Static Load Analysis of Twin-screw Kneaders

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A static load analysis of twin-screw kneaders is required not only for the dynamic analysis, but also because it is the basis of the stiffness and strength calculations that are essential for the design of bearings. In this paper, the static loads of twin-screw kneaders are analyzed, and a mathematical model of the force and torque moments is presented using a numerical integration method based on differential geometry theory. The calculations of the force and torque moments of the twin-screw kneader are given. The results show that the M_x and M_y components of the fluid resistance torque of the rotors change periodically in each rotation cycle, but the M_z component remains constant. The axis forces F_z in the female and male rotors are also constant. The static load calculated by the proposed method tends to be conservative compared to traditional methods. The proposed method not only meets the static load analysis requirements for twin-screw kneaders, but can also be used as a static load analysis method for screw pumps and screw compressors.

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1. Introduction

A static load analysis of twin-screw kneaders is required not only for the dynamic analysis, but also because it is the basis of the stiffness and strength calculations that are essential for the design of bearings. There will always be some vibration in the machine because the static load of the rotors changes periodically in each rotation cycle.[1-3] It is difficult to measure the static load of a screw rotor experimentally because of the complicated curve of the helicoids.[4] Most research on twin-screw extruders and kneaders focuses on the properties of the translated material, the fluid characteristics, and an analysis of the fluid load. Research also generally concentrates on the shear viscosity, shear rate, and shear stress of the fluid, as well as residence time distribution, moisture content, screw speed, nozzle diameter, and barrel temperature.[5-8]

Traditionally, the total axis force F_a of the rotors has been obtained by adding the axis force F_1 generated by the fluid and any additional dynamic loads F_2, as expressed by the following equation:[9]

$$\begin{cases} F_a = F_1 + F_2 \\ F_1 = p_{out} \cdot A_{ex} \\ F_2 = \mu F_1 \end{cases} \tag{1}$$

where

F_a is the axis force that acts on the bearing;

F_1 is the axis force generated by the fluid;

F_2 is the additional load due to dynamics;

μ is the dynamic load coefficient, generally in the range 0.15–0.25;

p_{out} is the fluid pressure at the exhaust end; and

A_{ex} is the projection cross-sectional area of the rotor.

Finite element analysis (FEA) software can be used to analyze the static load for most machine systems.[10,11] However, this not practical for analyzing the static load of a screw kneader without the complex screw rotor surface boundary conditions. In this study, the authors analyzed the static load of rotors for a new twin-screw kneader. A mathematical model for calculating the static load of the kneader was introduced based on differential geometry theory. This method can provide a theoretical foundation for a dynamic analysis, the stiffness and strength calculations, and the bearing design.

2. Analysis of rotor static load

The force and torque moments of rotors in a twin-screw kneader under static equilibrium conditions involve the fluid force F_g, the bearing reaction forces F_{b1} and F_{b2}, the input torque moment M_i, and the friction resistance torque moment M_f. The force and torque moments in the twin-screw rotors can be summarized as shown in Fig. 1. The input torque moment M_i is the sum of the fluid resistance torque M_g and the friction resistance torque M_f, *i.e.*,

$$\begin{cases} M_{if} = M_{gf} + M_{ff} \\ M_{im} = M_{gm} + M_{fm} \end{cases} \tag{2}$$

where

M_{if} is the input torque moment in the female rotor;

M_{gf} is the fluid resistance torque in the female rotor;

M_{ff} is the friction resistance torque in the female rotor;

M_{im} is the input torque moment in the male rotor;

M_{gm} is the fluid resistance torque in the male rotor; and

M_{fm} is the friction resistance torque in the male rotor.

Figure 1 shows the force and torque moments of rotors in the twin-screw kneader under static equilibrium conditions.

Because the input torque moment M_i is a known condition, if the fluid resistance torque M_g or the friction resistance torque M_f is also known, then another variable can be obtained from Eq. (2). The friction resistance torque M_f cannot be obtained directly because of the complexity of the system. The fluid resistance torque M_g in the rotors can be divided into three components along the x, y, and z axes (see Fig. 2), where

v is the fluid velocity along the axes of the rotors;

ω_f, ω_m are the angular velocities of rotation of the female and male rotor, respectively;

F_{fx}, F_{fy}, F_{fz} are the force components in the female rotor generated by the fluid;

M_{fx}, M_{fy}, M_{fz} are the fluid resistance torque M_{fg} components in the female rotor;

F_{mx}, F_{my}, F_{mz} are the force components in the male rotor; and

M_{mx} , M_{my} , M_{mz} are the fluid resistance torque M_{mg} components in the female rotor.

To calculate the fluid resistance torque M_g, the authors introduce a mathematical model for calculating the static load of the twin-screw kneader in the following sections.

