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Integrating Machine Reliability and Preventive Maintenance Planning in Manufacturing Cell Design

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Abstract. This paper presents a model for designing cellular manufacturing systems (CMS) by integrating system cost, machine reliability, and preventive maintenance (PM) planning. In a CMS, a part is processed using alternative process routes, each consisting of a sequence of visits to machines. Thus, a level of 'system reliability' is associated with the machines along the process route assigned to a part type. Assuming machine reliabilities to follow the Weibull distribution, the model assigns the machines to cells, and selects, for each part type, a process route which maximizes the overall system reliability and minimizes the total costs of manufacturing operations, machine under-utilization, and inter-cell material handling. The model also incorporates a reliability based PM plan and an algorithm to implement the plan. The algorithm determines effective PM intervals for the CMS machines based on a group maintenance policy and thus minimizes the maintenance costs subject to acceptable machine reliability thresholds. The model is a large mixed integer linear program, and is solved using LINGO. The results point out that integrating PM in the CMS design improves the overall system reliability markedly, and reduces the total costs significantly.

Keywords: CMS Design, Machine Reliability, Maintenance Planning, Integer Programming.

1. INTRODUCTION

The cell design problem may be described as the process of grouping the machines into a number of cells, each capable of processing independently a family of part types (with some parts types processed in more than one cell), based on the machining requirements of the parts. Interested readers are referred to Wemmerlov and Hyer (1986), Joines *et al.* (1996), Selim *et al.* (1998), and Mansouri *et al*. (2000) for reviews of the extensive literature on cell formation techniques. Modern production equipments are capable of performing more than one operation, and as such each part type may be processed using different process plans. The objectives of the cell formation have traditionally been to reduce throughput times, material handling costs, set up times, and to simplify production flow and control (Askin and Estrada 1999). However, most cell formation work has been based on the assumption that the machines are 100% reliable. Machine failures, however, have a significant impact on the system performance (e.g., due date compliance, utilization, etc.) even if there are options to reroute the parts to alternative machines. Often, it is not possible to handle a machine failure as quickly as the production requirements demand. Delays due to machine breakdowns not only impact the production rate, they also lead to scheduling problems which decrease the productivity of the entire manufacturing operation. This issue points out the importance of machine reliability consideration in cell formation decisions and during the operation allocation process.

Generally, in a cellular manufacturing system (CMS) the reliability configuration of the machines along a process plan is a series structure, whereas in a job shop the reliability structure is parallel, making it easier to reroute

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a part to another identical machine in case of any machine failure. In a CMS, however, intercellular transportation arrangements are needed to handle the same situation (Seifoddini and Djassemi 2001). Thus, in a CMS a higher level of machine reliability is needed to maintain a high CMS performance level. Furthermore, the CMS design process should include machine reliability consideration to effectively anticipate and plan for the adverse effects of machine breakdowns.

Machines are the major components which account for a significant share of the capital investment in CMS. Although machine reliability plays an important role in the performance of a CMS, it deteriorates over time as the machine ages. To stem the deterioration and to improve the reliability of machines, preventive maintenance (PM) plans are devised and put into effect to minimize the cumulative failure probability of a machine. In the multi-machine environment of a CMS this requirement is critical as unplanned breakdowns will halt the entire system and adversely impact the overall CMS performance. Thus we submit that the incorporation of appropriate PM systems in the planning of CMS is one of the most important requirements in the modern manufacturing sector. It is noted, however, that PM is justified only when it is cost effective, reduces random breakdowns, and extends the useful life of the equipment. Further, for PM to be effective, the failure rate of the equipment must be increasing with time (Jardine and Tsang 2006; Ebeling 1997), which is the case for manufacturing machinery (e.g., CNC machines).

The objective of this paper is, therefore, to develop a multi-objective CMS design model which minimizes the system costs and maximizes the overall system reliability along the selected process route, assuming the machine reliability to follow a Weibull distribution. The model also incorporates a PM plan in the CMS design process.

The paper is organized as follows. Relevant literature is reviewed in the next section. Section 3 presents a discussion of the CMS design model, along with the machine reliability analysis, the reliability-based group PM plan, and an algorithm to implement the PM plan. A numerical example is provided in Section 4 to demonstrate the applicability of the model, and some concluding remarks are given in Section 5.

2. RELEVANT LITERATURE

The number of research works dealing with the reliability aspects of CMS design is fairly small. Jeon *et al.* (1998) and Diallo *et al.* (2001) considered machine reliability in their analysis and development of the CMS. Jeon *et al.* (1998) considered alternative routes to develop cell configurations to handle the problem of a predefined number of machine breakdowns. Their model aimed to minimize waiting costs, late and early finish costs and machine investment costs to solve the machine breakdown problem. Diallo *et al.* (2001) considered the machines to be unreliable and consequently attempted to develop a cell configuration with alternative process plans to handle the machine failures.

Most of the machine reliability-based studies in CMS are directed towards performance evaluation. A number of studies (Seifoddini and Djassemi 2001; Logendran and Talkington 1997) emphasized the importance of machine reliability in relation to the desired output of the CMS.

In a CMS, the PM planning has to focus on the multi-machine environment of the cell to address the interdependent structure of the CMS. A number of studies have reviewed the various PM policies in manufacturing systems (e.g., Wang, 2002; Dekker *et al.*, 1997; Cho and Parlar 1991; Valdez-Flores and Feldman, 1989). Among the policies that may be applicable to CMS are the fixed group planned maintenance policy outlined by Dekker *et al.* (1997), or the group maintenance policy suggested by Wang (2002). Both plans are based on the concept of replacing a select group of components after a fixed interval of time, and addressing the unplanned failures of the components during the interval through repairs or minimal repairs. Another group maintenance policy studied by Wilderman *et al.* (1997) concerned the maintenance activities carried out on a group of equipment and involved a system-dependent set up cost that was the same for all the activities. The grouping of machines saved costs, since the execution of a group of activities required only one set up.

Talukder and Knapp (2002) developed a heuristic method for grouping equipment that would allow the application of PM in a series system with the goal of minimizing the total maintenance-related costs. The Weibull distribution was applied to represent the increasing failure rates of the equipment. The study derived a total cost model, and evaluated the PM intervals by minimizing the total cost for individual equipment groups.

