A Computationally Efficient Finite Element Analysis Algorithm Considering 2-D Magnetic Properties of Electrical Steel Sheet

Yingying Yao*, Wei Li, Hee Sung Yoon**, Koji Fujiwara*** and Chang Seop Koh†**

Abstract – For taking account of the two-dimensional magnetic properties of a grain-oriented electrical steel sheet, the effective anisotropic tensor reluctivity is examined, and a computationally efficient algorithm is suggested by using the response surface method to model the two-dimensional magnetic properties. It is shown that the reconstructed two-dimensional magnetic properties are fairly effective to stabilize the convergence characteristics of the Newton–Raphson iteration in the nonlinear magnetic field analysis.

Keywords: Finite element analysis, Grain-oriented, Tensor permeability, Two-dimensional magnetic property

1. Introduction

It is well known that anisotropic electrical steel sheet such as grain-oriented steel has two-dimensional magnetic properties even under alternating magnetic field condition, and that the magnetic field intensity, ***H***, is not always parallel to the magnetic flux density, ***B***. Modeling this property is very important in precise finite element magnetic field analysis. Enokizono proposed two kinds of models. One is to decompose the magnetic field intensity into effective isotropic field and effective anisotropic field, like model B in Fig.1, and define an effective isotropic reluctivity ν [1]. This model has two difficulties when used in FE analysis. One is how to determine the effective isotropic field and the other is that the magnetic field analysis should be carried out iteratively. The other method is to use the effective anisotropic reluctivity $\bar{\nu}$, where the independent variables are the magnitude and the direction of the magnetic flux density [2]. This method, when applied to non-linear finite element magnetic field analysis, also has problem because Newton-Raphson (NR) iteration often does not converge [3]. Therefore, in order to enhance the convergence characteristics of NR iteration, Fujiwara proposed a spline technique of the measured data using Bezier spline [3].

In this paper, the two-dimensional magnetic properties of a grain-oriented steel sheet are measured using a two dimensional single sheet tester, and smoothed by using response surface method (RSM). It is then incorporated with the finite element method to develop a computationally efficient magnetic field analysis algorithm considering two-dimensional magnetic properties. Through numerical examples with single-phase transformer core, the algorithm is proven to give fast convergence of the NR iteration.

2. Definition and Modeling of Reluctivity Tensor

2.1 Reluctivity of Anisotropic Material

In anisotropic material, the direction of ***H*** may be different from that of ***B*** as shown in Fig. 1. Enokizono proposed two kinds of models, as shown Fig. 1, to simulate it. In model A, H' is the rotation of ***H*** to ***B***. With the effective anisotropic reluctivity $\kappa = H'/B$, the relationship between ***B*** and ***H*** can be written as follows [2]:

$$\begin{Bmatrix} H_x \\ H_y \end{Bmatrix} = \begin{bmatrix} \cos\theta_{bh} & \sin\theta_{bh} \\ -\sin\theta_{bh} & \cos\theta_{bh} \end{bmatrix} \begin{bmatrix} \kappa & 0 \\ 0 & \kappa \end{bmatrix} \begin{Bmatrix} B_x \\ B_y \end{Bmatrix} = \bar{\nu} \begin{Bmatrix} B_x \\ B_y \end{Bmatrix} \quad (1)$$

where $\bar{\nu}$ is reluctivity tensor, which has non-linearity with respect to the magnitude B and inclination angle θ_B of ***B***. According to [3], after some algebra, the reluctivity tensor $\bar{\nu}$ can be expressed as follows:

$$\begin{aligned} \bar{\nu} &= \begin{bmatrix} \nu_{xx}(B,\theta_B) & \nu_{xy}(B,\theta_B) \\ \nu_{yx}(B,\theta_B) & \nu_{yy}(B,\theta_B) \end{bmatrix} \\ &= \begin{bmatrix} H\cos\theta_h / B\cos\theta_B & 0 \\ 0 & H\sin\theta_h / B\sin\theta_B \end{bmatrix} \end{aligned} \quad (2)$$

Based on the $\bar{\nu}$ defined in (2), Fujiwara deduced a

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Received 17 December, 2007 ; Accepted 5 June, 2007

complex expression of NR formula [3]. It is more effective for the convergence of NR iteration than its original definition in [1].