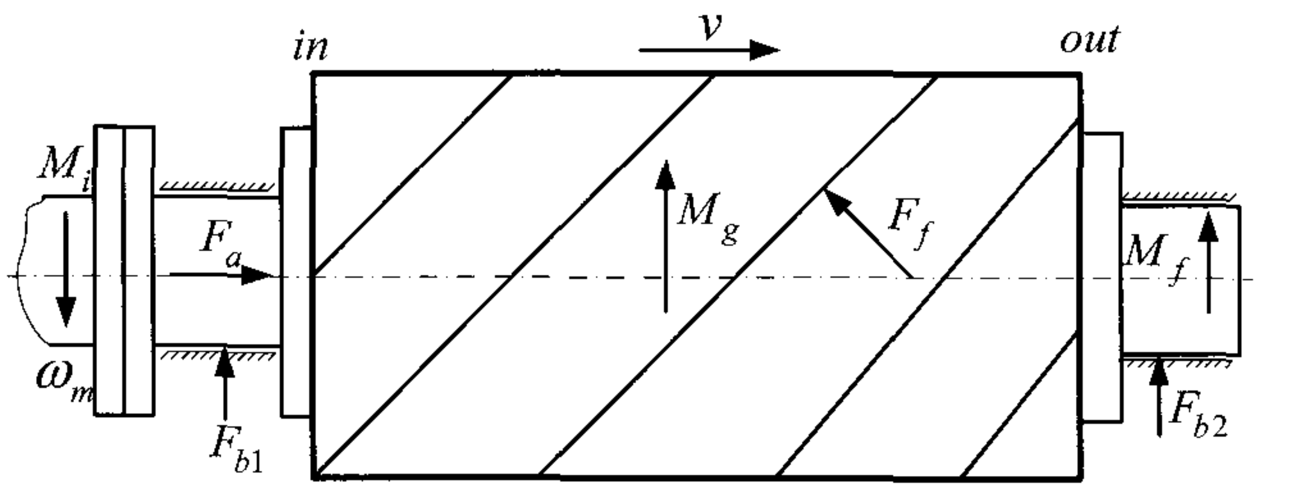


Fig. 1 Schematic diagram of the force and torque moments for screw rotors

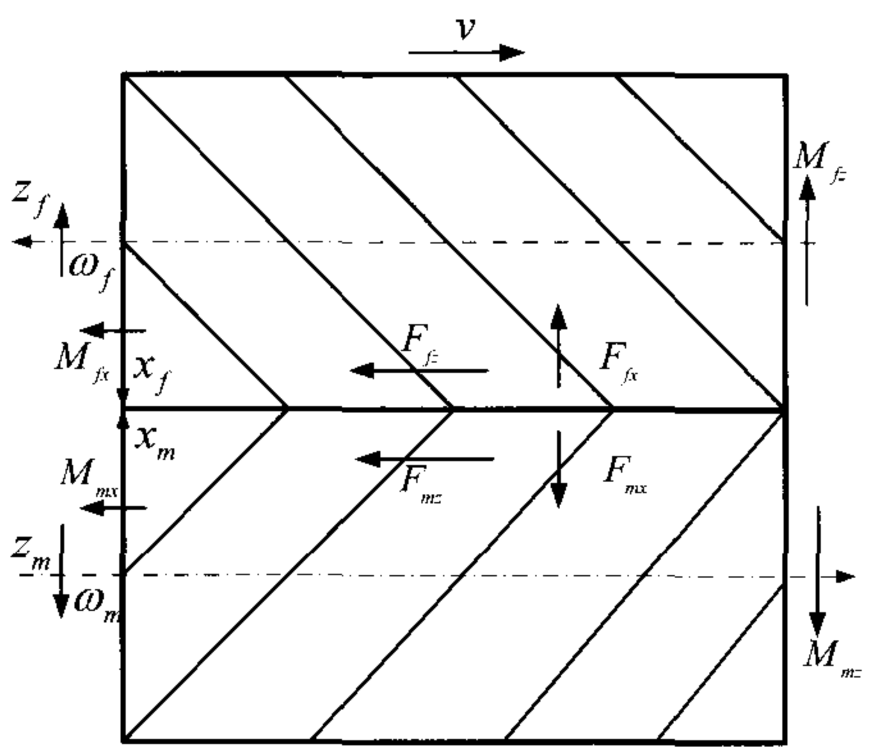


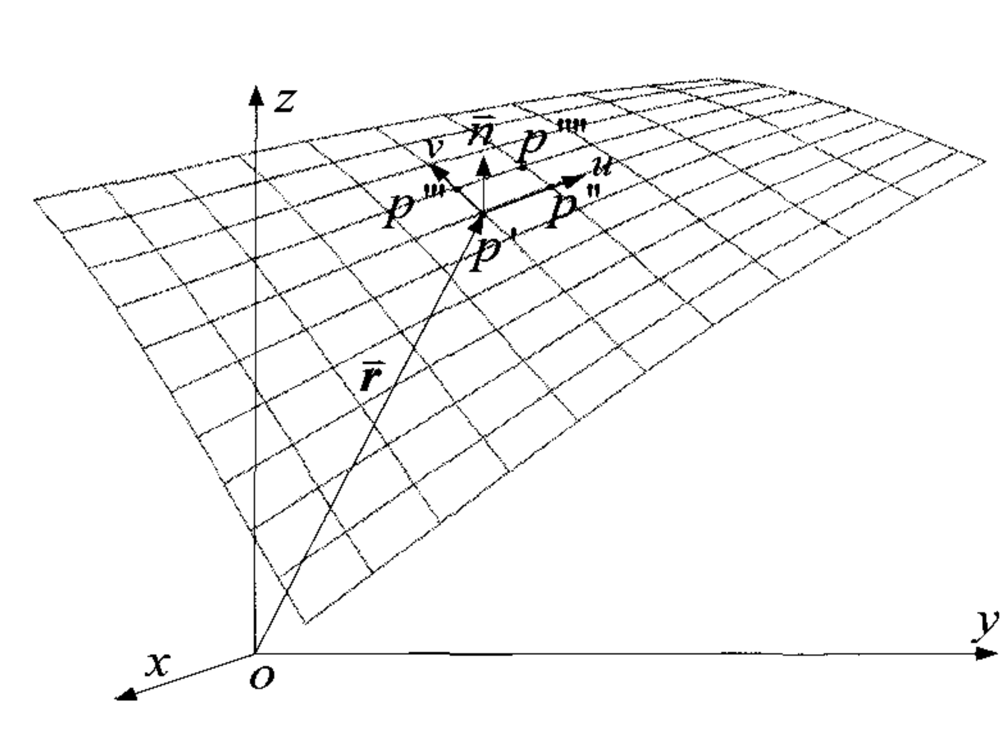
Fig. 2 Force and torque moments in a twin-screw rotor

