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채널 추정 오차를 고려한 WF-MIMO 시스템의 성능 분석

(Performance Analysis of WF-MIMO Systems with Channel Estimation Error)

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요 약

기존 WF(water-filling)-MIMO 시스템은 수신단에서 채널정보를 정확히 추정한다고 가정한다. 그러나, 일반적으로 정확한 채널 정보를 수신단에서 추정할 수 없기 때문에, 시스템에 채널 추정 오차를 고려해야 한다. 그래서 본 논문에서는 채널 추정 오차를 고려하여 WF-MIMO 시스템의 성능을 수식적으로 분석하였다. 이러한 분석 결과, 평균 오차 제곱이 10^{-4} 이하인 경우, 정확한 채널 추정이 이루어졌을 때와 동일한 시스템 성능을 갖게 됨을 확인할 수 있다.

Abstract

The conventional WF(water-filling)-MIMO systems assumes that the channel state information is perfectly known at receiver. However, since, generally, the perfect channel state information is not available at receiver, channel estimation error should be considered at the system. Therefore, in this paper, the performance of the conventional WF-MIMO systems is numerically analyzed when channel estimation error is considered. The analysis results show that mean square error of channel estimation up to 10^{-4} is tolerable to get the same performance obtained when perfect channel information is available.

Keywords : MIMO systems, water-filling, channel estimation error

I. Introduction

Recently, MIMO (Multiple-Input Multiple-Output) systems with several transmit and receive antennas have been spotlighted to increase system capacity among a number of technologies for next generation wireless communications[1]. Recent theory has shown that, in the case of transmitting signal with same power at each transmit antenna, called "uniform power allocation", system capacity is increased approximately in proportion to the number of transmit antennas[2].

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Several power allocation schemes on MIMO system have been studied, since performance of MIMO systems can be improved more than the uniform power allocation. Assuming that channel state information is known at transmitter, system performance can be improved by allocating different power at each antenna. Among them, the WF (water-filling)-MIMO system is known as an optimal solution to maximize system capacity[2~3].

However, practically, it is impossible to know the channel state information perfectly at transmitter as well as receiver. Therefore, performance of MIMO systems should be investigated with the consideration of channel estimation error[4].

Ⅱ. WF-MIMO Systems

1. MIMO Channel Modeling

A general channel model on MIMO system with M transmit antennas and N receiver antennas is considered. After Singular Value Decomposition (SVD)[5] of the channel matrix, the channel with the complex structure becomes virtual channel with parallel and orthogonal subchannels[2~3, 6]. Consider a transmitter and a receiver with N antennas, respectively. We can transform general MIMO channel, which consists of $N \times N$ channels, N to parallel subchannels by using unitary matrices obtained from SVD as transforming matrices at both transmitter and receiver. To get the unitary matrix used at the transmitter, receiver should feedback exact channel state information to transmitter.

Singular values obtained from SVD indicate the conditions of subchannel. Therefore, using the singular values, several power allocation schemes are available to improve the performance of MIMO systems.

In eigen mode channel, the received data vector is expressed as

$$\mathrm{y} = \mathrm{Hx} + \mathrm{n} = (\mathrm{U}\Lambda\mathrm{V}^{\mathrm{H}})\mathrm{x} + \mathrm{n}, \tag{1}$$

where H is an $N \times N$ channel matrix, and U, V is unitary matrices for HH^{H} and $\mathrm{H}^{\mathrm{H}}\mathrm{H}$, respectively, Λ is a diagonal matrix whose elements are square root of eigen values of HH^{H}, and n is $N \times 1$ AWGN(Additive White Gaussian Noise) vector.

By transforming (1) linearly with U, (2) and (3) are obtained[2].

$$\mathrm{U}^{\mathrm{H}}\mathrm{y} = \mathrm{U}^{\mathrm{H}}(\mathrm{U}\Lambda\mathrm{V}^{\mathrm{H}})\mathrm{x} + \mathrm{U}^{\mathrm{H}}(\mathrm{n}) = \Lambda\mathrm{V}^{\mathrm{H}}\mathrm{x} + \mathrm{U}^{\mathrm{H}}\mathrm{n} \tag{2}$$

$$\mathrm{y}' = \Lambda\mathrm{x}' + \mathrm{n}', \tag{3}$$

where $\mathrm{y}' = \mathrm{U}^{\mathrm{H}}\mathrm{y},\ \mathrm{x}' = \mathrm{V}^{\mathrm{H}}\mathrm{x},\ \mathrm{n}' = \mathrm{U}^{\mathrm{H}}\mathrm{n}$.

