한국도로학회 논문집
第10券1號 2008年 6月
pp.27~33

지반의 감쇠 거동을 위한 복합 모델 개발

Development of a Combined Model for Soil Damping Behavior

배윤신*
Bae, Yoon-Shin

요 지

다양한 지반의 모델들을 이용한 감쇠 거동을 실제 지반의 감쇠 거동과 비교하였다. 지반의 감쇠 거동을 예측하기 위해 몇가지 비선형 지반 모델들을 이용하고 평가하였다. 지반 거동을 대략적으로 잘 묘사하기 위해 비점성 감쇠 및 이력감쇠 거동을 모두 고려하는 복합 감쇠 모델이 개발 되었다. 이 모델의 장점 및 문제점이 논의 되었다.

핵심용어(Keyword) : 감쇠, 지반모델, 비점성 감쇠, 이력 감쇠

Abstract

Damping behavior of various soil models are compared to actual observed soil damping behavior. Several nonlinear soil models were used and evaluated to predict damping behavior of soils. A combined damping model incorporating both nonvisous and hysteretic damping behavior was developed to better approximate soil behavior. The strengths and limitations of this model are discussed.

Keyword: damping, soil models, nonviscous damping, hysteretic damping

* 정회원 · ㈜ 다산컨설턴트 기술연구소 부장 · 공학박사 (baeys@dasan93.co.kr)

1. INTRODUCTION

Soil damping behavior is described in several aspects. It is known that soils generally exhibit *constantdamping for strains in the linear range*, but increasing damping as strains increase into the nonlinear range. And damping in soil is generally constant with frequency of loading (Kokushu, 1987). Several models are used to describe these damping behaviors of soil but no single mechanistic damping model has all of these behaviors.

A viscous model is used to describe the energy loss due to nonhysteretic behavior, which is called viscous damping. Two weaknesses of the model is that it lacks in describing the hysteretic damping behavior and rate-independent nature of soil damping. On the other hand, hysteretic model describes the energy loss due to nonlinear stress-strain relationship. However, it does not contain the damping ratios at small strains.

Each of these models describes part of soils actual damping behavior but none of them completely describe the combined damping behavior. Hence, a combined model describing two separate phenomena is developed and the suitability of the model is presented.

2. DAMPING MODELS

Various models (viscous model, non-viscous model, and hysteretic model) have been developed to account for the dissipation of energy in soils during cyclic loading. Those models to account for damping in soil are described and the suitability of each model for soil dynamics modeling is briefly discussed in the following sections.

2.1 Viscous Model

Viscous damping using a Kelvin-spring-dashpot model is often used to represent the dissipation of elastic energy. This is the most widely used damping model because of its simplicity. In this model, the resistance of material to shearing deformation is the sum of an elastic part, τ_1 and a viscous part, τ_2. The Kelvin model is illustrated in Fig. 1.

Fig. 1

The elastic portion of the stress is partitioned to the spring and is equal to $G\gamma$. And the viscous portion of the stress is partitioned to the dashpot and is equal to $G'd\gamma/dt$, where G is a spring constant and G′ represents a dashpot constant. Hence, the total stress, $\tau = \tau_1 + \tau_2$, is given by:

$$\tau = G\gamma + G'\frac{d\gamma}{dt} \qquad (1)$$

For a harmonic shear strain of the form:

$$\gamma = \gamma_0 \sin\omega t \qquad (2)$$

the stress will be

$$\tau = \tau_1 + \tau_2 = G\gamma_0 \sin\omega t + \omega G'\gamma_0 \cos\omega t \qquad (3)$$

Another expression of shear stress in the dashpot is given by using trigonometric identity

$$\tau_2 = G'\dot{\gamma} = \pm G'\omega\sqrt{\gamma_0^{\,2} - \gamma^2} \qquad (4)$$

The relation $\tau_1 = G\gamma_0 \sin wt$ in Eq. (3) is depicted in Fig. 2 as a straight line with a slope G and the relation in Eq. (4) indicates an ellipse in the plot of τ_2 versus γ.