It is clear from Fig. 1 that the mean value of $\nu_{xx}(B,\theta_B)$ gives the relation between H_x and B_x, and $\nu_{yy}(B,\theta_B)$ between H_y and B_y. The major difference between the anisotropic and isotropic problem is that ν_{xx} and ν_{yy} depend on both B and θ_B in the former, but only B in the latter. For a grain-oriented silicon steel, the magnitude and direction of $\boldsymbol{H}$ can be measured, interpolated and extrapolated, then ν_{xx} and ν_{yy} can be expressed as functions of B and θ_B, which are used for the nonlinear finite element analysis.

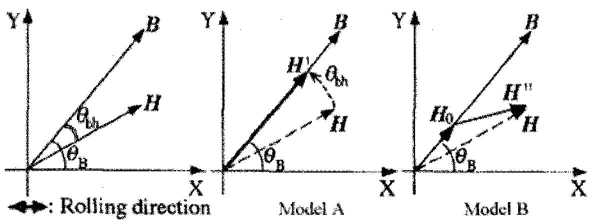


Fig. 1. The relationship between $\boldsymbol{B}$ and $\boldsymbol{H}$.

In this paper, similar to the numerical simulation for model A, a numerically efficient calculation method of an isotropic magnetic reluctivity in model B is proposed, where H_o is the magnitude of $\boldsymbol{H}$ when magnetic field is applied along the rolling direction i.e., when θ_B is zero. With the help of the effective anisotropic field H'', the relationship between $\boldsymbol{B}$ and $\boldsymbol{H}$ can be written as follows:

$$\begin{Bmatrix} H_x \\ H_y \end{Bmatrix} = \begin{bmatrix} \nu & 0 \\ 0 & \nu \end{bmatrix} \begin{Bmatrix} B_x \\ B_y \end{Bmatrix} + \begin{Bmatrix} H_x''(B,\theta_B) \\ H_y''(B,\theta_B) \end{Bmatrix} \tag{3}$$

From the measured data, the effective anisotropic field H'' is calculated as:

$$\begin{Bmatrix} H_x''(B,\theta_B) \\ H_y''(B,\theta_B) \end{Bmatrix} = \begin{Bmatrix} H\cos\theta_H - \nu B\cos\theta_B \\ H\sin\theta_H - \nu B\sin\theta_B \end{Bmatrix} \tag{4}$$

where the effective isotropic reluctivity ν is defined as:

$$\nu = |H_0|/|B| = |H_{\theta=0^\circ}|/|B_{\theta=0^\circ}| \tag{5}$$

Considering $B\cos\theta_B = B_x$ and $B\sin\theta_B = B_y$, the proposed $\boldsymbol{B}$-$\boldsymbol{H}$ relationship can be written as follows:

$$\begin{Bmatrix} H_x \\ H_y \end{Bmatrix} = \begin{bmatrix} \nu 0 \\ 0 \nu \end{bmatrix} \begin{Bmatrix} B_x \\ B_y \end{Bmatrix} + \begin{bmatrix} H\cos\theta_H / B\cos\theta_B - \nu & 0 \\ 0 & H\sin\theta_H / B\sin\theta_B - \nu \end{bmatrix} \begin{Bmatrix} B_x \\ B_y \end{Bmatrix} \tag{6}$$

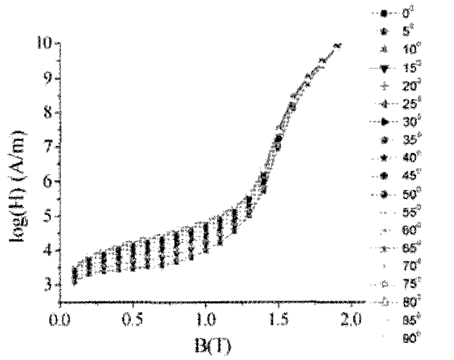
Comparing (1), (2), and (6), it is obvious that the suggested modeling (6) has the same form with (1) or (2). Therefore, both the FE formula and the Jacobian matrix of NR iteration will have the same form with (1) and (2). This will be given in detail at the following section.