3. Mathematical model of the rotor static load

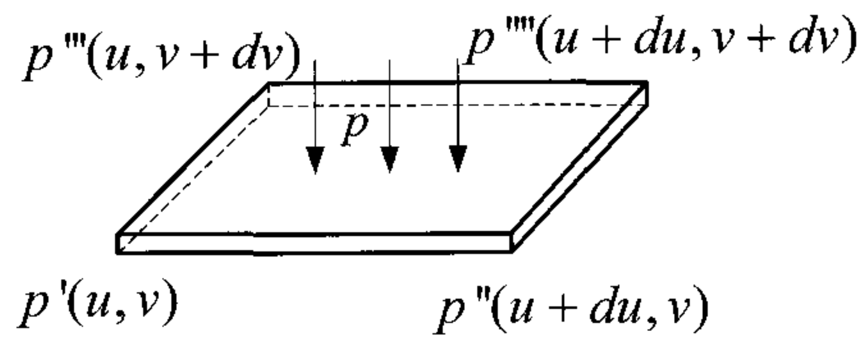
3.1 Mathematical model of the load in a minute area

A complex surface can be expressed as $r(x, y, z)$ in vector notation, as shown in Fig. 3(a). The normal vector $\vec{n}$ of point p' on the surface is:[12,13]

$$\vec{n} = r_u \times r_v \tag{3}$$



(a) Surface parameters



(b) Area parameters

Fig. 3 Surface and area parameters

The force on the surface can be regarded as the integration of fluid pressure over a minute area, *i.e.*,

$$F = \iint_R p ds \tag{4}$$

where p is the pressure on the minute surface, a scalar.

The crucial technique in a static load analysis is to calculate the force in the minute area. Assuming that there is a minute quadrangle area D with parameters (u, v), as shown in Fig. 3(b), then the two edges $p'p''$ and $p'p'''$ of the quadrangle should be [14,15]

$$\begin{cases} p'p'' = r(u+\Delta u, v) - r(u, v) = r_u \Delta u + \ldots \\ p'p''' = r(u, v+\Delta v) - r(u, v) = r_v \Delta v + \ldots \end{cases} \tag{5}$$

where "…" denotes the second- and higher-order Taylor expansion of Δu, Δv. The area of quadrangle $p'p''p'''p''''$ is approximately equal to the area of a parallelogram with edges $r_u du$ and $r_v dv$ in the tangent plane of point p'. Therefore, the area of parallelogram D is

$$ds = |r_u du \times r_v dv| = |r_u \times r_v| dudv \tag{6}$$

The force moment in the helical surface is

$$F = \iint_R p ds = \iint_R p |r_u \times r_v| dudv \tag{7}$$

where R is the integration region of parameters (u, v).

The force and torque moments in area D can be obtained using the Simpson' s rule double integral method and expressed as Eqs. (8) and (9)

$$\begin{cases} F_x = \iint_R p(r_u \times r_v) \cdot \vec{i} dudv \\ F_y = \iint_R p(r_u \times r_v) \cdot \vec{j} dudv \\ F_z = \iint_R p(r_u \times r_v) \cdot \vec{k} dudv \end{cases} \tag{8}$$

$$\begin{cases} M_x = \iint_R p[(r_u \times r_v) \cdot \vec{k} \cdot y - (r_u \times r_v) \cdot \vec{j} \cdot z] dudv \\ M_y = \iint_R p[(r_u \times r_v) \cdot \vec{i} \cdot z - (r_u \times r_v) \cdot \vec{k} \cdot x] dudv \\ M_z = \iint_R p[(r_u \times r_v) \cdot \vec{j} \cdot x - (r_u \times r_v) \cdot \vec{i} \cdot y] dudv \end{cases} \tag{9}$$

3.2 Mathematical model of the static load for the twin-screw kneader

The helical tooth profile of the screw rotor can be expressed as:

$$\begin{cases} \vec{r} = x\vec{i} + y\vec{j} + z\vec{k} \\ x = x_0(u)\cos v - y_0(u)\sin v \\ y = x_0(u)\sin v + y_0(u)\cos v \\ z = p_s v \end{cases} \tag{10}$$

where

u, v are the parameters of the helical tooth profile;

p_s is the screw parameter of the screw rotor, $p_s = H / 2\pi$, where H is the lead length of the screw rotor; and

$x_0(u)$, $y_0(u)$ are the cross-sectional components of the helical rotor.

The normal vector on the helical tooth profile can be expressed as

$$\begin{cases} \vec{n} = n_x\vec{i} + n_y\vec{j} + n_z\vec{k} \\ n_x = p_s[x_0'(u)\sin v + y_0'(u)\cos v] \\ n_y = p_s[x_0'(u)\cos v - y_0'(u)\sin v] \\ n_z = x_0(u)x_0'(u) + y_0(u)y_0'(u) \end{cases} \quad (11)$$

where

$x_0'(u)$ is the first-order derivative of $x_0(u)$;

$y_0'(u)$ is the first-order derivative of $y_0(u)$; and

n_x, n_y, n_z are the components of normal vector $\vec{n}$ on the x, y, and z axes.