Fig. 2

To quantify the damping characteristics, it has been customary to draw attention to the amount of energy which is lost during one cyclic loading (Ishihara, 1996). The energy loss per cycle is equal to the area enclosed by the hysteresis loop shown in Fig. 3, which is equal to the area of ellipse shown in Fig. 2.

$$\Delta W = \int_{t_0}^{t_0+2\pi} \tau \frac{\partial \gamma}{\partial t} dt = \pi G' \omega \gamma_0^{\ 2} \tag{5}$$

The peak energy stored in the cycle is,

$$W = \frac{1}{2} G \gamma_0^{\ 2} \tag{6}$$

Since the damping ratio, D is defined as:

$$D = \frac{1}{4\pi} \frac{\Delta W}{W} \tag{7}$$

then,

$$D = \frac{1}{4\pi} \frac{\pi G' \omega \gamma_0^{\ 2}}{\frac{1}{2} G \gamma_0^{\ 2}} = \frac{G' \omega}{2G} \tag{8}$$

It can be seen from Eq. (8) that damping is a linear function of frequency as shown in Fig. 4. Damping in soil is generally constant with fre quency, at least over the range if frequencies common in cyclic loading of soil. This makes the viscous damping model inappropriate for most soil model.

Fig. 4

Another weakness of viscous model for soil is that damping is constant with strain amplitude. Soils general exhibit constant damping for strains in the linear range, but increasing damping as strains increase into the nonlinear range.

2.2 Non-viscous model

To eliminate frequency dependence, Ishihara (1996) introduced rate independent dashpot shown in Fig. 5. He stated:

"It is to be recalled here, however, that in the classical theory thermodynamics, the change in entropy indicative of the energy dissipation has always been treated in terms of time rate of change in variables on the basis of the established lows of physics. Therefore even the existence of the rate-independent dashpot itself is very much in question and its physical basis may not be justified. Notwithstanding such a dilemma, it would be expedient to incorporate the rate-independent dashpot and to evolve a convenient model which can reflect actual behavior of soils to a good degree of accuracy."

Fig. 5

He called the simplest model satisfying this requirement the non-viscous type Kelvin model written as

$$\tau = (G + iG_0')\gamma \tag{9}$$

where G_0' = rate independent dashpot

The imaginary term is 90° out of phase with $\tau_1 = G\gamma$, and τ_2 is equal to $G_0'\gamma$.

As shown in Eq. (9), both moduli, G and G' have constant values independent of the frequency of applied loading. This makes the non-viscous model more appropriate for soil in cyclic loading than the viscous model. However, like the viscous model, damping is still constant with strain.

2.3 Hysteretic model

It has been shown that the response of soil to cyclic loading is non-linear, even at small strain levels as small as 10^{-5} %. The initial response of soil to shear loading is governed by the maximum shear modulus or low-amplitude shear modulus (G_{max}). However, *increasing the level of shear* stress or strain causes a decrease in the secant shear modulus, and consequently non-linear soil response, as shown in Fig. 6. During cyclic loading the equivalent linear shear modulus is equal to the secant modulus (G_1 or G_2 in Fig. 6) at a given strain level. Internal soil damping causes the soil stress-strain curve to exhibit a hysteresis loop. When hysteretic damping ratio is used to account for energy dissipation in the soil specimen, it is defined as the ratio of energy dissipated in a cycle of loading to the maximum energy stored during the cycle.

