

Rewriting (7), it becomes

$$\begin{aligned}C(s)=\frac{d_0'}{\sigma}\Big\{&1+\Big(\frac{d_1'}{d_0'}-\sigma\gamma_2\Big)s\\&+\Big[\frac{d_2'}{d_0'}-\sigma\gamma_2\frac{d_1'}{d_0'}-\sigma^2(\gamma_2^2-\gamma_3)\Big]s^2\\&+\Big[\frac{d_3'}{d_0'}-\sigma\gamma_2\frac{d_2'}{d_0'}-\sigma^2(\gamma_2^2-\gamma_3)\frac{d_1'}{d_0'}-\sigma^3(\gamma_2^3-2\gamma_2\gamma_3+\gamma_4)\Big]s^3\\&+\cdots\Big\}.\end{aligned}\tag{8}$$

For the sake of matching (8) with (2), the coefficients higher order than 4 must be zeros. However, it is not possible to make that by only one parameter σ. Thus, in the Kitamori method, the value of σ is determined so that the fourth coefficient of (8) is equal to zero. That is, $\hat{\sigma}$ is a real root of the following polynomial.

$$\begin{aligned}c_3(\hat{\sigma})=\frac{d_3'}{d_0'}-\hat{\sigma}\gamma_2\frac{d_2'}{d_0'}-\hat{\sigma}^2(\gamma_2^2-\gamma_3)\frac{d_1'}{d_0'}\\-\hat{\sigma}_3(\gamma_2^3-2\gamma_2\gamma_3+\gamma_4)=0.\end{aligned}\tag{9}$$

Substituting this $\hat{\sigma}$ into (8), we have the PID gains.

$$\begin{aligned}c_0&=\frac{d_0'}{\hat{\sigma}},\\c_1&=\frac{d_0'}{\hat{\sigma}}\Big(\frac{d_1'}{d_0'}-\hat{\sigma}\gamma_2\Big)\\c_2&=\frac{d_0'}{\hat{\sigma}}\Big[\frac{d_2'}{d_0'}-\hat{\sigma}\gamma_2\frac{d_1'}{d_0'}-\hat{\sigma}^2(\gamma_2^2-\gamma_3)\Big]\end{aligned}\tag{10}$$

As shown in [2], Kitamori's method is very simple and provides good PID control for a certain class of plant. It has been reported that his method was successfully applied to many industrial applications.

However, we point out that there are some problems in the Kitamori method as follows.

1) The PID gains (10) cannot guarantee the stability of closed-loop system.

2) The partial model matching by (10) may be poor. The reason is that $c_k(\hat{\sigma}) \neq 0$, for $k \geq 4$ since $\hat{\sigma}$ is computed by merely (9).

3) Since the reference model is composed by the Kitamori parameters which are constant, it is very difficult to make a reference APS that has satisfactory overshoot and the desired settling time.

4) The approach cannot be applied to the case where the plant has integrators more than or equal to one. Because if the plant is the case, d_0' in (8) must be zero.

5) It is difficult for this approach to extend to the problem of designing a fixed linear controller of arbitrary order in cascade structure.

2.2 Problem statement

Consider the unit feedback system shown in Fig. 1. The objective of the paper is to propose a new method of finding PID controller (or even fixed low-order controller) that meets the following two conditions for a given LTI, SISO plant;

1) The controller shall have to meet the given time response specifications such as the maximum overshoot and the desired settling time.

2) The controller must satisfy the closed loop stability.

As mentioned in the introduction, when we consider the cascade control structure, the zeros of the controller must appear in the numerator of closed loop transfer function, The zeros may be a substantial obstacle when we design a controller with the above time response requirements. Moreover, such low-order control design problems do not have analytical solution.

3. A NEW APPROACH FOR LOW ORDER CONTROLLER DESIGN

The new approach that we will propose consists of several key elements. The basic idea is based on the partial model matching (PMM) method, in which the overall transfer function is as close to the target model as possible. When we generate a reference model that meets a given time response specifications, the K-polynomial [7] is used. We will represent briefly how to synthesize it. The partial model matching problem between the resulting closed-loop transfer function and the reference model is formulated by an optimization problem with respect to the controller parameters. For the purpose of satisfying the closed loop stability, the sufficient condition by Lipatov [5] is constrained to the optimization problem. After all, it will be shown that the design problem in section 2.2 can be formulated by a non-convex optimization problem and can be solved by using the GloptiPoly software [6].