We present a finite element analysis formula to take account of the anisotropic effect based on the above reluctivity models. The convergence of the NR iteration will be investigated.

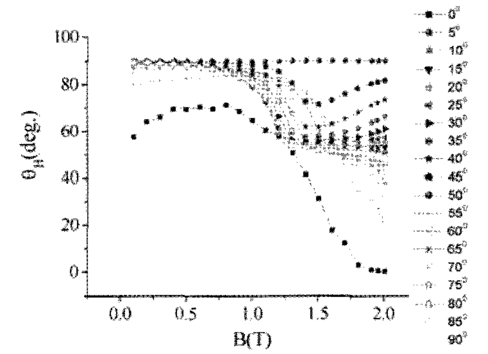
2.2 Modeling Method of 2D Magnetization Properties

The magnetization property of grain-oriented silicon steel poses nonlinearity, saturation, and anisotropy. Our measuring system can measure the magnitude H and the inclination angle θ_H of $\boldsymbol{H}$, the phase difference θ_{HB} between $\boldsymbol{B}$ and $\boldsymbol{H}$, and iron loss under different magnetic flux density B as well as different θ_B [4]. The measured data, shown in Fig. 2, are the H-B and θ_H-B relationships under different θ_B for a grain-oriented electrical steel sheet 35PG165 under the alternating 50Hz excitation with maximum value of the magnetic flux density of 2.0T, where θ_B indicates the inclination angle of $\boldsymbol{B}$ from the rolling direction.

In order to obtain a stable convergence of NR iteration in the finite element computation, it is necessary to precisely



(a) H-B curves under different θ_B



(b) θ_H -B curves under different θ_B

Fig. 2. Measured 2D magnetization properties of grain-oriented electrical sheet 35PG165.

present the magnetization property up to saturation. The smoothing model proposed in this paper is based on two parts. In the first part, the nonlinearities of a single *H-B*, *H-*θ_B, θ_H-*B* and θ_H-θ_B curves are splined by using C^1 continuous function. And in the second part, the data fitting procedure of anisotropic magnetic property is used.

For a magnetization curve, a smoothing model which consists of three intervals is suggested as follows:

$$H = \begin{cases} interpolation & B \le B_m \\ a_2B^2 + a_1B + a_0 & B_m \le B \le B_s \\ c + v_0 B & B \ge B_s \end{cases} \quad (7)$$

$$\theta_H = \begin{cases} interpolation & B \le B_m \\ b_2B^2 + b_1B + b_0 & B_m \le B \le B_s \\ \theta_B & B \ge B_s \end{cases} \quad (8)$$

where B_m is the maximum value of the measured magnetic flux density. For the measured region, ($B<B_m$), the measured data are smoothed by using respond surface method (RSM) with multi-quadric radial basis function. In *d*-dimensional space, the interpolating function in the RSM, is given as [5]:

$$f(\mathbf{x}) = \sum_{j=1}^{N} a_j \sqrt{\left\|\mathbf{x} - \mathbf{x}_j\right\|^2 + h} \quad (9)$$

where $\|\cdot\|$ is Euclidian distance, $\mathbf{x}_j$ is the *j*-th sampling points, which is measured data in this paper , *h* is a shape parameter to control the curvature of single basis function in the neighborhood of the centre $\mathbf{x}_j$, *N* is the number of sampling points, and coefficients a_j is calculated using point matching method as follows:

$$\left[a_j\right] = \left[g_{ij}\right]^{-1}\left[f_i\right], \quad i, j = 1, 2, \cdots, N \quad (10)$$

where $f_i = f(\mathbf{x}_i)$ and $g_{ij} = \sqrt{\left\|\mathbf{x}_j - \mathbf{x}_i\right\|^2 + h}$.