Kardon and Fredendall (2002) developed a maintenance approach for multi-machine situations. Using the Weibull distribution, the approach determines the PM intervals such that the cumulative failure probability of a machine stays below a specified limit set by the user organization. For multi-machine/multi-component systems the study considered a number of maintenance policies, two of which may be applicable to PM decision processes in a CMS. One policy is a block replacement approach in which the components are classified into categories or blocks, based on the similarity of their PM intervals, so as to maintain a maximum tolerable cumulative failure probability. The other policy is to replace all the components by determining the shortest maintenance interval that maintains a tolerable overall cumulative failure probability. A comparison of the policies leads to the suggestion that a trial and error approach is needed to adjust the interval to achieve a minimum possible total cost in a specific situation.

Das *et al*. (2005) developed a PM planning approach centered on an 'effective maintenance interval' for individual machines in a CMS. The approach has four basic steps: in the first step the model implements the reliability based PM approach and determines a common PM interval for the machines based on a cumulative failure probability *upper bound* as set by the organization. In the next step the model computes, for each machine, the maximum interval possible by allowing a cumulative failure probability *upper bound* for each machine. In the third step the model determines the effective interval for each machine as an integer multiple of the common interval so that the effective interval is less than or equal to the maximum possible interval. In the final step a maintenance schedule is developed depending on the effective interval for each machine.

Recently, Das *et al*. (2006) proposed a CMS design model incorporating manufacturing system cost, machine system reliability and effective PM interval based on the PM planning approach in Das *et al.* (2005). A numerical example was solved to investigate the applicability of the model in the manufacturing cell design. The results indicated that PM has the potential to reduce the manufacturing system cost and increase the machine system reliability performance when compared with the results of the cell design without any PM consideration. Extending the work of Das *et al.* (2006), the present paper integrates the PM plan with machine system reliability and manufacturing system cost in the form of a mathematical model of the cell design process. A numerical example problem is solved to investigate the CMS design performance in terms of machine system reliability, and maintenance cost of the CMS.

3. MODEL DEVELOPMENT

3.1 Machine Reliability Analysis in a Process Plan

To examine the concept of machine reliability in the context of a CMS, we consider a small cell consisting of four part types to be processed on five machines. Table 1 presents a typical routing table for the set of part types with information about the operations of the part types, and the machines capable of performing these operations.

In general, each part type may be processed under various process plans, and under a given process plan each operation of a part type may be performed on one or more machines, giving rise to a number of process routes. For instance, part type 1 may be processed using any of the eight process routes listed in Table 2. Considering process route #6 as an example, the system reliability corresponding to the machines along this route is:

$$R_{2-3-4}(t) = R_2(t)R_3(t)R_4(t) \tag{1}$$

where $R_j(t)$ is the reliability of machine j at time t. Assuming that machine failures follow a Weibull distribution with the characteristic life θ and the shape parameter β, the reliability function for a machine j is:

$$R_j(t) = \exp[-(\frac{t}{\theta_j})^{\beta_j}] \tag{2}$$

and equation (1) is now written as:

$$R_{2-3-4}(t) = \prod_{j=\{2,3,4\}} \exp[-(\frac{t}{\theta_j})^{\beta_j}] \tag{3}$$

Or,

$$R_{2-3-4}(t) = \exp[-\sum_{j=\{2,3,4\}} (\frac{t}{\theta_j})^{\beta_j}] \tag{4}$$

After conversion to logarithmic scale may be written as:

$$LIR_{2-3-4}(t) = \ln\frac{1}{R_{2-3-4}(t)} = \sum_{j\in\{2,3,4\}} \left(\frac{t}{\theta_j}\right)^{\beta_j} \tag{5}$$

where $LIR_{2\text{-}3\text{-}4}(t)$, hereafter referred to as the reliability index, is the natural log inverse of the reliability of the machine sequence M2-M3-M4 corresponding to process route #6. For the Weibull distribution we have:

$$\theta_j = \frac{MTBF_j}{\Gamma(1+1/\beta_j)} \tag{6}$$

which, upon substitution in equation (5), results in:

$$LIR_{2-3-4}(t) = \sum_{j\in\{2,3,4\}} \left(\frac{t\cdot\Gamma(1+1/\beta_j)}{MTBF_j}\right)^{\beta_j} \tag{7}$$

where *MTBF* is the mean time between failures for machine *j*.

In a similar fashion, the reliability indices for each process route corresponding to each (*ip*) combination may be evaluated. Since for each part only one process route may be chosen, the objective would be to select process routes such that the sum of their reliability indices would result in an optimum level of overall reliability for the

Table 1. A typical routing table for a set of part types.

Part types	Process plans	Operations		
		1	2	3
1	1	M3, M2	M4, M5	
	2	M2, M4	M3	M1, M4
2	1	M2	M4, M5	M3
	2	M1, M3	M2	M5
3	1	M1, M4	M3, M2	M2
	2	M4, M5	M2, M4	M1, M3
4	1	M1, M3	M2, M4	M5
	2	M4, M5	M1	M4

CMS. This is in fact one of the objectives of the CMS design model to follow.

Table 2. Process routes for part type 1 in Table 1.

Process Routes	Process Plan	Machine Sequence in Process Route
1	1	M3-M4
2		M3-M5
3		M2-M4
4		M2-M5
5		M2-M3-M1
6	*2*	*M2-M3-M4*
7		M4-M3-M1
8		M4-M3-M4

3.2 Reliability-Based Group PM Plan

A reliability-based PM planning for CMS is developed with the aim of determining the largest possible PM interval to minimize the total maintenance cost by reducing the number of maintenance actions while keeping the individual machine failure probabilities below a predefined *Upper Bound* as may be specified by an organization. To set the limit on the cumulative failure probability of machines we have followed the approaches of Johnson (1959) and Kardon and Fredendall (2002). Assuming that *tp* is the interval at which PM is carried out, the cumulative failure probability of a machine *j* at time *tp* may be expressed as:

$$F_j(tp) = 1 - \exp[-(tp/\theta_j)^{\beta_j}] \tag{8}$$

Using equation (8), we may determine the PM interval *tp* when an *Upper Bound* on the cumulative failure probability of machine *j* at time *tp* is set by the organization; that is, determine:

$$tp \le \theta_j \{\ln\left[\frac{1}{1-F_j\,(tp)}\right]\}^{1/\beta_j}, \quad j=1,\ 2,\ \cdots; \tag{9}$$

such that,

$$F_j\,(tp) \le Upper\ Bound, \quad j=1,\ 2,\ \cdots,\ m \tag{10}$$

Based on these equations, the following optimization model (to be identified as *OptimInterval* model henceforth) may be proposed to determine the optimal PM interval:

OptimInterval Model *Maximize tp*

subject to:

$$tp \le \theta_j \{\ln\left[\frac{1}{1-F_j\,(tp)}\right]\}^{1/\beta_j} \quad j=1,\ 2,\ \cdots,\ m \tag{11}$$

$$F_j\,(tp) \le Upper\ Bound, \qquad j=1,\ 2,\ \cdots,\ m \tag{12}$$

The solution to the *OptimInterval* model is illustrated through numerical example 1.

Numerical Example 1.

We consider an example involving 14 machines. The

Table 3. Machine data for the example problem.

Machine	*b*	*cp*	*MTBF*	*MTTR*	*β*	*θ*	*cfr*	*cpr*	*Co*
M1	2000	185	299	117	1.64	334.29	1334	249	$150
M2	2000	70	231	55	1.24	247.61	400	206	
M3	2000	100	462	30	1.40	506.92	320	169	
M4	2000	104	123	35	1.72	137.73	321	141	
M5	2000	73	428	71	1.86	482.15	434	173	
M6	2000	91	80	12	1.39	87.70	231	185	
M7	2000	103	416	127	1.94	469.04	888	237	
M8	2000	70	232	72	2.00	261.48	504	249	
M9	2000	135	406	106	1.87	457.58	944	228	
M10	2000	105	293	31	1.19	310.94	308	147	
M11	2000	130	101	16	1.27	108.81	224	122	
M12	2000	53	405	126	1.37	443.25	653	317	
M13	2000	66	177	68	1.06	181.39	429	205	
M14	2000	179	258	62	1.53	286.20	796	244	

Note: *b* is machine capacity in hours; *cp* is the machine non-utilization penalty cost; *cfr* is the failure maintenance cost; *cpr* is the PM cost; $C_o$is the fixed cost of PM, to be explained later.

reliability data including *MTBF*, *MTTR* (mean time to repair), as well as the cost data are displayed in Table 3[1]. The *MTBF*, *MTTR*, β, and θ values are generated randomly. The β values are assumed to be greater than one to consider increasing failure rate of machines, and an *Upper Bound* cumulative failure probability of 0.25 is assumed. The solution to the *OptimInterval* model is presented in Table 4.

The optimal PM interval *tp* is 40.3 hours. To get an insight into the solution, the cumulative failure probability of each machine at *tp* = 40.3 hours is computed and displayed in column 3 of Table 4.

Table 4. Solution of *OptimInterval* Model.

Machine	Common Interval *tp*	*F(tp)*	$Tmax_j$
M1	40.3 hours	0.03	156.3
M2		0.10	90.9
M3		0.028	208.2
M4		0.112	66.9
M5		0.094	246.7
M6		0.25	40.3
M7		0.008	246.8
M8		0.020	140.4
M9		0.010	234.9
M10		0.084	109.0
M11		0.247	40.8
M12		0.037	178.4
M13		0.184	56.0
M14		0.048	126.9

It may be observed that only the cumulative failure probability of machine M6 has reached the *Upper Bound* of 0.25, whereas for other machines the cumulative failure probability is less than the *Upper Bound.* This implies that, by implementing PM actions after every 40.3 hours, the failure probabilities of the machines are maintained at or below the reliability threshold set by the *Upper Bound* level.

In column 4 of Table 4 the maximum possible PM interval, $Tmax_j$, for each machine is displayed. The $Tmax_j$ value for a machine *j* is obtained by solving equation (11) using the *Upper Bound* as the value of *F(tp)*. It is evident that for all the machines, other than M6, $Tmax_j \geq 40.3$ hours. For instance, for machine M1 the cumulative failure probability at *tp* = 40.3 hours is 0.03; however, from equation (11), at a value of $F_j(tp) = 0.25$, we obtain $Tmax_j = 156.3$, implying that machine M1 may be maintained at intervals of 156.3 hours without violating the cumulative failure probability *Upper Bound* of 0.25.Maintaining machines such as M1 at intervals of 40.3 hours results in too many maintenance actions unnecessarily.

By defining an 'effective' maintenance interval for a machine, we can avoid the unnecessary PM actions and still maintain a threshold on the machine failure probabilities. This idea underlies the development of the following algorithm which addresses the above limitation.

3.3 Algorithm for Effective Maintenance Planning

Step 1. Specify the values of *Co* (the fixed cost of carrying out each PM action), cpr_j (estimated average PM cost for machine *j* to take it back to as-good–as new condition), cfr_j (estimated failure repair cost for machine *j*), *Upper Bound,* θ_j, and β_j

Step 2. Compute the optimum PM interval *tp* using the *OptimInterval* model as described above

Step 3. Compute the maximum PM interval, $Tmax_j$, for each machine by setting $F_j(tp)$ = *Upper Bound* in equation (11):

$$Tmax_j = \theta_j \{\ln\left[\frac{1}{1-(Upper\,Bound)}\right]\}^{1/\beta_j} \quad (13)$$

Step 4. Compute the total cost *TC(T)* over the planning period *T* using the above inputs in the following sequence.

$$Y_j = \left\lfloor \frac{Tmax_j}{tp} \right\rfloor \quad \forall j \quad (14)$$

$$efftp_j = tp \cdot Y_j, \quad \forall j \quad (15)$$

$$N_j = \left\lfloor \frac{T}{efftp_j} \right\rfloor \quad \forall j \quad (16)$$

$$N_{\max} = \max\{N_j, j = 1, 2, \cdots, m\} \quad (17)$$

$$CPMcell = N_{\max} Co + \sum_{j=1}^{m} N_j cpr_j \quad (18)$$

$$CFMcell = \sum_{j=1}^{m} N_j cfr_j \left(\frac{efftp_j}{\theta_j}\right)^{\beta_j} \quad (19)$$

$$TC(T) = CPMcell + CFMcell \quad (20)$$

Y_j, N_j *integer*

In this model, Y_j computes the equivalent number of optimum intervals corresponding to $Tmax_j$ for machine *j*, $efftp_j$ represents the effective PM interval applicable to machine *j,* and N_j is the number of preventive maintenance actions to be scheduled for machine *j*. *CPMcell* and *CFMcell* represent, respectively, the total PM cost and the total failure repair cost for the CMS machines. *cpr* is the PM cost per occasion; *cfr* is the failure maintenance cost; and $C_o$is the fixed cost of PM.