Since $\mathbf{\Lambda}$ is a diagonal matrix, (3) can be expressed as an element-wise form:

$$y'_m = \lambda_m x'_m + n'_m, \quad m = 1, 2, \cdots, N, \tag{4}$$

where λ_m is the mth singular value of H.

One of system performance evaluations obtained by eigen mode channel is the mutual information . Generally, the mutual information τ is obtained from singular values by SVD[2~3].

$$\begin{aligned}\tau &= \log_2\det(\mathrm{K}^{\mathrm{d}} + \mathrm{K}^{\mathrm{n}}) - \log_2\det(\mathrm{K}^{\mathrm{n}}) \\ &= \log_2\det(\mathrm{K}^{\mathrm{d}} + \mathrm{K}^{\mathrm{n}})(\mathrm{K}^{\mathrm{n}})^{-1} \\ &= \log_2\det(\mathrm{K}^{\mathrm{d}}(\mathrm{K}^{\mathrm{n}})^{-1} + \mathrm{I})\end{aligned} \tag{5}$$

where $\mathrm{K}^{\mathrm{d}} = \mathrm{H}E[\mathrm{xx}^H]\mathrm{H}^H$ and $\mathrm{K}^n = E[\mathrm{nn}^H]$.

For WF-MIMO system, the mutual information is obtained as

$$\tau = \sum_{m=1}^{N} \log_2(1 + \frac{p_m g_m}{\Gamma}), \tag{6}$$

where $p_m\,(m = 1, 2, \cdots, N)$ is an allocated power to the mth subchannel, $g_m = \frac{\lambda_m}{\sigma_n^2}$ is Signal-to-Noise Ratio (SNR) for the mth subchannel, and Γ is the SNR gap which is a constant value by error rate and modulation methods[7].

2. WF-MIMO Systems

Virtual subchannel conditions can be easily seen from singular values of MIMO channel. Therefore, several MIMO systems use these singular values to allocate power to transmit antennas and improve performance of MIMO systems. Among them, the WF-MIMO system uses power allocation schemes to maximize channel capacity under the required Bit Error Rate (BER).

Assuming that total transmit power is fixed, the WF-MIMO system is a typical link optimal algorithm[7]. To maximize system capacity under the constraint of the fixed total transmit power, the optimal solution for the WF-MIMO systems[3] is given by

$$\tau = \sum_{m=1}^{N} \log_2(1 + \frac{p_m g_m}{\Gamma}) \ \text{subject to} \ \sum_{m=1}^{N} p_m = P_T \tag{7}$$

where P_T is total transmit power. Using Largrange multiplier, the optimal solution of an allocated power on each subchannel is obtained as

$$p_m = \left(\nu - \frac{\Gamma}{g_m}\right)^+, \tag{8}$$

where ν is a constant value satisfying total transmit power constraint and $(\cdot)^+$ is an operator to choose a positive value.

From (8), the WF-MIMO systems allocate power on each subchannel in proportional to subchannel SNR to satisfy total power constraint. Therefore, if a subchannel condition is good, relatively large power is allocated on the subchannel and vice versa. Applying different modulation schemes such as 4QAM, 16QAM, etc, the subchannel with larger power can exploit more high-level modulation orders to satisfy a required BER.

Ⅲ. Performance Analysis

1. Subchannel SINR Evaluation

One of the fundamental assumptions for the WF-MIMO systems is that channel state information is perfectly known at transmitter by using feedback channel. However, in practical system it is impossible to estimate channel state information, exactly. Therefore, channel estimation error always exists between currently true channel and estimated channel. Consequently, the channel estimation error causes degradation of system capacity and BER performance.

To numerically induce the performance degradation of the water-filling scheme due to channel estimation error, channel model is expressed as

$$\mathrm{H} = \hat{\mathrm{H}} + \Delta\mathrm{H}, \tag{9}$$

where H is an $N\times N$ current true channel matrix, $\hat{\mathrm{H}}$ is an estimated channel matrix, and a channel error $\Delta\mathrm{H}$ is a random matrix whose elements are normally distributed. Therefore, a magnitude of $\Delta\mathrm{H}$ is referred as Mean Square Error (MSE) value, ϵ between true matrix and estimated matrix. In other words,

$$E_{\Delta\mathrm{H}}\left[\Delta\mathrm{H}\Delta\mathrm{H}^{\mathrm{H}}\right] = \epsilon\mathrm{I}, \tag{10}$$

where

$$\epsilon = \left(Tr\Delta\mathrm{H}\Delta\mathrm{H}^{H}\right)/N^2 \cong \left(TrE_{\Delta\mathrm{H}}\left[\Delta\mathrm{H}\Delta\mathrm{H}^{H}\right]\right)/\mathrm{N}$$

since $\frac{1}{N}\left(\Delta\mathrm{H}\Delta\mathrm{H}^{H}\right) \cong E_{\Delta\mathrm{H}}\left[\Delta\mathrm{H}\Delta\mathrm{H}^{H}\right]$ assuming that the dimension of $\Delta\mathrm{H}$ is sufficiently large. Tr denotes the trace value.