In general, since the number of sampling points for anisotropic *B-H* data, that is measured data, is not sufficient, finding a pleasant fitted surface is very difficult. Therefore, the proposed algorithm includes the following aspects:

1) Smooth the measured 2D data in each direction by using 1D interpolation. For example, find the C-1 continuous function of *B* using $H(B,\theta_B)$ under specified θ_B as original data, and then find the C-1 continuous function of θ_B using the interpolated $H(B,\theta_B)$ under specified *B*. During interpolating the data, take the mid-point of the neighboring measured data to check the fitting error.

2) After smoothing $H(B,\theta_B)$ along each direction, a 2-D data surface is, then, constructed by 2D RSM.

3) 1D or 2D interpolation is suggested by using adaptive RSM, which adaptively inserts new sampling points to minimize the fitting error as much as possible.

The adaptive RSM with multi-quadric radial basis function includes the following three steps:

Step 1: With initial sampling points ($\mathbf{x}_i$, f_i), $i = 1, 2, \cdots, N$ find proper shape parameter *h* based on the condition number of the coefficient matrix in (10);

Step 2: With existing sampling points ($\mathbf{x}_i$, f_i) and shape parameter *h*, solve (10) to obtain $[a_j]$ and the interpolating function;

Step 3: Check construction error. If it is not small enough, insert a set of new sampling points, which is 3×3 in 2D, around the point where there is the biggest interpolation error.

The smoothed data surface by using RSM is compared with other interpolation methods such as Cubic, Bezier, and Spline function. Spline function failed in the fitting of BH data in this paper. The comparison of the fitting results of the *B-H* data among different methods for $\theta_B = 5^o$ and $\theta_B = 20^o$ is shown in Fig. 3 and Table I, where the error is defined as follows:

$$Error = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(f_{\text{interpolated}}(\mathrm{x}_i) - f_{\text{measured}}(\mathrm{x}_i)\right)^2} \quad (11)$$

It is clear that the proposed RSM with adaptive insertion of sampling points gives more accurate fitting curves than any other methods.

The measured *B-H* data are, then, extrapolated. When magnetic flux density is between B_m and B_s, *H* is extrapolated approximately by a quadratic function of *B*. When *B* is bigger than B_s, the gradient of the *B-H* curve is equal to the reluctivity of the vacuum. When the maximum measurable magnetic flux density is not very high, the extrapolation in the second interval is difficult. Therefore, the quadratic function is used as piecewise C^1 continuous form for smoothing the second interval of the magnetization property.

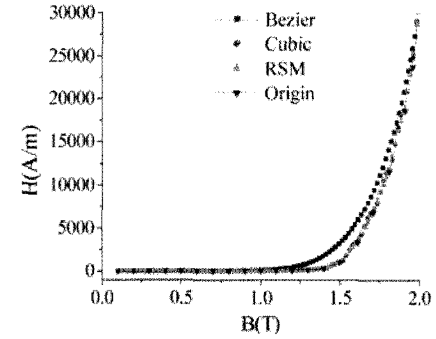
The effective anisotropic reluctivity $\overline{\nu}$ in (2), or the effective anisotropic field H'' in (4), can be calculated using the interpolated and extrapolated curves. The RSM model is, then, adopted to model the effective anisotropic field H'' or reluctivity $\overline{\nu}$ as a function of the magnitude of magnetic flux density *B* and inclination angle θ_B as follows:

$$\nu_{xx}(B,\theta_B) = \sum_{j=1}^{N} a_{xj}\sqrt{\left(B - B_j\right)^2 + \left(\theta_B - \theta_{Bj}\right)^2 + h_{\nu x}} \quad (12)$$