[1] This example will be used in the sections that follow; therefore, Table 3 presents the complete set of data.

The expressions for *CPMcell* and *CFMcell* are derived based on the total cost model in Jardine and Tsang (2006). For details, the interested readers may refer to Das *et al*. (2006)

It may be noted here that if the planning period T is not exactly an integer multiple of the effective interval $efftp_j$, equation (16) will result in the last PM interval being a partial one, and as such the corresponding failure maintenance cost (equation 19) as well as the PM cost for this period will be underestimated; however, this cost decrease is, for all practical purposes, negligible and will not affect the design outcome.

Step 5. Record *CPMcell, CFMcell, TC(T)*, N_j, N_{max}

Step 6. Develop the PM schedule for the group of machines according to N_j

Numerical Example 2

The algorithm is illustrated using the data in Table 3 for numerical example 1. At the cumulative failure probability *Upper Bound* of 0.25, the solution of the *OptimInterval* model in Step 2 of the algorithm is the same as that presented in Table 4. For practical considerations, the optimum PM interval of 40.3 hours is set as $tp \approx 40$ hours, at which the cumulative failure probability of each machine is computed and displayed in column 3 of Table 4. As was pointed out earlier, the optimum *tp* corresponds to machine M6 whose cumulative failure probability is at the *Upper Bound* level of 0.25. The failure probabilities of all the other machines computed at $tp \approx 40$ are less than 0.25; equivalently, the corresponding *tp* for these machines would be higher than 40 hours if their failure probabilities are set at the *Upper Bound* value. This is done by implementing *Step 3* of the algorithm, which computes the maximum PM intervals, $Tmax_j$, for machines other than M6, as given in the last column of Table 4.

The detailed output from *Step 4* of the algorithm is presented in Table 5. Equations (14) and (15) evaluate the equivalent number of common PM intervals corresponding to the $Tmax_j$ values, and the effective PM interval for each machine *j*, respectively. For example, for machine M3, the maximum preventive maintenance interval of 208.2 hours can be written as $208.2 \geq (40)(5)$, implying that $Y_3 = 5$, and therefore, the effective PM interval for machine M3 is 200 hours. Equation (16) computes the number of times PM action is carried out on each machine *j* during the planning period *T*. In our case, T = 2000 hours, thus, M3 undergoes a total of 2000/200 = 10 PM actions during the planning period of 2000 hours. Equation (17) computes $N_{max} = 50$, the maximum number of times PM is carried out in the cell.

Based on the Y_j values, a PM schedule may now be defined. In this case there are 50 PM actions, therefore, when $Y_j = 1$, the PM schedule for machine *j* is in periods 1, 2, ⋯, 50 . When $Y_j = 2$, the PM schedule for machine *j* is in periods 1, 3, 5, ⋯, 49, and so on.

Equation (20) computes the total maintenance cost, $TC(T)$ = \$108,126, for the cell over the planning period *T*. The components of $TC(T)$ are the PM costs, *CPMcell* = \$73,370 (equation 18), and the total failure repair costs, *CFMcell* = \$ 34,756 (equation 19).

3.4 PM and Machine Reliability Analysis

We consider the CMS discussed in Section 3.1, where there is a PM schedule defined by the organization based on the algorithm introduced in Section 3.3. Having determined *tp*, the common maintenance interval, a machine *j* will undergo a PM action after every $Y_j \cdot tp$ time units, for a total of N_j times during the planning period *T*,

Table 5. Preventive maintenance plan determined by the algorithm.

Machines	Effective PM interval ***effint***	# of PM Actions (N)	# of PM intervals (***Y***)	Schedule of maintenance	*CPMcell* (\$)	*CFMcell* (*\$*)
M1	120	17	3	1, 4, 7, ⋯, 49	73,370	34,756
M2	80	25	2	1, 3, 5, ⋯, 49		
M3	200	10	5	1, 6, 11, ⋯, 49		
M4	40	50	1	1, 2, 3, ⋯, 50		
M5	240	9	6	1, 7, 13, ⋯, 49		
M6	40	50	1	1, 2, 3, ⋯, 50		
M7	240	9	6	1, 7, 13, ⋯, 49		
M8	120	17	3	1, 4, 7, ⋯, 49		
M9	200	10	5	1, 6, 11, ⋯, 49		
M10	80	25	2	1, 3, 5, ⋯, 49		
M11	40	50	1	1, 2, 3, ⋯, 50		
M12	160	13	4	1, 5, 9, ⋯, 49		
M13	40	50	1	1, 2, 3, ⋯, 50		
M14	120	17	3	1, 4, 7, ⋯, 49		

after which its reliability may be written as:

$$R_j(T) = [R_j(Y_j \cdot tp)]^{N_j} R_j(T - N_j \cdot Y_j \cdot tp), \quad (21)$$

assuming that the machine is restored to its original condition after a PM action is administered (Ebeling 1997).

For the Weibull distribution equation (21) becomes:

$$R_j(T) = \exp[-N_j \left(\frac{Y_j \cdot tp}{\theta_j}\right)^{\beta_j}] \times$$

$$\exp[-\left(\frac{T - N_j \cdot Y_j \cdot tp}{\theta_j}\right)^{\beta_j}] \quad (22)$$

Again, using process route #6 in Table 2 as an example, we now substitute equation (22) in equation (1) to obtain:

$$R_{2-3-4}(T) = \prod_{j=2,3,4} \exp[-N_j \left(\frac{Y_j \cdot tp}{\theta_j}\right)^{\beta_j}] \times$$

$$\prod_{j=2,3,4} \exp[-\left(\frac{T - N_j \cdot Y_j \cdot tp}{\theta_j}\right)^{\beta_j}] \quad (23)$$

which may be simplified as:

$$\ln \frac{1}{R_{2-3-4}(T)} = \sum_{j \in \{2,3,4\}} [N_j \left(\frac{Y_j \cdot tp}{\theta_j}\right)^{\beta_j} +$$

Table 6. Typical part type information for the numerical example.