After SVD, the estimated channel matrix, $\hat{\mathrm{H}}$ can be expressed as

$$\hat{\mathrm{H}} = \hat{\mathrm{U}}\hat{\Lambda}\hat{\mathrm{V}}^{H}. \tag{11}$$

For general MIMO systems with power allocation schemes, when channel estimation error is considered, the relationship between transmitted data vector x and received data vector y is modeled as

$$\begin{aligned}\mathrm{y} &= \mathrm{H}\hat{\mathrm{V}}\mathrm{P}^{1/2}\mathrm{x} + \mathrm{n} = (\hat{\mathrm{H}} + \Delta\mathrm{H})\hat{\mathrm{V}}\mathrm{P}^{1/2}\mathrm{x} + \mathrm{n} \\ &= \hat{\mathrm{H}}\hat{\mathrm{V}}\mathrm{P}^{1/2}\mathrm{x} + \Delta\mathrm{H}\hat{\mathrm{V}}\mathrm{P}^{1/2}\mathrm{x} + \mathrm{n},\end{aligned} \tag{12}$$

where P is a diagonal matrix whose elements are powers allocated on parallel subchannels.

Then, (12) can be linearly transformed to

$$\begin{aligned}\hat{\mathrm{U}}^{\mathrm{H}}\mathrm{y} &= \hat{\mathrm{U}}^{\mathrm{H}}\hat{\mathrm{H}}\hat{\mathrm{V}}\mathrm{P}^{1/2}\mathrm{x} + \hat{\mathrm{U}}^{\mathrm{H}}\Delta\mathrm{H}\hat{\mathrm{V}}\mathrm{P}^{1/2}\mathrm{x} + \hat{\mathrm{U}}^{\mathrm{H}}\mathrm{n} \\ &= \hat{\Lambda}\mathrm{P}^{1/2}\mathrm{x} + \hat{\mathrm{U}}^{\mathrm{H}}\Delta\mathrm{H}\hat{\mathrm{V}}\mathrm{P}^{1/2}\mathrm{x} + \hat{\mathrm{U}}^{\mathrm{H}}\mathrm{n}.\end{aligned} \tag{13}$$

From (13), the influence of channel estimation error can be analyzed by evaluating each subchannel Signal to Interference plus Noise power Ratio ($SINR$), that is, a ratio of signal component ($\hat{\Lambda}\mathrm{P}^{1/2}\mathrm{x}$) power to interference and AWGN component ($\hat{\mathrm{U}}^{\mathrm{H}}\Delta\mathrm{H}\hat{\mathrm{V}}\mathrm{P}^{1/2}\mathrm{x} + \hat{\mathrm{U}}^{\mathrm{H}}\mathrm{n}$) power.

A signal component power for the mth subchannel is expressed as

$$\sigma_m^2 = \hat{\lambda}_m^2 p_m^2, \tag{14}$$

where p_m is the mth subchannel power and $\hat{\lambda}_m^2$ is the mth eigenvalue of $\hat{\mathrm{H}}$. Each subchannel power for the channel interference component ($\hat{\mathrm{U}}^{\mathrm{H}}\Delta\mathrm{H}\hat{\mathrm{V}}\mathrm{P}^{1/2}\mathrm{x}$) is evaluated by total interference power.

The total interference power σ_i^2, can be expressed as

$$\sigma_i^2 = TrE_{\Delta H, x}\left[(\hat{U}^H \Delta H \hat{V} P^{1/2} x)(\hat{U}^H \Delta H \hat{V} P^{1/2} x)^H\right] \\ = TrE_{\Delta H}\left[\hat{U}^H \Delta H \hat{V} P \hat{V}^H (\Delta H)^H \hat{U}\right]. \tag{15}$$

Consider a matrix G to evaluate interference power for each subchannel from total interference power.

$$\hat{U}^H \Delta H \hat{V} P \hat{V}^H (\Delta H)^H \hat{U} = G, \tag{16}$$

From (16) and (10) for a sufficient large system, each subchannel interference power is evaluated as follows:

$$\left(\sigma_i^2\right)_m = N\epsilon p_m, \tag{17}$$

where $\left(\sigma_i^2\right)_m$ is an interference power for the mth subchannel.