3.1 Synthesis of a reference transfer function using the K-polynomial

Consider a polynomial with real positive coefficients.

$$\delta(s) = \delta_0 + \delta_1 s + \cdots + \delta_{n-1}s^{n-1} + \delta_n s^n,\ (\delta_i > 1) \tag{11}$$

The characteristic ratios and the generalized time constant are defined as:

$$\alpha_1 = \frac{\delta_1^{\,2}}{\delta_0\delta_2},\ \alpha_2 = \frac{\delta_2^{\,2}}{\delta_1\delta_3},\ \cdots \alpha_{n-1} = \frac{\delta_{n-1}^2}{\delta_{n-2}\delta_n},\ \tau = \frac{\delta_1}{\delta_0}. \tag{12}$$

Reversely, every coefficient δ_i of $\delta(s)$ can be represented in terms of α_ks, τ and δ_0 as follows:

$$\delta_1 = \delta_0\tau,\ \delta_2 = \frac{\delta_0\tau^2}{\alpha_1},\ \delta_i = \frac{\delta_0\tau^i}{\alpha_{i-1}\alpha_{i-2}^2\alpha_{i-3}^3\cdots\alpha_2^{i-2}\alpha_1^{i-1}}. \tag{13}$$

The K-polynomial is defined as a polynomial of which its characteristic ratios obey the following two equations;

1) $\alpha_1 > 2$,

2) $\alpha_k = \dfrac{\sin(\frac{k\pi}{n}) + \sin(\frac{\pi}{n})}{2\sin(\frac{k\pi}{n})}\alpha_1,\ (k=2, 3, \cdots, n-1)$. (14)

Therefore, a K-polynomial can be generated by choosing only α_1 for any given τ and δ_0 [see (13)]. Let $T_k(s)$ be an all-pole transfer function whose denominator is a K-polynomial $\delta_k(s)$:

$$T_k(s) = \frac{\delta_0}{\delta_k(s)} = \frac{\delta_0}{\delta_0 + \delta_1 s + \cdots + \delta_{n-1}s^{n-1} + \delta_n s^n}. \tag{15}$$

It was shown in [7] that the K-polynomial is stable and its frequency magnitude function is monotonically decreasing. Moreover, it is seen in [8] that the damping of $\delta_k(s)$ increases as α_1 increases. This means, the step response of $T_k(s)$ shall give rise to smaller overshoot as α_1 increases. On the other hand, the speed of step response of $T_k(s)$ can be exactly controlled by the generalized time constant τ [7]. The procedure synthesizing a reference model $T_k(s)$ is as follows. We first choose $\delta_0=1$ and $\tau_1=1$ arbitrary. Using (13) and (14), one can generate a K-polynomial $\delta_k(s)$ by selecting a proper α_1^*. It is easily seen from [8] that α_1 can be determined a priori so that the corresponding APS $T_k(s)$ has satisfactory overshoot. Then, from the step response of the first trial model $T_k(s)$ with $\tau_1=1$, we find its settling time. Let the settling time be t_{s1}. If the desired settling time of target model is given by t_s^*, the generalized time constant τ^* that yields the desired settling time is computed by [7]

$$\tau^*=\frac{t_s^*}{t_{s1}}\cdot\tau_1. \tag{16}$$

Finally, the denominator polynomial $\delta_k^*(s)$ of the target model $T_k^*(s)$ is obtained by using (13) with α_1^*, α_2^*, $\cdots$, α_{n-1}^* and τ^*. Thus, it is possible to construct a reference model $T_k(s)$ that satisfies the desired damping and settling time by means of choosing only two parameters (α_1, τ). This approach has been extended to the synthesis of transfer function involving a fixed numerator [9]. Let the numerator be $N(s)$. A reference model is written in the form of the followings;

$$T_N(s)=\frac{K_0N(s)}{\delta_N(s)}=\frac{N(s)}{\delta^*(s)}. \tag{17}$$

where $\delta_N(s)$ is constructed by choosing α_1 and τ. K_0 may be determined so as to $T_N(0)=K_0N(0)/\delta_N(0)=1$. $T_N(s)$ can be regarded as $N(s)T_k(s)$. However, because of the effect of $N(s)$, we need to make a compromise between response speed and damping when τ is selected. The larger τ is, the slower the time response of $T_N(s)$ is. It is generally known that the to make a system be slow response corresponds to make its overshoot be reduced. The details are referred to [9].

3.2 Design of PID controller in cascade structure

We will now formulate the PID control design problem subject to the conditions that shall satisfy both closed loop stability and the given time response requirements simultaneously. Let us consider a unit feedback system shown in Fig. 1, where the controller $G_c(s)$ to be designed is a PID type. Suppose that we have already obtained a reference model (either $T_k(s)$ or $T_n(s)$) by means of the method in section 3.1.

The closed-loop transfer function is written by:

$$T_0(s)=\frac{G_p(s)G_c(s)}{1+G_p(s)G_c(s)}=\frac{N(s)C(s)}{sD(s)+N(s)C(s)}. \tag{18}$$

According to the partial model matching, we match (18) with the reference model (17) as in

$$\frac{N(s)C(s)}{sD(s)+N(s)C(s)}\cong\frac{N(s)}{\delta^*(s)}, \tag{19}$$

where

$$\delta^*(s)=\delta_0^*+\delta_1^*s+\cdots+\delta_{l-1}^*s^{l-1}+\delta_l^*s^l. \tag{20}$$

Equivalently, (19) can be written as

$$(\delta^*(s)-N(s))C(s)\cong sD(s). \tag{21}$$

From (21), the degree of $\delta^*(s)$ is chosen by

$$\mathrm{degree}(\delta^*(s))\ge\mathrm{degree}(sD(s))-\mathrm{degree}(C(s))=n-1 \tag{22}$$