Part (Demand)	Parameters	Operations according to process plan 1								Operations according to process plan 2							
		1		2		3		4		1		2		3		4	
1(3162)	Machine	M1	M4	M13	M7	M8	M3	M3	M6	M5	M13	M9	M8	M14	M2		
	Time (min)	2.65	4.40	3.40	3.45	2.95	4.22	4.83	3.70	2.89	2.96	3.90	3.40	3.50	3.97		
	Cost($)	3.09	5.13	3.96	4.03	3.44	4.92	5.64	4.31	3.38	3.45	4.56	3.96	4.09	4.63		
2(2976)	Machine	M3	M10	M8	M11	M6	M13	M12	M10	M9	M6	M8	M7	M8	M9	M8	M9
	Time (min)	4.46	3.48	2.55	5.00	4.48	4.42	2.72	3.50	3.04	2.80	2.68	4.92	2.90	3.16	3.92	4.62
	Cost($)	5.20	4.06	2.98	5.83	5.23	5.16	3.17	4.08	3.54	3.27	3.13	5.74	3.39	3.68	4.57	5.39
3(1881)	Machine	M10	M6	M2	M11	M13	M12			M9	M8	M3	M14	M7	M14		
	Time (min)	2.74	4.38	3.90	3.15	3.86	2.66			4.15	3.27	4.20	3.85	3.17	4.55		
	Cost($)	3.20	5.11	4.55	3.68	4.50	3.11			4.84	3.82	4.91	4.49	3.70	5.30		
4(2202)	Machine	M2	M1	M9	M3	M8	M12			M11	M13	M4	M10	M13	M7	M8	M9
	Time (min)	4.35	3.34	2.57	3.56	4.66	4.73			3.90	4.18	3.52	3.67	4.65	4.12	4.06	4.32
	Cost($)	5.08	3.90	3.00	4.15	5.43	5.52			4.55	4.87	4.11	4.29	5.42	4.81	4.73	5.03
5(2946)	Machine	M13	M3	M9	M6					M12	M14	M8	M4	M6	M11		
	Time (min)	3.44	4.07	2.96	3.19					4.33	3.83	3.58	3.46	3.37	4.93		
	Cost($)	4.01	4.75	3.46	3.72					5.05	4.47	4.18	4.04	3.93	5.75		
6(1935)	Machine	M12	M5	M7	M9					M13	M1	M14	M7				
	Time (min)	3.21	4.58	2.63	2.73					3.21	3.14	3.21	2.91				
	Cost($)	3.74	5.34	3.06	3.18					3.74	3.67	3.74	3.40				
7(2388)	Machine	M9	M13	M2	M9	M6	M5			M4	M12	M13	M7	M4	M10	M6	M1
	Time (min)	3.53	2.75	4.52	3.64	2.58	3.45			4.03	3.29	4.85	2.54	2.89	3.06	3.64	2.62
	Cost($)	4.12	3.21	5.28	4.25	3.01	4.03			4.70	3.84	5.66	2.97	3.37	3.57	4.25	3.05
8(2766)	Machine	M4	M7	M14	M12	M2	M4			M10	M2	M2	M12	M3	M4	M4	M1
	Time (min)	4.84	3.02	4.46	2.51	4.04	3.75			4.78	4.84	4.73	4.97	4.88	4.28	4.66	4.99
	Cost($)	5.64	3.52	5.20	2.93	4.71	4.38			5.58	5.65	5.52	5.80	5.70	4.99	5.44	5.82
9(2151)	Machine	M12	M10	M2	M5	M13	M6	M3	M9	M7	M12	M8	M10	M11	M1		
	Time (min)	3.52	3.20	3.45	2.10	3.20	3.85	3.62	3.17	3.50	3.58	1.69	2.80	1.71	3.11		
	Cost($)	4.10	3.74	4.03	2.45	3.73	4.50	4.23	3.70	4.08	4.18	1.97	3.26	2.00	3.63		
10(1986)	Machine	M12	M2	M5	M13	M3	M12			M4	M13	M3	M5	M7	M2		
	Time (min)	2.45	3.51	3.69	3.04	3.03	3.49			2.84	2.87	1.90	3.28	3.79	2.06		
	Cost($)	2.85	4.10	4.30	3.55	3.53	4.07			3.32	3.34	2.22	3.83	4.43	2.41		

$$\left(\frac{T - N_j \cdot Y_j \cdot tp}{\theta_j}\right)^{\beta_j}] \tag{24}$$

Recalling that, for each machine, the planning period T is divided into a number of **effective intervals** (equation (16)), and that we ignore the last (partial) PM interval in case the planning period T is not an integer multiple of the effective interval, equation (24) now reduces to:

$$\ln \frac{1}{R_{2-3-4}(T)} = \sum_{j \in \{2,3,4\}} N_j \left(\frac{Y_j \cdot tp}{\theta_j}\right)^{\beta_j} \tag{25}$$

Using equation (6), equation (25) is now written as:

$$LIR_{2-3-4}(T) = \sum_{j \in \{2,3,4\}} N_j \left(\frac{Y_j \cdot tp \cdot \Gamma(1+1/\beta_j)}{MTBF_j}\right)^{\beta_j}$$

$$= \sum_{j = \{2,3,4\}} LIR_j(tp) \tag{26}$$

where we have defined:

$$LIR_j(tp) = N_j \left(\frac{Y_j \cdot tp \cdot \Gamma(1+1/\beta_j)}{MTBF_j}\right)^{\beta_j} \tag{27}$$

3.5 CMS Design Model

In this section, we describe the integration of the machine reliability and maintenance planning concepts into the multi-objective design model for a cellular manufacturing system.