According to simulation results for a flow field analysis of the twin-screw kneader, the fluid pressure in the screwed groove increases along the axis direction of the rotors from the inlet to the outlet (Fig. 4).[9] To simplify the calculation, the authors regard the fluid pressure in the screwed groove along the axis direction of the rotors as a linear distribution, and the pressure in the screwed groove along the radial direction of rotors as a constant. The p_{in} and p_{out} are the fluid pressures at the entrance and exit sections, respectively.

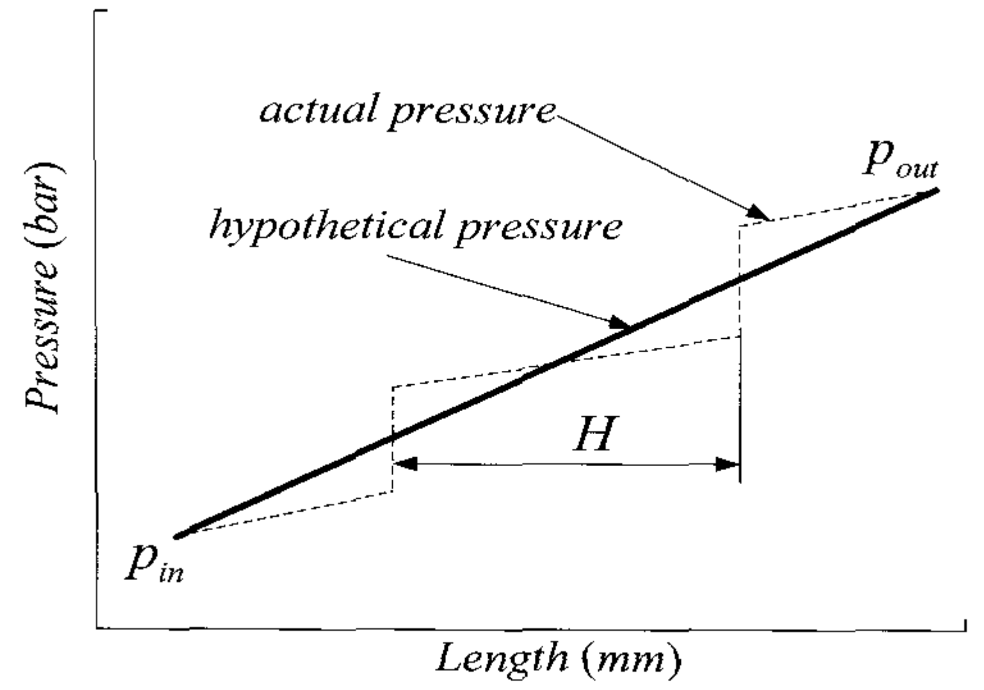


Fig. 4 Relationship between pressure and rotor length

Figure 5 shows the calculation model for a minute area of the helicoids. According to the mathematical model of load in a minute area, the helicoids can be regarded as the parameter surface described above. Based on the above theories and the properties of helicoids, a static load analysis of the twin-screw kneader can be presented as follows.

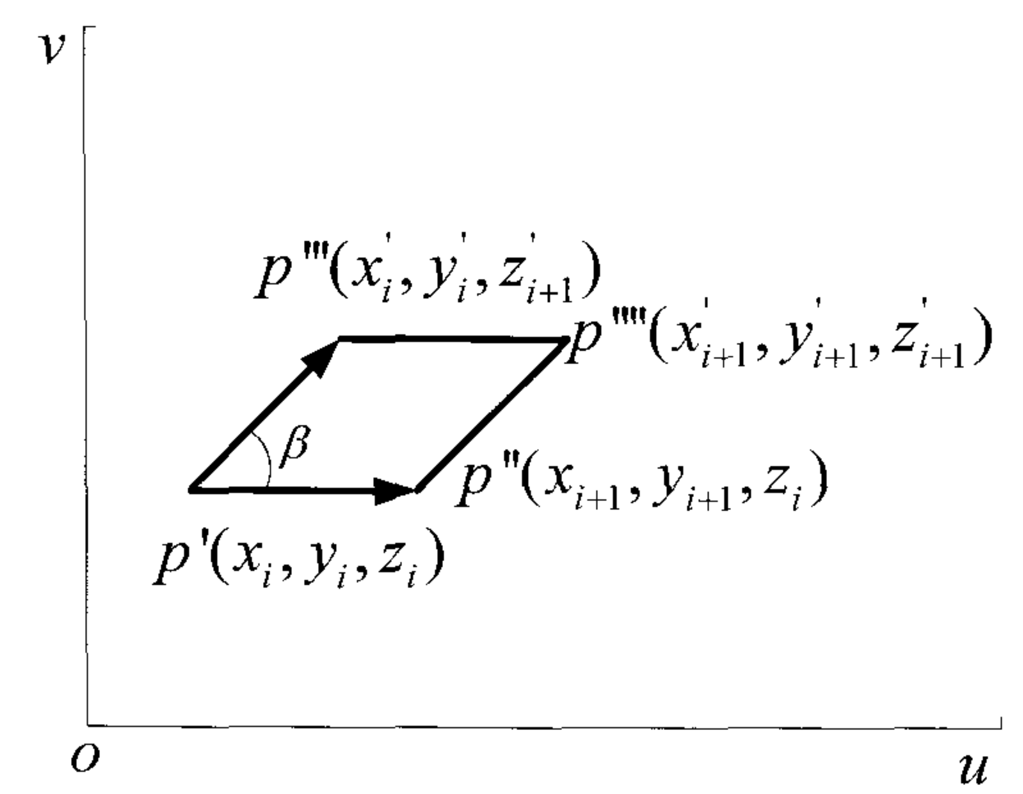


Fig. 5 Calculation model of a quadratic polynomial area

Considering adjacent points p', p'', and p''', the vectors op', op'', and op''' in the minute parallelogram area can be expressed as:

$$\begin{cases} op' = x_i\vec{i} + y_i\vec{j} + z_i\vec{k} \\ x_i = x_{0i}\cos v_i - y_{0i}\sin v_i \\ y_i = x_{0i}\sin v_i + y_{0i}\cos v_i \\ z_i = p_s v_i \end{cases} \quad (12)$$

$$\begin{cases} op'' = x_{i+1}\vec{i} + y_{i+1}\vec{j} + z_{i+1}\vec{k} \\ x_{i+1} = x_{0i+1}\cos v_{i+1} - y_{0i+1}\sin v_{i+1} \\ y_{i+1} = x_{0i+1}\sin v_{i+1} + y_{0i+1}\cos v_{i+1} \\ z_{i+1} = p_s v_{i+1} \end{cases} \quad (13)$$

$$\begin{cases} op''' = x_i'\vec{i} + y_i'\vec{j} + z_i'\vec{k} \\ x_i' = x_{0i}\cos v_{i+1} - y_{0i}\sin v_{i+1} \\ y_i' = x_{0i}\sin v_{i+1} + y_{0i}\cos v_{i+1} \\ z_i' = p_s v_{i+1} \end{cases} \quad (14)$$

$$\begin{cases} \Delta l_1 = p'p'' = op' - op'' \\ \Delta l_2 = p'p''' = op' - op''' \\ \Delta s = \Delta l_1 \Delta l_2 \sin\beta \\ \beta = \arctan(H / \pi d_0) \end{cases} \quad (15)$$

where

Δl_1 is the length of point p' and p'';

Δl_2 is the length of point p' and p''';

Δs is the area of the minute parallelogram;

d_0 is the pitch circle diameter of the screw rotor; and

β is the helical angle of the pitch circle.

The force in the parallelogram can be expressed as

$$\begin{cases} \Delta F_i = p_i \Delta s \\ p_i = p_{in} + \dfrac{p_{out} - p_{in}}{L} \cdot \dfrac{H}{2\pi} \cdot v_i \end{cases} \quad (16)$$

where

ΔF_i is the force on the minute parallelogram; and

p_i is the pressure on the minute parallelogram.

The components of ΔF_i and ΔM_i in x, y, z can be expressed as

$$\begin{cases} \Delta F_{xi} = \Delta F_i \cdot e_{xi} \\ \Delta F_{yi} = \Delta F_i \cdot e_{yi} \quad (i = 1,2,3,...,n) \\ \Delta F_{zi} = \Delta F_i \cdot e_{zi} \end{cases} \quad (17)$$

and

$$\begin{cases} \Delta M_{xi} = \Delta F_{zi} \cdot y_i - \Delta F_{yi} \cdot z_i \\ \Delta M_{yi} = \Delta F_{xi} \cdot z_i - \Delta F_{zi} \cdot x_i \quad (i = 1,2,3,...,n) \\ \Delta M_{zi} = \Delta F_{yi} \cdot x_i - \Delta F_{xi} \cdot y_i \end{cases} \quad (18)$$

where

ΔF_{xi}, ΔF_{yi}, ΔF_{zi} are the ΔF_i components in x, y, and z;

ΔM_{xi}, ΔM_{yi}, ΔM_{zi} are the ΔM_i components in x, y, and z;

e_{xi}, e_{yi}, e_{zi} are the unit normal vector $\vec{e}$ components in x, y, and z;

and

$$\vec{e} = \vec{n} / |\vec{n}|$$

Therefore, the force and torque moments in the screw rotor can be expressed as:

$$\begin{cases} F_x = \sum_{j=1}^{m}(\sum_{i=1}^{n} p_{ji}\Delta l_{1ji}\Delta l_{2ji}\sin\beta \cdot e_{xji}) \\ F_y = \sum_{j=1}^{m}(\sum_{i=1}^{n} p_{ji}\Delta l_{1ji}\Delta l_{2ji}\sin\beta \cdot e_{yji}) \\ F_z = \sum_{j=1}^{m}(\sum_{i=1}^{n} p_{ji}\Delta l_{1ji}\Delta l_{2ji}\sin\beta \cdot e_{zji}) \end{cases} \quad (19)$$

$$\begin{cases} M_x = \sum_{j=1}^{m}(\sum_{i=1}^{n} p_{ji}\Delta l_{1ji}\Delta l_{2ji}\sin\beta \cdot (e_{zji} \cdot y_{ji} - e_{yji} \cdot z_{ji})) \\ M_y = \sum_{j=1}^{m}(\sum_{i=1}^{n} p_{ji}\Delta l_{1ji}\Delta l_{2ji}\sin\beta \cdot (e_{xji} \cdot z_{ji} - e_{zji} \cdot x_{ji})) \\ M_z = \sum_{j=1}^{m}(\sum_{i=1}^{n} p_{ji}\Delta l_{1ji}\Delta l_{2ji}\sin\beta \cdot (e_{yji} \cdot x_{ji} - e_{xji} \cdot y_{ji})) \end{cases} \quad (20)$$