Expanding (21) and equating every coefficient in the same power of s, the following algebraic equation is obtained.

$$Ax=b \tag{23}$$

where

$$A=\begin{bmatrix}\delta_0^*-b_o & 0 & 0\\ \delta_1^*-b_1 & \delta_0^*-b_0 & 0\\ \delta_2^*-b_2 & \delta_1^*-b_1 & \delta_0^*-b_0\\ \vdots & \vdots & \vdots\\ \delta_m^*-b_m & \delta_{m-1}^*-b_{m-1} & \delta_{m-2}^*-b_{m-2}\\ \vdots & \vdots & \vdots\\ \delta_l^* & \delta_{l-1}^* & \delta_{l-2}^*\\ 0 & \delta_l^* & \delta_{l-1}^*\\ 0 & 0 & \delta_l^*\end{bmatrix}, \tag{24}$$

$$x=[c_0\ \ c_1\ \ c_2]^T,$$

$$b=[0\ \ a_0\ \ a_1\ \ a_2\ \ \cdots\ \ a_{n-1}\ \ 1]^T.$$

Obviously, (23) can not be solved analytically because the number of equations is more than that of unknown variables. Let us define an error cost function as follows:

$$\epsilon:=Ax-b, \tag{25}$$

$$J(x):=\frac{1}{2}\epsilon^T\epsilon. \tag{26}$$

The PMM problem (19) can be regarded as an optimization problem in the sense that $J(x)$ is minimized with respect to the controller parameter x. In other words, the PID control design problem satisfying condition 1) in the problem statement is reduced to the following problem:

$$\hat{x}=\min_x J(x) \tag{27}$$

Recall that the solution (27) does not guarantee the closed loop stability.

Now, we will formulate the stability condition in parameter space. Since it is very difficult to apply the Routh-Hurwitz criterion directly, we employ the sufficient condition for stability by Lipatov and Sokolov [5].

From (19), the closed-loop characteristic polynomial is

$$\begin{aligned}\delta_c(s,x) &= s\,D(s)+N(s)\,C(s)\\ &= p_0(x)+p_1(x)s+\cdots+p_{n+1}(x)s^{n+1}.\end{aligned} \tag{28}$$

Each coefficient of $\delta_c(s)$ can be described by

$$p_i(x)=h_i^{\,T}x+\zeta_i,\ \ i=0,\ 1,\ 2,\cdots,\ n+1 \tag{29}$$

where

$$M=\begin{bmatrix} b_o & 0 & 0\\ b_1 & b_0 & 0\\ b_2 & b_1 & b_0\\ \vdots & \vdots & \vdots\\ b_m & b_{m-1} & b_{m-2}\\ 0 & b_m & b_{m-1}\\ 0 & 0 & b_m \end{bmatrix}=\begin{bmatrix} h_0^T\\ h_1^T\\ \vdots\\ h_{n+1}^T\end{bmatrix}, \tag{30}$$

$$\zeta=[0\ a_0\ a1\cdots\ a_n]^T.$$

According to the stability condition by Lipatov and Sokolov [5], a real polynomial $\delta_c(s)$ is stable if

$$p_k\,p_{k+1}>2.148\,p_{k-1}\,p_{k+2},\ \ \text{for}\ k=1,\ 2,\cdots,n-2 \tag{31}$$

Since coefficients p_i's of the characteristic polynomial are linear functions of controller's parameters as seen in (29), it turns out that (31) is a quadratic functions with respect to x. Therefore, using (29), the sufficient condition (31) for $\delta_c(s)$ to be stable can be expressed by the following quadratic inequalities.

$$f_k(x)=x^TH_kx+2g_k^{\,T}x+e_k\geq 0\ \ \text{for}\ \ k=1,\ 2,\cdots,\ n-1 \tag{32}$$

where

$$\begin{aligned}H_k &:= h_kh_{k+1}^{\,T}-\eta h_{k-1}h_{k+2}^{\,T},\\ g_k &:= \frac{1}{2}\left[\zeta_{k+1}h_k^{\,T}+\zeta_kh_{k+1}^{\,T}-\eta(\zeta_{k+2}h_{k-1}^{\,T}+\zeta_{k-1}h_{k+2}^{\,T})\right],\\ e_k &:= \zeta_k\zeta_{k+1}-\eta\zeta_{k-1}\zeta_{k+2},\\ \eta &:= 1.4656^2.\end{aligned}$$

In the sequel, the PID design problem with two conditions in problem statement can be regarded as the problem of finding $\hat{x}$ that minimizes $J(x)$ subject to (32). That is,

$$\begin{aligned}&\hat{x}=\min_x J(x)\\ &\text{subject to (i)}\ f_k(x)\geq 0,\ \ k=1,\ 2,\cdots,\ n-1\\ &\qquad\qquad\ \text{(ii)}\ x_i>0,\ \ i=1,\ 2,\ 3\end{aligned} \tag{33}$$

The constraint (ii) is imposed on when a minimum phase PID controller is required. If there exists a solution of (33), the PID controller must guarantee the closed-loop stability. Note that (33) is a non-convex optimization problem.