Fig. 6

Ishihara (1996) decomposed the nonlinear hysteretic curve into elastic and energy dissipating components shown in Fig. 7. He did not specify whichmodel was used to describe stress-strain relationship. In the stress-strain curve, it is seen that there are two types of curves: one associated with monotonic loading, doa, in Fig 7 (a), and the other associated with hysteresis acdef, in Fig. 7(b). The monotonic loading curve is called a backbone curve and the loop is called a hysteresis loop, and its shape is determined by assuming Masing behavior (1926). If the stress value in the backbone curve is subtracted from that of the hysteresis curve, a loop a'b'c'd'e'f' or gh'j'k'l' is obtained as shown in Fig. 7 (a). This indicates that energy is dissipated by nonlinear stress-strain behavior.

Fig. 7

At a small strain, the hysteresis curve is *almost a line.* Hence, the damping ratio at small strains approach zero, which is a shortcoming of the hysteretic damping model for soil. Real soil has some damping even at very small strains. At a large strain, unrealistic large damping is obtained, which is another shortcoming of the hysteretic damping model for soil.

A hysteretic damping model requires a non-linear stress-strain model. The hyperbolic model is a common soil model to describe the non-linear stress-strain behavior. The following equation describes the hyperbolic model (Hardin and Drnevich, 1972):

$$\tau = \frac{G_{max}\gamma}{1+\dfrac{\gamma}{\gamma_r}} \qquad (10)$$

where, G_{max} is the maximum shear modulus or low-amplitude shear modulus, and, γ_r, called reference strain is defined as:

$$\gamma_r = \frac{\tau_f}{G_{max}} \qquad (11)$$

in which, τ_f *is the maximum shear stress.*

In this study, the hyperbolic model was employed to partition the nonlinear hysteresis curve into elastic and energy dissipating components. The model used a reference strain, γ_r, of 0.01% and has small strain shear modulus, G_{max}, of 1.0×10^6 kPa.

Fig. 8 (a) shows the energy dissipating component due to nonlinear stress-strain curve and backbone curve for the case of hysteretic damp ing ratio equal to 14.5 %. Fig. 8 (b) shows the hysterisis loop and backbone curve at a max imum shear strain of 9.977×10^{-3} %. Fig. 8 (c) shows the energy dissipating component due to nonlinear stress-strain curve and backbone curve in case of hysteretic damping ratio equal to 57 %. And Fig. 8 (b) shows the hysterisis loop and backbone curve at maximum shear strain of 0.63 %.

Fig. 8

Masing (1926) assumed that the stress-strain path during cyclic loading could be related to the monotonic loading stress-strain path, which is called the backbone curve. He states that: 1) on reversals the stress-strain relationship goes back to a slope of G_{max}, and 2) after the first quarter cycle of loading, the scale of the stress-strain re lationship changes by a factor of two. As illus trated in Fig. 9, Masing rule assumes that hyste resis loop for two-way cyclic loading can be constructed by scaling the backbone curve by a factor of two. After initial loading, the scaled curve is flipped on the horizontal and vertical ax es, respectively, and placed at the end of the backbone curve to represent the unloading path. In order to represent reloading, the scaled curve is placed at the end of the unloading path.

Fig. 9

Figs 10 and 11 shows comparisons between measured hysteresis loops and the hysteresis loop from assuming Masing behavior, from torsional shear (TS) tests on sand cycled to 0.0019 and 0.0035 rad, respectively. The same sands were used for both 0.0019 and 0.0035 rad tests. In Fig.10, the sample was tested at a confining pressure equal to 25.3 kPa for first cycle of loading. In Fig.11 the same soil specimen was tested at a confining pressure equal to 101.3 kPa for first cycle of loading. The damping ratio from the actual hysteresis loop in Fig.10 is 20.5 %. The damping determined assuming Masing behavior is 21.2 %. The damping ratio from the actual hysteresis loop in Fig.11 is 17.4 %. The damping determined assuming Masing behavior is 16.7 %. This shows that assuming Masing behavior can be an appropriate assumption for damping calculations.

Fig.10

Fig. 11

In order to represent soils actual damping behavior,a combined hysteretic and nonviscous soil model was developed. This model has small strain damping, damping increasing with strain level, and the damping is independent of cyclic frequency.