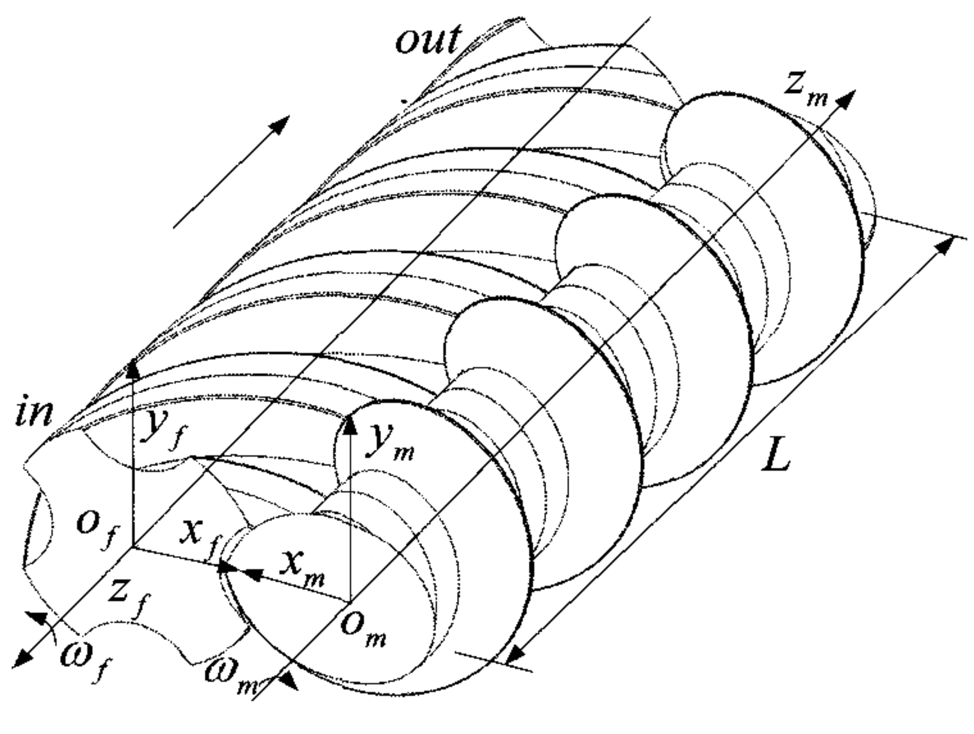
where

F_x, F_y, F_z are the F components in x, y, and z; and

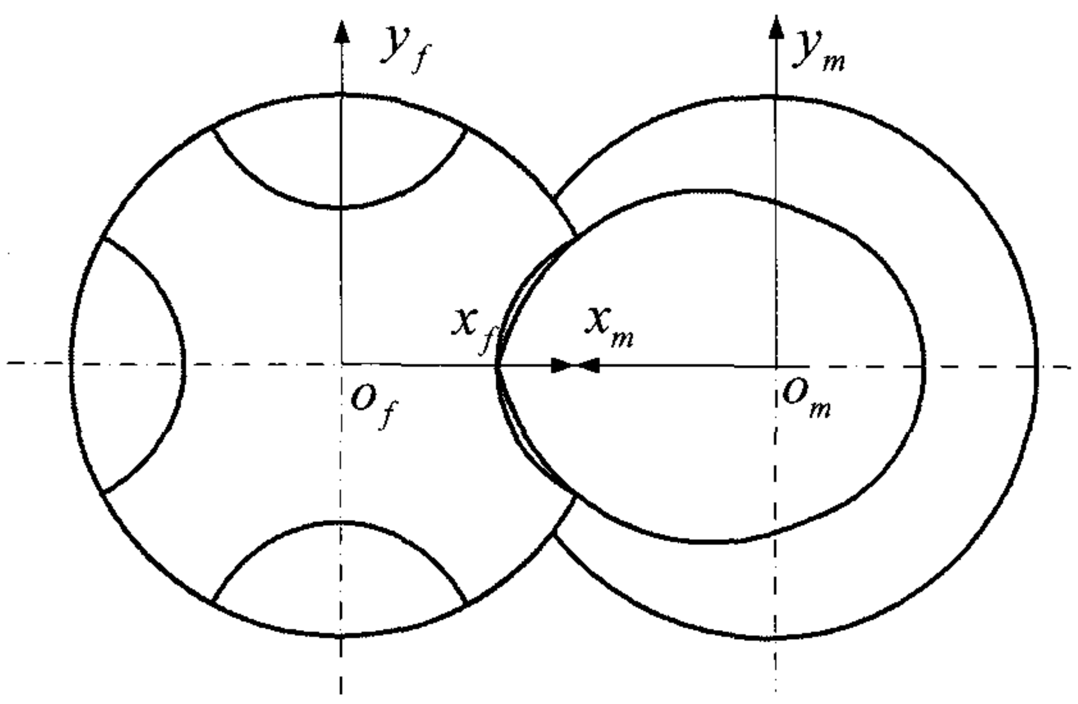
M_x, M_y, M_z are the M components in x, y, and z.