By (17), (15) is evaluated as

$$\sigma_i^2 = TrE_{\Delta H}[G] = N\epsilon P_T. \tag{18}$$

Therefore, the subchannel *SINR* can be calculated by transmitted signal power, interference power and AWGN power as

$$SINR_m = \frac{\sigma_m^2}{\left(\sigma_i^2\right)_m + \sigma_n^2} = \frac{\hat{\lambda}_m^2 p_m}{N\epsilon p_m + \sigma_n^2}, \tag{19}$$

where σ_n^2 is AWGN noise power.

2. Capacity Performance

The mutual information τ, considering channel estimation error, is obtained as

$$\tau = \log_2 \det(K^d (K^n)^{-1} + I), \tag{20}$$

where

$K^d = E\left[(\hat{\Lambda} P^{1/2} x)(\hat{\Lambda} P^{1/2} x)^H\right]$ and

$$K^n = E\begin{bmatrix}(\hat{U}^H \Delta H \hat{V} P^{1/2} x + \hat{U}^H n) \\ \times (\hat{U}^H \Delta H \hat{V} P^{1/2} x + \hat{U}^H n)^H\end{bmatrix}.$$

By exploiting the above results, τ is finally given as

$$\tau = \log_2 \prod_{m=1}^{N} (1 + \frac{\hat{\lambda}_m^2 p_m}{N\epsilon p_m + \sigma_n^2}) \\ = \sum_{m=1}^{N} \log_2 (1 + \frac{\hat{\lambda}_m^2 p_m}{N\epsilon p_m + \sigma_n^2}) \tag{21}$$

So far, we calculated theoretically the effects of channel estimation error for the conventional WF-MIMO systems, such as *SINR* and capacity performance. In the next section, the performances on WF-MIMO systems are evaluated by using (19) and (21).

Ⅳ. Simulation Results

Throughout the simulations, under rich-scattering environments, channel was assumed to be Rayleigh distributed[2~3] and modulation schemes available at all transmit antennas is 4QAM, 16QAM or 64QAM. For simplicity, the numbers of both transmit and receive antennas are same.

Capacity performance, throughput on various average SNRs and 4×4 MIMO system were evaluated in aspects of numerical analysis and computer simulations.

1. Capacity Performance

From (21), we can get numerical capacity performance given in Fig. 1 and Fig. 2. Fig. 1 shows the difference ratio between numerical capacity (21) and the exact capacity (20). From Fig. 1, up to $\epsilon = 10^{-4}$, numerical capacity is almost same as exact capacity within 3% error at 35 dB SNR. When MSE (ϵ) is equal to or greater than 10^{-3}, the capacity difference is seriously large.

Fig. 2 shows the relationship between average SNR and capacity. Generally, the larger average SNR is, the more increased capacity is. Furthermore, capacity becomes saturated, as average SNR goes high. From Fig. 2, if average SNR increases, the effect of channel estimation error from the view point of capacity is more and more increased. The capacity performance is influenced not only by MSE (ϵ) but

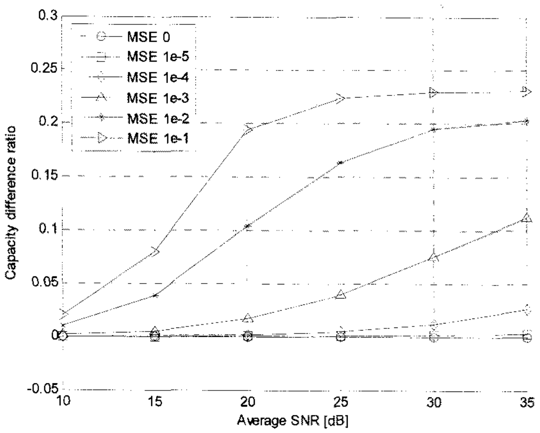

그림 1. 유도된 채널용량과 실제 채널용량의 오차 비율
Fig. 1. Capacity difference between numerical channel capacity and exact capacity.