3.3 Design of low-order controller

The previous approach for PID control design problem can be extended to the problems of designing the general form of low order controllers. As in the section 3.2, the same assumptions are given here. Consider the following low order controller.

$$G_c(s)=\frac{B(s)}{A(s)}=\frac{k_qs^q+k_{q-1}s^{q-1}+\cdots+k_1s+k_0}{s^p+v_{p-1}s^{p-1}+\cdots+v_1s+v_0} \tag{34}$$

The closed-loop transfer function is

$$T_f(s)=\frac{G_p(s)\,G_c(s)}{1+G_p(s)\,G_c(s)}=\frac{N(s)B(s)}{A(s)D(s)+B(s)N(s)}. \tag{35}$$

Matching $T_f(s)$ with $T_N(s)$ in (17), that is,

$$\frac{N(s)B(s)}{A(s)D(s)+B(s)N(s)}\cong\frac{N(s)}{\delta^*(s)}. \tag{36}$$

Rearranging (36), we have

$$(\delta^*(s)-N(s))B(s)-A^*(s)D(s)\cong s^pD(s) \tag{37}$$

where

$$A(s)=s^p+A^*(s), \tag{38}$$

$$A^*(s)=v_{p-1}s^{p-1}+\cdots+v_1s+v_0. \tag{39}$$

From (37), we choose the degree of $\delta^*(s)$ by

$$\mathrm{degree}(\delta^*(s)) \geq n+p-q. \tag{40}$$

Equating each coefficient in the same power of s from (37), the following algebraic equation is obtained.

$$A_f x_f = b_f, \tag{41}$$

where

$$A_f = \begin{bmatrix} \delta_0^* - b_0 & 0 & 0 & \cdots - a_0 & 0 & 0 \\ \delta_1^* - b_1 & \delta_0^* - b_0 & 0 & \cdots - a_1 & -a_0 & 0 \\ \delta_2^* - b_2 & \delta_1^* - b_1 & \delta_0^* - b_0 & \cdots - a_2 & -a_1 & -a_0 \\ \vdots & \vdots & \vdots & \cdots \vdots & \vdots & \vdots \\ \delta_{n-1}^* - b_{n-1} & \delta_{n-2}^* - b_{n-2} & \delta_{n-3}^* - b_{n-3} & \cdots -1 & -a_{n-1} & -a_{n-2} \\ 0 & \delta_{n-1}^* - b_{n-1} & \delta_{n-2}^* - b_{n-2} & \cdots 0 & -1 & -a_{n-1} \\ 0 & 0 & \delta_{n-1}^* - b_{n-1} & \cdots 0 & 0 & -1 \end{bmatrix},$$

$$x_f = [k_0 \ \cdots \ k_q \ v_0 \ \cdots \ v_{p-1}]^T, \tag{42}$$

$$b_f = [\underbrace{0 \cdots 0}_{p} \ a_0 \ a_1 \ a_2 \cdots \ 1]^T.$$

We define the following error function.

$$\epsilon_f := A_f x_f - b_f \tag{43}$$

$$J_f(x) := \frac{1}{2} \epsilon_f^T \epsilon_f \tag{44}$$

Then, remaining formulation of this problem are almost the same as those [see (28)~(33)] in the previous section. Hence, the details are omitted here.

3.4 A solver for the non0convex optimization problem

In section 3.2 and 3.3, we have formulated the design problems for PID and low-order controllers into non-convex optimization problems, wherein the objective function $J(x)$ (or $J_f(x)$) is a quadratic function of controller parameter x, and $f_k(x)$ are stability constraints and the other constraints are given for minimum phase controller.

Now, the remaining question is how one can solve (33). Recently, a general purpose solver for the non-convex optimization problem, so called "GloptiPoly," has been developed by Henrion and Lasserre [6, 11]. The software GloptiPoly requires the Matlab together with the free ware LMI solver SeDuMi [10]. For more details about GloptiPoly, readers are referred to [11].

The following Matlab script is the main program of the GloptiPoly which can be used for solving any non-convex optimization problems like (33).

```
mset clear; % Initialization
NoCtr=3; % Number of controller parameters
mpol('x',NoCtr); % Declaration of variables
Obj=min(x'Ax+2qx+c);  % Objective function
QCont=(x'Mx+2px+e>=0);  % Quadratic constraint
LCont=(rx+d>=0);  % Linear constraint (r is a vector)
Ord=1; % Relaxation order
P=msdp(Obj,QCont,LCont,Ord); % Problem definition
[status, obj]=msol(P) % Solve the problem
xsol=double(x); % Optimal solution
```

4. ILLUSTRATIVE EXAMPLES

In this section, several examples are given in order to demonstrate how the proposed approach is applied. The results will be compared with those by the Kitamori approach.