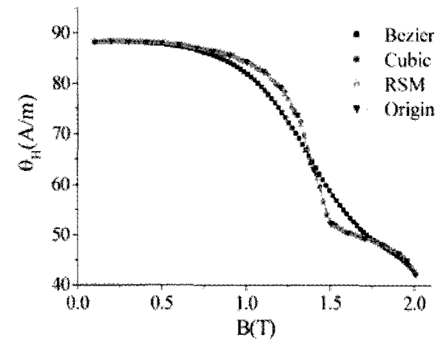
$$\nu_{yy}(B,\theta_B) = \sum_{j=1}^{N} a_{yj}\sqrt{\left(B - B_j\right)^2 + \left(\theta_B - \theta_{Bj}\right)^2 + h_{\nu y}} \quad (13)$$

$$H^{*}_{x}(B,\theta_B)=\sum_{j=1}^{N} b_{xj}\sqrt{(B-B_j)^2+(\theta_B-\theta_{Bj})^2+h_{Hx}} \quad (14)$$

$$H^{*}_{y}(B,\theta_B)=\sum_{j=1}^{N} b_{yj}\sqrt{(B-B_j)^2+(\theta_B-\theta_{Bj})^2+h_{Hy}} \quad (15)$$



(a) H-B curves under $\theta_B = 5^{\circ}$



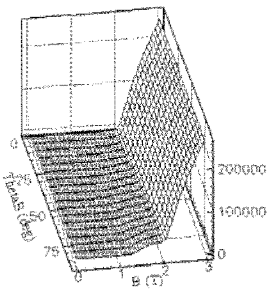
(b) θ_H -B curves under $\theta_B = 20^{\circ}$

Fig. 3. Interpolated *H-B* and θ_H*-B* curves with different methods.

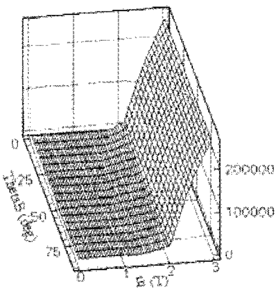
Table 1. Interpolation Error of Different Methods

Error	Bezier	Cubic	RSM	Adap.RSM
H at $\theta_B = 5^{\circ}$	9836.85	8851.68	136.5	10.89
θ_H at $\theta_B = 20^{\circ}$	23.74	74.12	0.321	0.027

Fig. 4. shows the effective anisotropic reluctivity computed using (12) ~ (15) with the measured BH data after interpolation and extrapolation.



(a) $\nu_{xx}(B,\theta_B)$



(b) $\nu_{yy}(B,\theta_B)$

Fig. 4. Interpolated and extrapolated results

3. Finite Element Formulation and Numerical Application

As we know, Maxwell's equations for a static magnetic field using the relationship between ***H*** and ***B*** in grain-oriented electrical steel sheet can be expressed as follows:

$$\begin{aligned}\nabla\times\boldsymbol{H}&=\boldsymbol{J}_s\\ \boldsymbol{B}&=\nabla\times\boldsymbol{A}\\ \boldsymbol{H}&=\bar{\nu}\boldsymbol{B}\end{aligned} \quad (16)$$

where $\boldsymbol{J}_s$ is the exciting current density, $\bar{\nu}$ reluctivity tensor, and $\boldsymbol{A}$ is vector magnetic potential. Galerkin approximation of the governing equation in an element is expressed in matrix form as follows:

$$\int_e \{[\nabla\times][N]\}^T[\bar{\nu}]\{[\nabla\times][N]^T\}[A]d\Omega=\int_e [N][\boldsymbol{J}_s]d\Omega \quad (17)$$

The left hand side of (17) is rewritten as:

$$[S]^e[A]^e=\int_e \left\{[\nabla\times][N]^T\right\}^T[\boldsymbol{H}]d\Omega \quad (18)$$