It is assumed that there is a set of machines $j = 1, 2, \cdots, m$ to process a set of part types $I = 1, 2, \cdots, n$ with uniform demands d_i during the planning period T. The reliability data for the machines are available in terms of $MTBF$, $MTTR$, β, and θ. A part type i may be processed under any of the process plans $p = 1, 2, \cdots, P(i)$. For the sake of brevity, a part type-process plan combination will be represented by (ip) from hereon. The operations performed on an (ip) combination are $o = 1, 2, \cdots, O(ip)$, and the machines that can perform operation o of (ip) are represented by the set J_{ipo}. The corresponding refixturing cost and the operation cost are represented by $CR_{oj}(ip)$ and $CO_{oj}(ip)$,, respectively. The 0-1 decision variable $X_{ojc}(ip)$ equals 1 if operation o of (ip) is performed on machine j in cell c, and zero otherwise. The objective is to group the machines into a number of cells, and to assign each part type to one or more cell for processing so as to minimize the total costs and maximize the overall system reliability index as defined in section 3.4.

3.5.1 Objective Functions

The first objective function F_1 computes the total system costs consisting of the variable cost of manufacturing operations (VCM), the inter-cell material handling cost (MHC) and the penalty cost associated with machine non-utilization (MNC):

$$\text{Minimize } F_1 = VCM + MHC + MNC \tag{28}$$

The variable cost of manufacturing operations, VCM, may be expressed as:

$$VCM = \sum_{i=1}^{n} d_i \sum_{p=1}^{P(i)} \sum_{o=1}^{O(ip)} \sum_{j \in J_{ipo}} \{CO_{oj}(ip) + CR_{oj}(ip)\} \sum_{c=1}^{C} X_{ojc}(ip) \tag{28a}$$

The inter-cell material handling cost MHC computes the cost of moving the parts from cell c to cell $\hat{c}$:

$$MHC = \sum_{i=1}^{n} d_i \sum_{p=1}^{P(i)} \sum_{o=1}^{O(ip)-1} \sum_{j \in J_{ipo}} \sum_{\hat{j} \in J_{ip(o+1)}} \sum_{1 \le c, \hat{c} \le C} H_{ijc\hat{j}\hat{c}} X_{ojc}(ip) X_{(o+1)\hat{j}\hat{c}}(ip)$$

where, $H_{ijc\hat{j}\hat{c}}$ is the cost of moving a unit of part type i from machine j, after performing operation o in cell c, to machine $\hat{j}$ in cell $\hat{c}$ for the next operation (o+1). It is noted that MHC is a non-linear function, which may be linearized by replacing the product term $X_{ojc}(ip) X_{(o+1)\hat{j}\hat{c}}(ip)$ by a binary linearization variable, $Y_{ojc\hat{j}\hat{c}}(ip)$, which satisfies constraints (36) and (37) below. It is evident that $Y_{ojc\hat{j}\hat{c}}(ip)$ takes the value of 1 if and only if a unit of part type i is moved from machine j in cell c, after performing operation o, to machine $\hat{j}$ in cell $\hat{c}$ for operation (o+1). Thus, the resulting expression for MHC is:

$$MHC = \sum_{i=1}^{n} d_i \sum_{p=1}^{P(i)} \sum_{o=1}^{O(ip)-1} \sum_{j \in J_{ipo}} \sum_{\hat{j} \in J_{ip(o+1)}} \sum_{1 \le c, \hat{c} \le C} H_{ijc\hat{j}\hat{c}} Y_{ojc\hat{j}\hat{c}}(ip) \tag{28b}$$

Finally, the term MNC computes the penalty cost for the proportion of the time machine j is not utilized:

$$MNC = \sum_{j=1}^{m} cp_j \Big(1 - [\sum_{i=1}^{n} d_i \sum_{p=1}^{P(i)} \sum_{o=1}^{O(ip)} \frac{TO_{oj}(ip) + TR_{oj}(ip)}{A_j(T) b_j}] \Big) \sum_{c=1}^{C} X_{ojc}(ip) \tag{28c}$$

where b_j is the capacity of machine j, $A_j(T)$ is the inherent availability of machine j, and $A_j(T)b_j$ represents the effective capacity of machine j. In addition, $TO_{oj}(ip)$ and $TR_{oj}(ip)$ are, respectively, the operation and refixturing times corresponding to operation o of (ip) on machine j, and cp_j is the penalty cost of non-utilizing the capacity of machine j.

The second objective function F_2 computes a measure of the inverse of the system reliability, in natural logarithmic scale, over the set of all the (ip) combinations:

$$\text{Minimize } F_2 = \sum_{i=1}^{n} \sum_{p=1}^{P(i)} \sum_{o=1}^{O(ip)} \sum_{j \in J_{ipo}} \sum_{c=1}^{C} LIR_j(tp)\ X_{ojc}(ip) \tag{29}$$

where $LIR_j(tp)$ was defined in equation (27). Equation (29) generates a composite expression by adding up the reliability indices along all the feasible process routes for each (ip) combination. During the optimization process, the operation allocation variable $X_{ojc}(ip)$ is compelled to assign only one machine to each operation of the (ip) in order to comply with constraints (30) and (31), which will follow. Consequently, for each (ip), the solution will include the reliability indices of the machines for only one selected process route.

3.5.2 Constraints

1. Each part type is assigned to a single process plan. The binary variable $Z(ip)$ equals one if and only if part type i is processed under process plan p.

$$\sum_{p=1}^{P(i)} Z(ip) = 1, \quad \forall i \tag{30}$$

2. For a given (ip) combination, each operation of the process plan is assigned to one of the available machines in one of the cells.

$$\sum_{j \in J_{ipo}} \sum_{c=1}^{C} X_{ojc}(ip) = Z(ip), \quad \forall i, p, o \tag{31}$$

3. A machine j is assigned to at most one cell. The variable M_{jc} equals 1 if machine j is assigned to cell c, and 0 otherwise.

$$\sum_{c=1}^{C} M_{jc} \le 1 \quad \forall j \tag{32}$$

4. There is a user-defined upper limit on the number of machines allowed in a cell.

$$\sum_{j=1}^{m} M_{jc} \le UM \quad \forall c \tag{33}$$

5. A machine j has to be assigned to a cell c before any operation could be allocated to that machine.

$$\sum_{i=1}^{n} \sum_{p=1}^{P(i)} \sum_{o=1}^{O(ip)} X_{ojc}(ip) \ge M_{jc}, \quad \forall j, c \tag{34}$$