Example 1: An all pole plant

For the sake of fair comparison, we consider the PID controller design problem that was used in [2].

$$G_P(S) = \frac{1}{1+4s+2.4s^2+0.448s^3+0.0256s^4} \tag{45}$$

A. The solution by Kitamori's approach

Since the plant is already all pole system, we can apply the model to (9) directly. We have $\sigma = 0.436$. From (3) and (4), the reference model was obtained. Its characteristic polynomial of degree 5 results in

$$P(s) = 4.727 \times 10^{-5} s^5 + 0.1084 \times 10^{-3} s^4 + 0.01243 s^3 + 0.09505 s^2 + 0.436 s + 1.$$

The PID controller obtained by (10) is

$$G_c(s) = \frac{3.542s^2 + 8.664s + 2.291}{s}. \tag{46}$$

B. The solution by the proposed approach

According to the synthesis method presented in section 3.1, we can construct a reference model that meets the desired damping and settling time. However, for the sake of fairly comparing the proposed method with the Kitamori one, we have generated a reference model that gives rise to the similar settling time as Kitamori's reference, which is 1.25sec, and however overshoot less than 1%. Such a $T_N(s)$ was generated using the K-Polynomial with $\alpha_1 = 2.5$ and $\tau = 0.6$. That is, from

(13) and (14), the denominator of $T_N(s)$ is determined as

$$\delta^*(s) = 0.01382s^3 + 0.144s^2 + 0.6s + 1. \qquad (47)$$

From (24) and (25), we have

$$\epsilon = Ax - b \qquad (48)$$
$$= \begin{bmatrix} 0.6 & 0 & 0 \\ 0.144 & 0.6 & 0 \\ 0.013824 & 0.144 & 0.6 \\ 0 & 0.013824 & 0.144 \\ 0 & 0 & 0.013824 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ 4 \\ 2.4 \\ 0.448 \\ 0.0256 \end{bmatrix}.$$

The characteristic polynomial (28) is written by

$$\begin{aligned} \delta_c(s) &= p_0(x) + p_1(x)s + \cdots + p_5(x)s^5 \\ &= c_0 + (c_1+1)s + (c_2+4)s^2 + 2.4s^3 \\ &\quad + 0.448s^4 + 0.0256s^5. \end{aligned} \qquad (49)$$

Then, the cost function (26) becomes

$$J(x) = x^T H_0 x + 2q_0{}^T x + e_0 \qquad (50)$$

where

$$H_0 = \begin{bmatrix} 0.1905 & 0.0442 & 0.0041 \\ 0.0442 & 0.1905 & 0.0442 \\ 0.0041 & 0.0442 & 0.1905 \end{bmatrix},$$
$$q_0^T = [-0.6046 \ \ -1.3759 \ \ -0.7524], \ e_0 = 11.4807.$$

From (49), the stability constraints (32) are given by;

$$f_1(x) = x^T \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x + 2[-2.5776 \ \ 2 \ \ 0.5]\, x + 4,$$
$$f_2(x) = [0 \ \ -0.9623 \ \ 2.4]\, x + 8.6377,$$
$$f_3(x) = [0 \ \ 0 \ \ -0.05499]\, x + 0.85525,$$
$$f_4 = x_1 = c_0 \ge 0,\ f_5 = x_2 = c_1 \ge 0,\ f_6 = x_3 = c_2 \ge 0.$$

Therefore the PID controller design problem with time response specifications has been formulated into a global optimization problem (33). To solve this problem, the software Gloptipoly was used. The PID controller is given by

$$G_c(s) = \frac{2.4602s^2 + 6.2662s + 1.667}{s}. \qquad (51)$$

The step responses of both controllers (46) and (51) are shown in Fig. 2, where y_k^*, y_k, y_N^*, and y_N denote the step responses of the Kitamori reference model, Kitamori's PID controller, the target model and PID controller derived by the present method, respectively.

It is seen that the closed loop transient response with (46) is considerably different from that of its reference model. The overshoot and the settling time of y_k are 8.44% and 1.27 sec, respectively. Recall that the Kitamori approach does not allow us to adjust the damping and settling time of the target model.