4. Results and discussion

For multiphase and non-Newtonian fluids that cannot be transferred by conventional pumps, a novel twin-screw kneader and its profiles were developed,[16] as shown in Fig. 6. During operation, the contact line of the female and male rotors moves to the exhaust end along with the rotation of the rotors. Fluid in the screw groove will be discarded at the exhaust end evenly under the fluid pressure difference. In this study, the pressure p_{in} at the entrance section was 0.5 bar, and p_{out} at the exit section was 2.0 bar. The length L was 400 mm, and the outer diameter of the rotors d was 60 mm. The static load in the male and female rotors can be obtained using the static load mathematical model for the twin-screw kneader above. The force and torque moments are shown in Fig. 7.

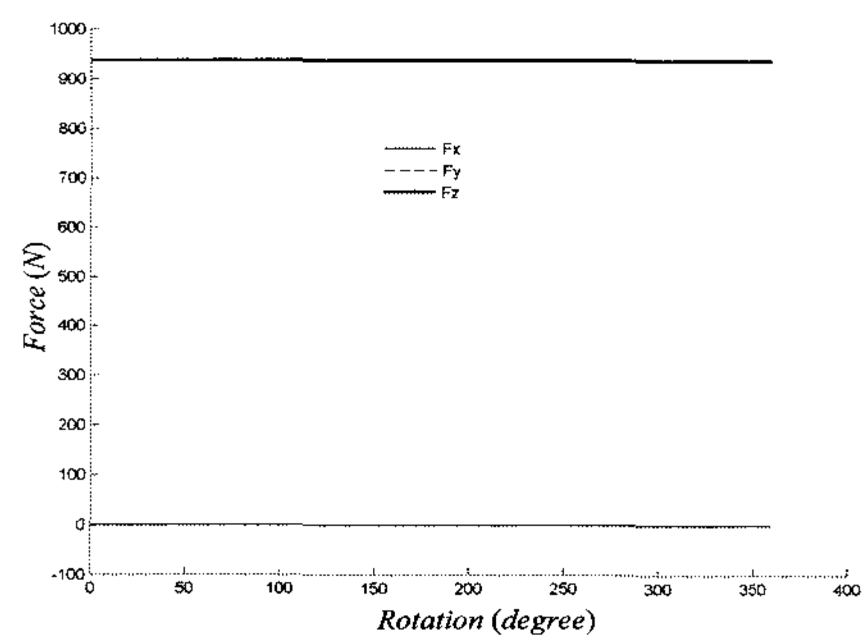


(a) D model of screw rotors in the kneader showing the defined coordinates

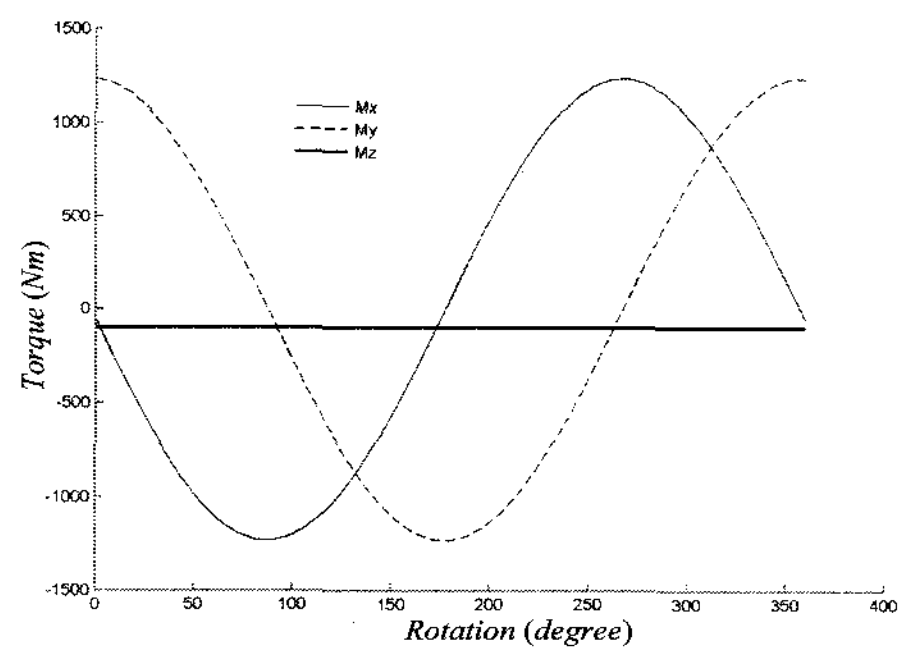


(b) Cross-section of the twin-screw kneader

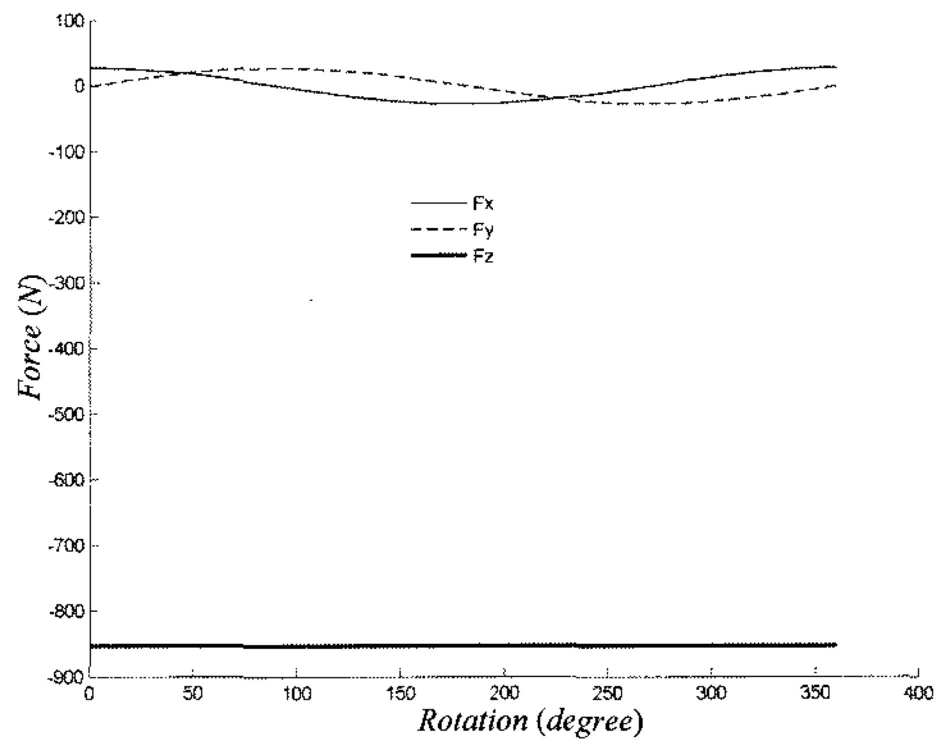
Fig. 6 Profiles of the new twin-screw kneader



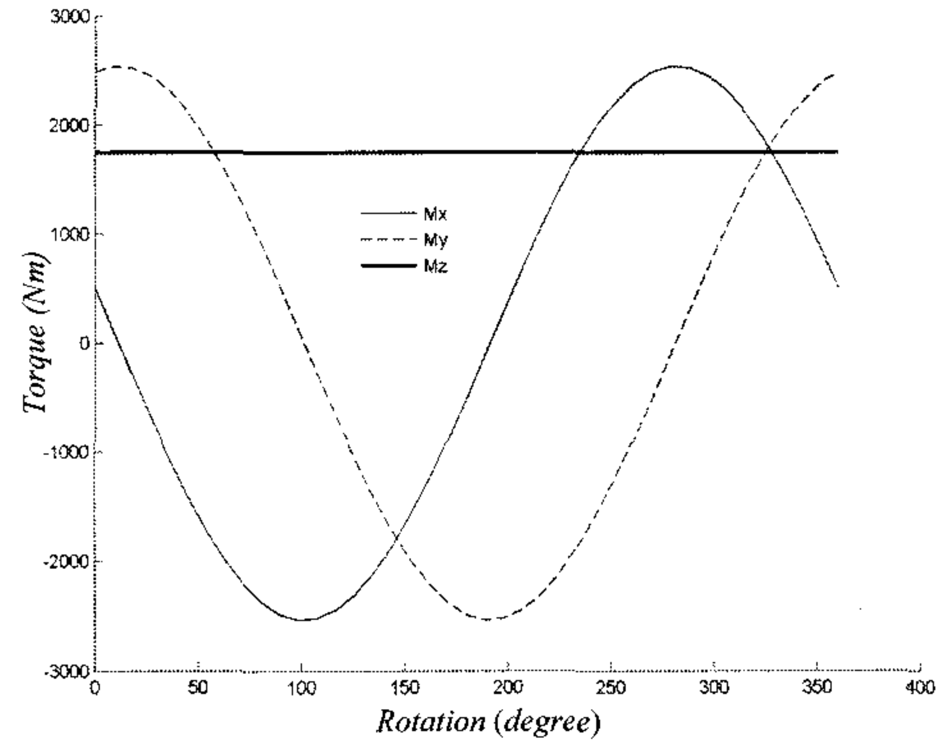
(a) Force of the female rotor



(b) Torque of the female rotor



(c) Force of the male rotor



(d) Torque of the male rotor

Fig. 7 Force and torque moments of the rotors

Figures 7(a) and (c) show that the forces F_z in the female and male rotors are constant, and the orientation of force F_z is opposite to the fluid flow direction. The M_x and M_y components of fluid resistance torque in the female and male rotors change periodically in each rotation cycle, but the M_z component in each rotor is constant.