6. The allocated operations to a machine will not exceed its effective capacity.

$$\sum_{i=1}^{n} d_i \sum_{p=1}^{P(i)} \sum_{o=1}^{O(ip)} [\ TO_{oj}(ip) + TR_{oj}(ip)\ ] X_{ojc}(ip) \le b_j M_{jc} A_j(T), \quad \forall j, c \tag{35}$$

where $A_j(T)$, the inherent availability of machine j, is approximated as (Ebeling 1997):

$$A_j(T) \approx \frac{MTBF_j}{MTBF_j + MTTR_j}$$

7. Constraint for linearizing non-linear function for MHC in equation (28b) as described above:

$$X_{ojc}(ip) + X_{(o+1)\hat{j}\hat{c}}(ip) - 2Y_{ojc\hat{j}\hat{c}}(ip) \ge 0,$$
$$\forall i, p, o \in \{1,2,\cdots,O(ip)-1\},\ j \in J_{ipo}, \hat{j} \in J_{ip(o+1)}, c, \hat{c} \tag{36}$$

$$X_{ojc}(ip) + X_{(o+1)\hat{j}\hat{c}}(ip) - Y_{ojc\hat{j}\hat{c}}(ip) \le 1,$$
$$\forall i, p, o \in \{1,2,\cdots,O(ip)-1\},\ j \in J_{ipo}, \hat{j} \in J_{ip(o+1)}, c, \hat{c} \tag{37}$$

8. The last constraint set imposes integrality on relevant variables

$$X_{ojc}(ip),\ M_{jc},\ Z(ip),\ Y_{ojc\hat{j}\hat{c}}(ip) \in \{0, 1\},$$
$$\forall i, p, o, j \in J_{ipo}, \hat{j} \in J_{ip(o+1)}, c, \hat{c} \tag{38}$$

4. A NUMERICAL EXAMPLE

To illustrate the applicability of the model, a numerical example involving 14 machines and 22 part types is presented. The solution is obtained using the commercial solver LINGO 9. The total number of variables, integer variables and the constraints are, respectively, 553217, 553140, and 8216.

Table 6 displays a portion of the processing data for the first 10 part types. The relevant machine information for this example was already given in Table 3. As can be seen in Table 6, each part type may be processed using one of the two process plans. For example, under process plan 1, part type 1 has four operations; operation 1 may be assigned to either machine M1 or machine M4; the operation time on M1 is 2.65 minutes, and the corresponding cost is \$3.09. Also, the demand for part type 1 is 3162 units during the planning period. The information in Table 3 includes, for each machine, the machine capacity

(b), and the penalty cost (cp) associated with the non-utilization of the machine capacity, as well as the reliability parameters *MTBF*, *MTTR*, β, θ, and the related maintenance costs. For example, for machine M1, the capacity is 2000 hours, the penalty cost is $185 per percentage non-utilization of machine capacity, *MTBF* is 299 hours, and *MTTR* is 117 hours; the parameters of the Weibull distribution are β = 1.64, and θ = 334.29; the preventive

Table 7. Comparison of model results with and without preventive maintenance consideration.

Comparison Factors	Scenario 1 Integrated machine reliability and PM	Scenario 2 Machine reliability only, no PM
CASE 1		
Minimize F_1 (Objective function I) only		
F_1 value	$755,842.60	$755,842.60
F_1 components		
VCM ($)	$754,129.50	$754,129.50
MHC($)	$950.00	$950.00
MNC($)	$763.10	$763.10
F_2 value	376.17	1,906.25
Cell Configuration		
Cell 1	M1, M2, M3, M5	M1, M9, M10, M13
Cell 2	M4, M7, M12,	M4, M7, M12,
Cell 3	M6, M9, M10, M13	M2, M3, M6
Cell 4	M8, M11, M14,	M5, M8, M14, M11
CASE 2		
Minimize F_2 (Objective function II) only		
F_1 value	916,237.10	919,232.30
F_1 components		
VCM ($)	914,203.70	917,278.20
MHC($)	1,450.00	1,300.00
MNC($)	583.40	654.10
F_2 value	214.61	780.83
Cell configuration		
Cell 1	M1, M3, M5	M2, M3, M7
Cell 2	M8, M9	M9, M2
Cell 3	M3, M10, M14	M1, M5, M10, M13
Cell 4	M4, M7, M12	
CASE 3		
Minimize F_1 s.t. $F_2 \leq \epsilon$	ϵ = 214.61	ϵ = 780.83
F_1 value	915,887.10	885,651.10
F_1 components		
VCM ($)	914,203.70	883,868.30
MHC($)	1100.00	1100.00
MNC($)	583.40	682.80
F_2 value	214.61	780.83
Cell configuration		
Cell 1	M3, M7, M9, M12	M2
Cell 2	M4, M5	M3, M7, M9, M12
Cell 3	M1, M8, M10, M14	M1, M5, M10, M13
Maintenance Activity Costs:		
CPMcell ($)	73, 370.00	---
CFMcell ($)	34,756.00	184,142.00
TC(T) ($)	108126.00	184,142.00

and failure maintenance costs are \$249 and \$1334, respectively.

To evaluate the second objective function F_2, we need to compute LIR_j (equation (27)) for each (ip) combination which in turn depends on the values of tp, N_j, and Y_j. These parameters are already computed and listed in Table 5.

To examine the effects of the PM integration on the CMS design, the numerical example is also solved without the consideration of preventive maintenance in the CMS design. This is achieved by setting $N_j = 1$, $Y_j = 1$ and $tp = T = 2000$ hours in equation (27) and thereby transforming it into the following form:

$$LIR_j(tp) = \left(\frac{2000 \cdot \Gamma(1+1/\beta_j)}{MTBF_j} \right)^{\beta_j} \tag{39}$$

Accordingly, Table 7 summarizes the solution results under two scenarios: scenario 1 when PM is considered, and scenario 2 when it is not. Under each scenario, three cases are considered. In the first case, the multi-objective model is solved using a hierarchical approach to optimize the first objective function, F_1, only, subject to constraints (30)-(38), and ignoring the second objective function, F_2. The solution corresponds to an F_1 value of \$755, 842.60 (which is a lower bound on this objective function); the value of second objective function, F_2, evaluated at this solution point is 376.17 under scenario 1, and 1906.25 under scenario 2. Furthermore, under either scenario, the solution generates four cells, although the cell compositions in the two scenarios are different. For example, under scenario 1, cell 1 consists of machines M1, M2, M3, and M5, whereas under scenario 2 cell I consists of machines M1, M9, M10, and M13.