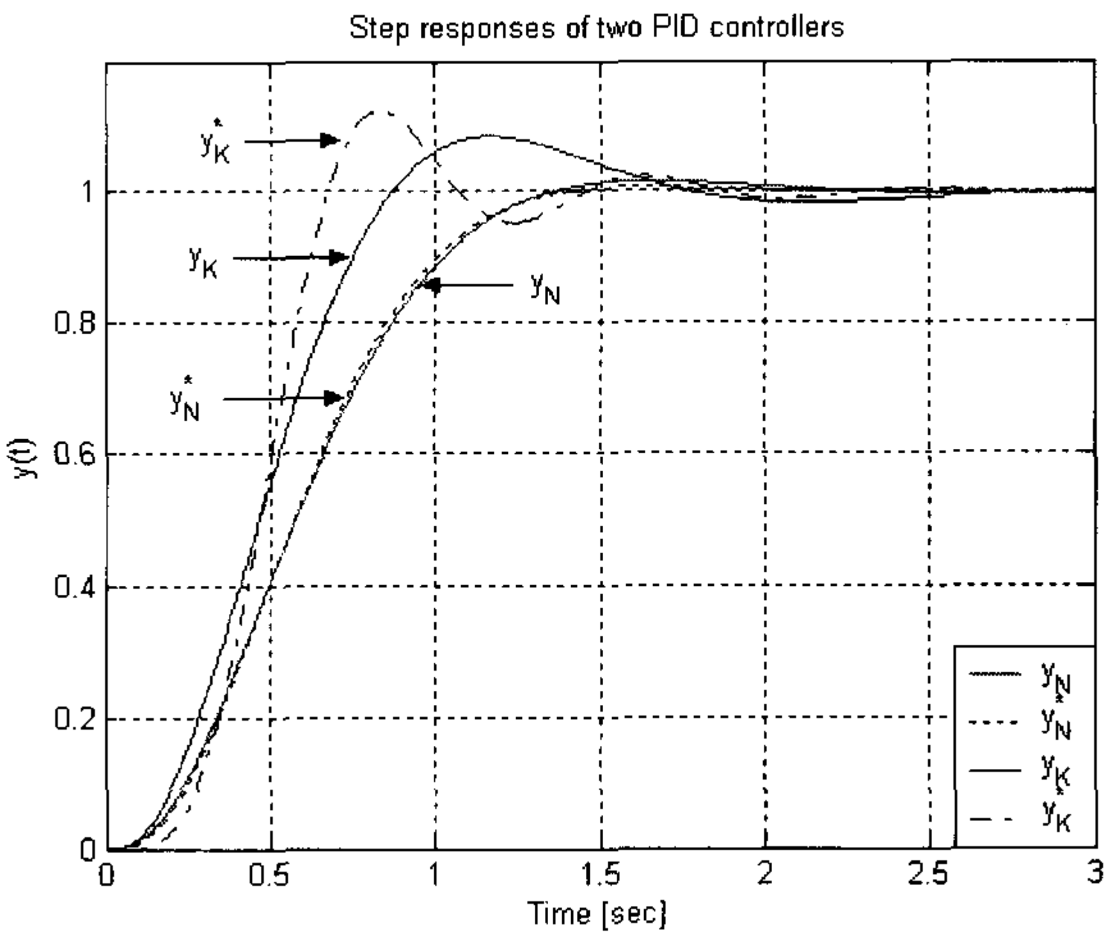

Fig. 2 Step responses of both approaches in Example1.

On the other hand, the step response of the PID controller by the proposed method coincides with that of its target model very closely. The maximum overshoot and setling time of y_N are 1.7 % and 1.25 sec. We can conclude that the proposed PID controller (51) is successfully designed.

Example 2: A non-minimum phase plant.

Now, consider the following non-minimum phase plant.

$$G_p(s) = \frac{N(s)}{D(s)} = \frac{12 - 3s + 0.25s^2}{12 + 15s + 3.25s^2 + 0.25s^3} \qquad (52)$$

A. The solution by Kitamori's approach

From (3) and (9), we obtain $\sigma = 0.69$. Also, the denominator of target model is computed by the following Kitamori's polynomial.

$$\begin{aligned} P(s) &= 0.6793 \times 10^{-2}s^5 + 0.4924 \times 10^{-1}s^4 \\ &\quad + 0.2379s^3 + 0.6898s^2 + 0.436s + 1 \end{aligned}$$

Then, (10) yields the following PID controller.