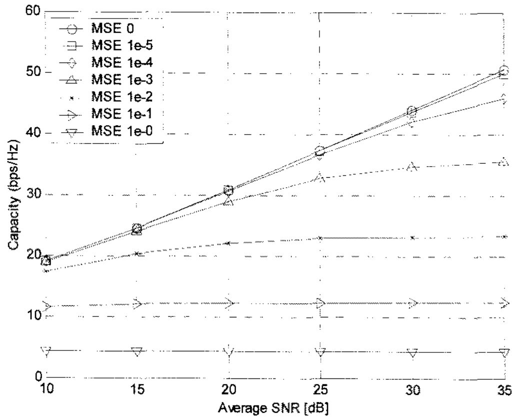

그림 2. 4x4 MIMO 시스템의 SNR에 따른 채널 용량
Fig. 2. Capacity versus average SNR for 4 transmit antennas and 4 receive antenna system.

also by the product of transmit power on subchannel and MSE (ϵ). Therefore, when MSE (ϵ) is same, the higher SNR is, the more degraded system is.

2. Throughput Performance

Since the water-filling scheme is adaptive to channel state to satisfy required BER, data throughput, which shows the number of correctly detected bits among allocated bits for each subchannel at receiver, is more effective performance measure than BER performance. The expression for data throughput performance at each subchannel[8~9] is given as

$$Throughput = (1 - BER)^{FL} \times \log_2 M, \tag{22}$$

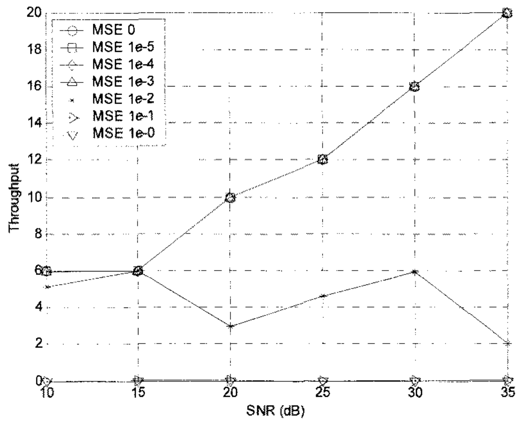

그림 3. 4x4 MIMO 시스템의 SNR에 따른 수식적 분석에 의한 throughput
Fig. 3. Throughput vs. average SNR for 4 transmit and receive antenna systems for numerical analysis.

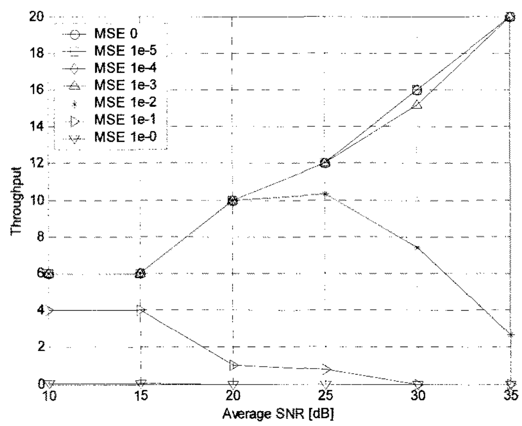

그림 4. 4x4 MIMO 시스템의 SNR에 따른 실험적인 throughput
Fig. 4. Throughput vs. average SNR for 4 transmit and receive antenna systems for computer simulation.

where $\log_2 M$ is the number of allocated bits at each subchannel and FL is the frame length. Throughout simulations, FL was 1024, and 4QAM, 16QAM and 64QAM were used for modulation schemes. Data throughputs evaluated from (22) are shown in Fig. 3. Fig. 4 shows the results of computer simulations to compare with numerical results of Fig. 3.

Both figures are almost same up to $\epsilon = 10^{-3}$. However, in the case of larger error than $\epsilon = 10^{-3}$, the results are more and more different. The differences between numerical results and simulation results are caused by the facts that, in simulations, the dimension of $\Delta \mathrm{H}$ cannot be assumed to be sufficiently large and that MSE (ϵ), equal to or larger than 10^{-3}, is unreliable as shown in the previous result. Thus, it is known that the expressions derived in section Ⅲ. Performance Analysis is valid.

Ⅴ. Conclusions

The conventional WF-MIMO systems require the assumption that the channel state information is perfectly known at receiver. However, practically, differences between exact channel state and estimated channel state degrade performance of the WF-MIMO systems.

In this paper, the effect of channel estimation error on the WF-MIMO systems is theoretically evaluated and analyzed. From the results, it is shown that, if MSE is same or smaller than 10^{-4}, the performance maintains almost same quality compared to the case when the perfect channel information is available at receiver.

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