To considering both hysteretic damping and nonviscous damping, the linear spring (G) in Fig. 5 was replaced by a non-linear spring with a varying spring constatnt, G_{sec} shown in Fig.12. The dashpot constant, G_0', can be calculated using G_{sec} determined for specific shear strain using:

$$G_0' = 2 \cdot G_{sec} \cdot D_{min} \qquad (11)$$

where, D_{min} is damping ratio at small strains.

Fig.12

Hence, the total stress, τ is the sum of stress in the non-linear spring, τ_1, and the stress in the dashpot, τ_2. These are

$$\tau = \tau_1 + \tau_2 = (G_{sec} + iG_0')\gamma \qquad (13)$$

Fig.13 illustrates the stress -strain relationship based only on the hyperbolic model (τ_1) at several strain amplitudes. The model uses a reference strain, γ_r, of 0.01 % and has small strain shear modulus, G_{max}, of 1.0×10^6 kPa. As shown in Fig.13, the hysteretic damping approaches zero at low strain levels. Fig.14 presents the stress -strain relationship based only upon the nonviscous model (τ_2) with D_{min} equal to 4 %. At low strain levels, the plot of τ_2 versus γ is elliptical. As strain increases, the shape becomes non-elliptical. Fig.15 illustrates the combined stress-strain model from both the hyperbolic and the nonviscous elements. At low strains, the stress-strain plot describes a hysteresis loop, which results in low strain damping. Fig.16 shows damping ratio, D %, with respect to strain from both the hyperbolic and nonviscous model of Figs. 13 and 14, respectively.

Fig. 13
Fig. 14
Fig. 15

4. CONCLUSIONS

A combined model was developed to describe soil damping behavior. This model has small strain damping, increasing damping with strain level, and frequency independent damping.

The combined model was developed assuming that the theoretical soil behaves according to Masing rules and employing rate-independent dashpot. Using Masing rules, unrealistic large damping is obtained at high strain levels and the existence of the rate-independent dashpot itself is in question. These limitations require further study.

5. ACKNOWLEDGMENTS

This project was supported by the Geotechnical Engineering Center at the Utah State University. Its support is greatly appreciated. Encouragement and help from Dr. Bay is gratefully acknowledged.

6. REFERENCES

Hardin, B. O (1965). *"The nature of damping in sands."* J. Soil Mech. And Found. Div., ASCE, 91 (1), 63-97.

Hardin, B. O and Drenevich, V. P. (1972). *"Shear modulus and damping in soils: design and equations and curves."* J. Soil Mech. And Found. Div., ASCE, 98 (7), 667-692.

Isenhower, W. M. (1979). *"Torsional simple shear / resonant column properties of San Francisco bay mud,"* MS thesis, Univ. of Texas, Austin, Texas.

Ishihara, K. (1996). Soil behavior in earthquake geotechnics. Oxford University Press, Oxford.

Kokushu, T. (1987) "In situ dynamic soil properties and their evaluation," *Proceedings of the 8th Asian Regional Conference on Soil Mechanics and Foundation Engineering, pp. 215-435.*

Kramer, S. L. (1996). *Geotechnical earthquake* *engineering.* Prentice Hall, Upper Saddle River, New Jersey

Pyke, R.M. (2004), *"Evolution of soil models since the 1970s,"* International Workshop on the Uncertainties in Nonlinear Soil Properties and their Impact on Modeling Dynamic Soil Response, University of California, Berkeley.

7. SYMBOLS

D = damping ratio (%)
D_{min} = damping ratio at small strains (%)
G = shear modulus
G_{max} = the maximum shear modulus
G_{sec} = secant shear modulus
G_0' = rate independent dashpot
τ_{max} = the maximum shear stress
γ_r = reference strain

접 수 일: 2007. 11. 9
심 사 일: 2007. 11. 20
심사완료일: 2008. 5. 13