$$G_c(s) = \frac{0.225s^2 + 1.674s + 1.45}{s} \qquad (53)$$

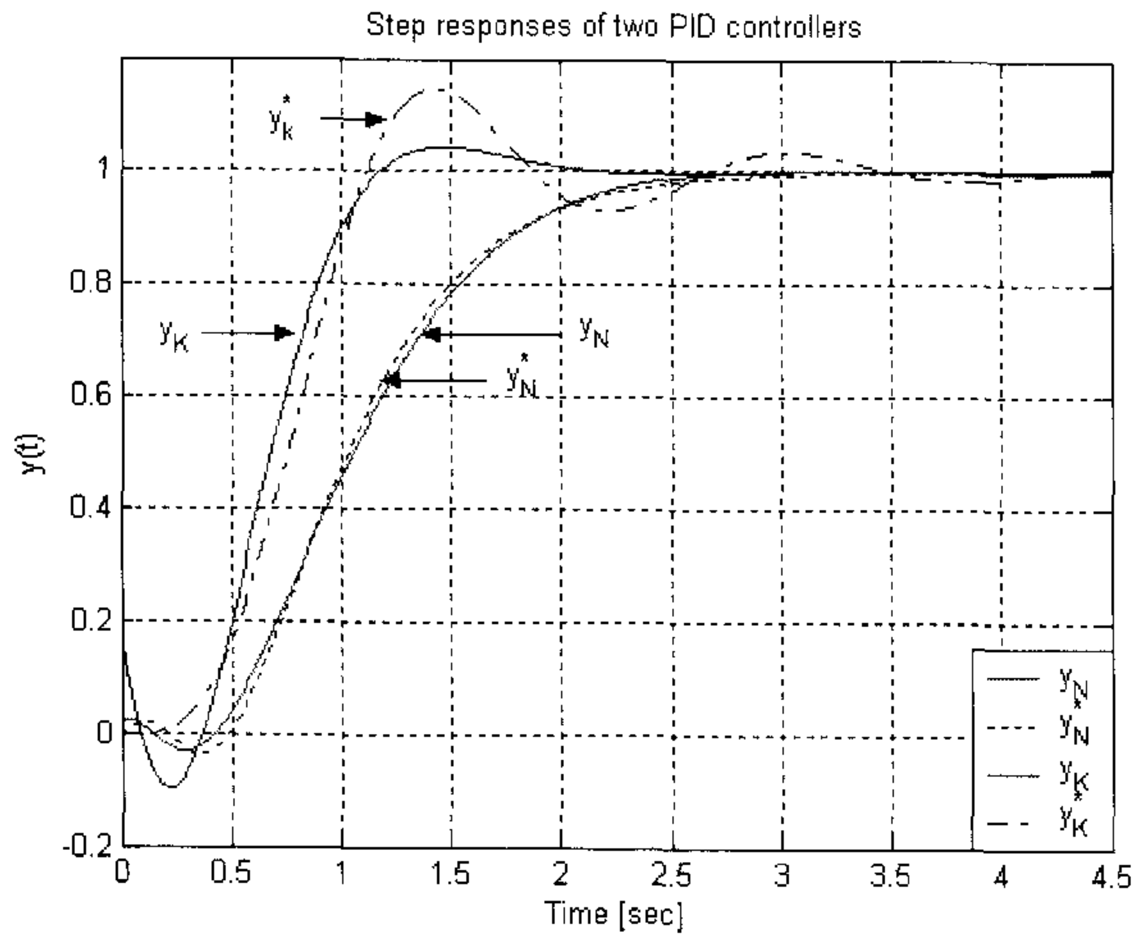

Fig. 3 Step responses of both approaches in Example 2.

B. The solution by the proposed approach

We will first design a PID controller by using the present method and will compare the result with Kitamori's PID (53). Secondly, we will design a first order controller (FOC) that meets the same time response requirements as the PID case. Similar to Example 1, for the sake of fairly comparing the proposed method with the Kitamori one, we have selected a reference model so that it has settling time less than 2.5 sec and no overshoot. However, if the other time responses are required, it is possible to construct such a reference model by using the method in section 3.1. As doing this, the following $\delta^*(s)$ was generated by choosing $\alpha_1=3$ and $\tau=0.8986$.

$$\delta^*(s)=0.01473s^4+0.3778s^3+3.23s^2+10.78s+12 \quad (54)$$

Applying the GloptiPoly to (33), the PID controller is given by

$$G_c(s)=\frac{0.0149s^2+0.9004s+0.8708}{s}. \quad (55)$$

The step responses of both PID controllers are shown in Fig. 3. The Kitamori's PID controller gives rise to the overshoot of 8.5% and the settling time of 1.8 sec. Moreover, the transient response of y_k does not track the trajectory of y_k^* well. On the other hand, It is seen that the proposed PID controller achieves the good matching between the given target model and resulting closed loop system. This controller attains almost no overshoot and the setting time of 2.5 sec, which have been already set as design objectives. It is obvious that the proposed controller gives much better performances than Kitamori's PID.

Now, Let us find a first order controller that meets the same design purposes as the PID case. To do this, the procedure in section 3.3 is applied. Let the FOC be

$$G_c(s)=\frac{k_1s+k_0}{s+v_0}. \quad (56)$$

We use the same reference model $T_N(s)=N(s)/\delta^*(s)$. Thus, $\delta^*(s)$ in (54) is employed. Then (43) and (44) can be computed below.

$$\epsilon_f=A_fx_f-b_f = \begin{bmatrix} 0 & 0 & -12 \\ 13.78 & 0 & -15 \\ 2.98 & 13.78 & -3.25 \\ 3.778 & 2.98 & -0.25 \\ 0.01473 & 0.3788 & 0 \\ 0 & 0.01473 & 0 \end{bmatrix}\begin{bmatrix} k_0 \\ k_1 \\ v_0 \end{bmatrix}-\begin{bmatrix} 0 \\ 12 \\ 15 \\ 3.25 \\ 0.25 \\ 0 \end{bmatrix} \quad (57)$$

$$J_f(x)=x_f^T\left[\frac{1}{2}A_f^TA_f\right]x_f-b_f^TA_fx_f+\frac{1}{2}b_f^Tb_f \quad (58)$$

The constraints for stability are derived from the characteristic polynomial as follows:

$$f_1(x_f)=x_f^T\begin{bmatrix} -0.75 & 2.56 & -16.19 \\ 3 & -36 & 39 \\ 3.75 & -45.54 & 48.21 \end{bmatrix}x_f +2[-62.89\ \ 72\ \ 128.51]\,x_f+180,$$

$$f_2(x_f)=x_f^T\begin{bmatrix} 0 & 0.063 & 0.063 \\ 0 & -0.75 & -0.75 \\ 0 & 0.813 & 0.813 \end{bmatrix}x_f +2[1.212\ \ -6.222\ \ 3.129]\,x_f+42.31,$$

$$f_3=x_{f1}=k_0\ge 0,\ f_4=x_{f2}=k_1\ge 0,\ f_5=x_{f3}=v_0\ge 0.$$

Applying the GloptiPoly to the problem (33) associated with the above objective function (58) and constraints, we have obtained the following FOC.

$$G_c(s)=\frac{0.90365s+0.8709}{s+3.2334\times 10^{-4}} \quad (59)$$

Fig. 4 shows the step responses of the target model and the closed loop system including (59). As seen in Fig. 4, we can conclude that the proposed first order controller (59) is also successfully designed. In contrast with PID controller (55), the (59) seems to be a PI controller because of small v_0 compared with the other parameters.