The axis force of the female and male rotors can be calculated as shown in Fig. 7 using the traditional method. From Table 1, the axis forces on each rotor calculated by traditional methods are less than those calculated by the method presented in this paper, which means the calculation results are inclined to be conservative. Therefore, the static loads calculated by the proposed method also tend to be conservative.

Table 1 Axis force calculated using the traditional method

A_{ex} (mm^2)	F_1 (N)	μ	F_2 (N)	F_a (N)
2714.894	542.979	0.2	108.596	651.575

According to the design results of the twin-screw kneader, the ratio of fluid transferred by the male and female rotors is about 1.762, which was calculated using the total screw groove area of each rotor cross-section. Similarly, the ratio of input torque moment in the male and female rotors is also about 1.762. Therefore, Figs. 7(b) and (d) are in close agreement.

5. Conclusions

The force and torque moments in twin-screw rotors were calculated using numerical integration based on the differential geometry method. The method proposed in this paper can simplify the numerical analysis calculations for discrete points.

The static loads can be solved if equations or discrete points of the helical tooth profile of the screw rotors are given. The method can be used for the static load analysis of twin-screw pumps, compressors, and kneaders.

The axis force calculated using the proposed method was greater than that obtained using the traditional method. Therefore, the static loads obtained using this method tends to be conservative. Compared with traditional methods, the static loads calculated by the proposed method meet the analysis requirements for twin-screw kneaders.

ACKNOWLEDGMENT

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