As is evident, there is a significant decrease in the value of the second objective function under scenario 1, implying an improved reliability performance when PM is considered.

In the second case, the model is solved to optimize the second objective function, F_2 only, subject to the same constraints as before, and ignoring the first objective function F_1. Under scenario 1, the solution results in an F_2 value of 214.61 (which is a lower bound on this objective function), and the value of first objective function, F_1, evaluated at this solution point is \$916,237.10. Under scenario 2, the respective values of the two objective functions are 780.83 and \$919,232.30. Once again, there is a significant decrease in the F_2 value under scenario 1 compared to scenario 2, indicating that the reliability performance may be greatly improved by integrating PM planning into the CMS design process.

The solution in the second case involves four cells in scenario 1 and three cells in scenario 2, with widely different cell compositions.

In the third case, the multi-objective model is solved using a pre-emptive solution approach, placing priority on the second objective function, and optimizing the first objective function subject to constraints (30)-(38) and the additional constraint:

$$\textit{Objective function } F_2 \quad \leq \quad \varepsilon$$

where $\varepsilon = 214.61$ under scenario 1, and $\varepsilon = 780.83$ under scenario 2. As may be observed in Table 7, under either scenario, the solution achieves the F_2 target values (i.e., 214.61 under scenario 1 and 780.83 under scenario 2) and results in an F_1 value of \$915887.10 under scenario 1, and \$885, 651.10 under scenario 2. Once again, the results indicate the CMS reliability performance improvement as a consequence of the PM consideration in the cell design. The solution in this case involves three cells under each scenario, although the cell compositions are widely different as expected.

Finally, Table 7 compares the total maintenance cost under the two scenarios as well. The maintenance costs are computed by following the group PM planning approach and the corresponding algorithm in Section 3.3. Under scenario 1, when PM is considered, the total maintenance cost $TC(T)$ for the CMS during the planning period is: \$108,126.00, which consists of the PM cost of $CPMcell$ = \$73,370.00 (equation (18)) and the failure repair cost of $CFMcell$ = \$34,756.00 (equation (19)). When PM is ignored, the machines are only subject to random failures, and there are no PM-related costs. Therefore, $CPMcell = 0$, and to compute $CFMcell$ in equation (19), we set $tp = T$, $N_j = 1$, and $Y_j = 1$. The total maintenance cost for the CMS in this case is \$184,142.00. As is evident, the integration of PM planning concepts into the CMS design process entails substantial benefits by improving the system reliability performance and thus reducing the unplanned machine downtime cost to a large extent.

5. CONCLUSIONS

We have presented a CMS design model which considers machine reliability and preventive maintenance planning assuming that machine failure times follow a Weibull distribution with increasing failure rate. The model is in the form of a large scale multi-objective 0-1 integer program.

This is a new approach that integrates machine reliability, system costs, and preventive maintenance planning in the overall design of the CMS. The model considers the alternative process routes to process a part type, evaluates the system reliability corresponding to the machines along a process route, and seeks to maximize the overall reliability of the cell while minimizing the overall system costs.

A numerical example is provided to demonstrate the application of the model. The results indicate that the consideration of PM planning in the CMS design process leads to a significant improvement in the reliability performance of the system, and a sizeable reduction in the total maintenance cost.

Finally, the model is computationally feasible, and as is demonstrated here, it may be solved using commercial software (e.g., Lingo 9).

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APPENDIX

Indices

$c \in \{1, 2, \cdots, C\}$	cells
$i \in \{1, 2, \cdots, n\}$	part types
$j \in \{1, 2, \cdots, m\}$	machines
$p \in \{1, 2, \cdots, P(i)\}$	process plan for part type i
Ip	a part type-process plan combination
$o \in \{1, 2, \cdots, P(i)\}$	operations of (ip)
$J_{ipo} \subset \{1, 2, \cdots, m\}$	set of machines that can perform operation o of *(ip)*

Parameters

$A_j(t)$	availability of machine j at time t
b_j	available time on machine j during the planning period
$CO_{oj}(ip)$	cost of performing operation o of (ip) on machine j
$CR_{oj}(ip)$	cost of refixturing a unit of (ip) for operation o on machine j
cp_j	penalty cost of non-utilization of the capacity of machine j
$CPMR_j$	average cost of PM per occasion for machine j
d_i	demand for part type i during planning period
$H_{ijc\hat{j}\hat{c}}$	cost of moving part type i from machine j in cell c to machine $\hat{j}$ in cell $\hat{c}$ to perform the next operation
$MTBF_j$	mean time between failures for machine j
$MTTR_j$	mean time to repair for machine j
N_j	number of PM intervals for machine j during the planning period
tp	common PM interval for the machines in a cell
$TO_{oj}(ip)$	time to perform operation o of (ip) on machine j
$TR_{oj}(ip)$	time to refixture (ip) for operation o on machine j
UM	maximum number of machines in a cell
Y_j	equivalent number of common intervals tp applicable to machine j (an integer)
β_j	shape parameter of Weibull distribution for machine j
θ_j	characteristic life of Weibull distribution for machine j

Decision variables

M_{jc}	= 1 if machine j is assigned to cell c; 0 otherwise
$X_{ojc}(ip)$	= 1 if operation o of (ip) is performed on machine j in cell c; 0 otherwise
$Y_{ojc\hat{j}\hat{c}}(ip)$	= 1 if (ip) moves to machine $\hat{j}$ in cell $\hat{c}$ to perform operation $(o+1)$ after performing operation o on machine j in cell c; 0 otherwise
$Z(ip)$	= 1 if part type i is processed under process plan p; 0 otherwise