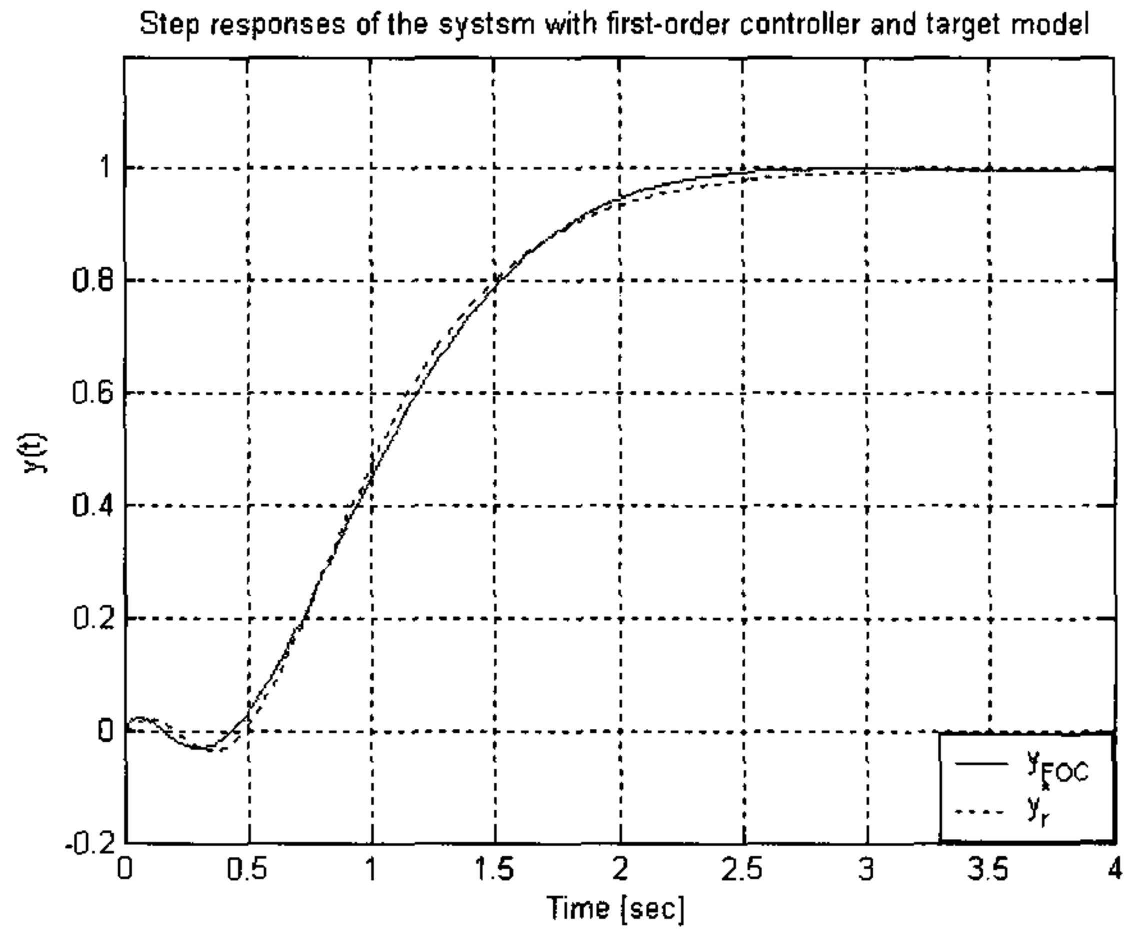

Fig. 4 Step responses of the system with first order controller (59) and the selected target model in Example2

5. Conclusion

As mentioned by P. Dorato [1], the practical control design problem that we have to carry out under the condition of a simple, fixed-structure controller is in general very difficult to solve. In this paper, we have considered such a problem. For a given linear time invariant plant with delay free, we have proposed a new approach of designing PID or any low-order controllers in cascade structure under the conditions that it shall satisfy the time response specifications such as desired overshoot and settling time, as well as the closed-loop stability. We showed that this problem could be formulated by a non-convex optimization problem. Then, a solution can be computed by means of a free software solver, the GloptiPoly, which has been developed by D. Henrion et al. [6]. It is important to note that any low-order controllers obtained by using the present method guarantee the closed loop stability, if any. Through several examples, we have shown that the proposed approach provides good low-order controller for this problem. We have also compared the present method with the Kitamori approach. As a result, we expect that the proposed method is very useful for practical control design.

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