The nonlinear equation (18) is solved by using NR iteration. The Jacobian matrix is given as:

$$[J]^e=\int_e \left\{[\nabla\times][N]^T\right\}^T[\bar{\nu}_d]\left\{[\nabla\times][N]^T\right\}d\Omega \quad (19)$$

where $[\bar{\nu}]$, $[N]$ and $[A]$ have their usual meaning, and in 3D anisotropic problems we have the following:

$$[\nabla\times]=\begin{bmatrix}0 & -\frac{\partial}{\partial z} & -\frac{\partial}{\partial y}\\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x}\\ -\frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0\end{bmatrix},\quad \bar{\nu}_d=\frac{\partial\vec{H}}{\partial\vec{B}}=\begin{bmatrix}\frac{\partial H_x}{\partial B_x} & \frac{\partial H_x}{\partial B_y} & \frac{\partial H_x}{\partial B_z}\\ \frac{\partial H_y}{\partial B_x} & \frac{\partial H_y}{\partial B_y} & \frac{\partial H_y}{\partial B_z}\\ \frac{\partial H_z}{\partial B_x} & \frac{\partial H_z}{\partial B_y} & \frac{\partial H_z}{\partial B_z}\end{bmatrix} \quad (20)$$

In an application to the 2D magnetostatic problem, we have $A_x = A_y = 0$, $\partial/\partial z = 0$, and $\nu_{kz} = \nu_{zk} = 0$ $(k=x,y,z)$. Therefore, NR formula can be easily obtained based on (18) and (19).

A grain-oriented silicon steel sheet having a rectangular hole is used for a verification model [3]. The rolling direction θ_{RD} is the inclination angle from *x* axis. By changing the average flux density B_{leg} in the leg and θ_{RD}, the computational efficiency of the nonlinear and anisotropic field computation is investigated. Fig. 5 and Table 2 show the computed magnetic flux distribution and the required nonlinear iterations for convergence, respectively. From the table, we can see that the NR iteration converges efficiently.

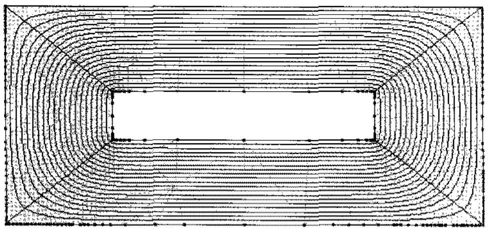


Fig. 5. Flux distribution (B_{leg} =1.0T, θ_{RD} =0deg)

Table 2. Number of Nonlinear Iterations

θ_{RD} = 45 deg				B_{leg} = 0.5(T)			
B_{leg}	0.1	0.5	1.0	θ_{RD}	15	45	75
Iterations	15	38	87	Iterations	97	35	28

θ_{RD} : angle from the rolling direction. B_{leg} : average flux density.

4. Conclusion

A uniform modeling method of 2D magnetization properties of a grain-oriented electrical steel sheet for two kinds of proposed models is presented in this paper. The measured data of the magnetization properties are fitted by proposed adaptive RSM and extrapolated with C^1 continuous function up to high saturation. This makes FE implementation easy and gives faster convergence of the NR iteration.

Acknowledgements

This work is supported by Basic Research Program of the Korea Science and Engineering Foundation under Grant R01-2007-000-11471-0.

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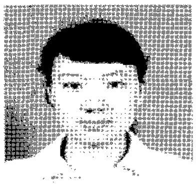

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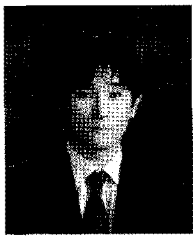

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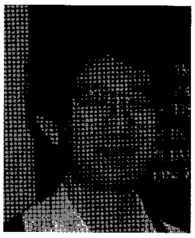

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