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Common fixed point theorem and example in intuitionistic fuzzy metric space

박종서+ · 김선유
Jong Seo Park and Seon Yu Kim

진주교육대학교 수학교육과

Abstract

Park et.al.[10] defined the intuitionistic fuzzy metric space in which it is a little revised in Park[4], and Park et.al.[7] proved a fixed point theorem of Banach for the contractive mapping of a complete intuitionistic fuzzy metric space.

In this paper, we will establish common fixed point theorem for four self maps in intuitionistic fuzzy metric space. These results have been used to obtain translation and generalization of Grabiec's contraction principle.

Keywords : Common fixed point theorem, t-norm, t-conorm, intuitionistic fuzzy metric space.

1. Introduction

Kramosil and Michalek[3] introduced the concept of fuzzy metric space. George and Veeramani[2] studied this concept of fuzzy metric space and defined Hausdorff topology on fuzzy metric space. Grabiec[1] obtained the Banach contraction principle in setting of fuzzy metric spaces introduced by Kramosil and Michalek[3].

Recently, Park et.al.[10] defined the intuitionistic fuzzy metric space in which it is a little revised in Park[4], and Park et.al.[7] proved a fixed point theorem of Banach for the contractive mapping of a complete intuitionistic fuzzy metric space.

In this paper, we will establish common fixed point theorem for four self maps in intuitionistic fuzzy metric space. These results have been used to obtain translation and generalization of Grabiec's contraction principle.

2. Preliminaries

We give some definitions, properties of the intuitionistic fuzzy metric space as following :

Definition 2.1. ([5]) A operation $* : [0,1] \times [0,1] \rightarrow [0,1]$ is continuous $t-$norm if $*$ is satisfying the following conditions:

(a) $*$ is commutative and associative,
(b) $*$ is continuous,
(c) $a * 1 = a$ for all $a \in [0,1]$,
(d) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ $(a, b, c, d \in [0,1])$.

Definition 2.2. ([5])A operation $\diamond : [0,1] \times [0,1] \rightarrow [0,1]$ is continuous $t-$conorm if $\diamond$ is satisfying the following conditions:

(a) $\diamond$ is commutative and associative,
(b) $\diamond$ is continuous,
(c) $a \diamond 0 = a$ for all $a \in [0,1]$,
(d) $a \diamond b \geq c \diamond d$ whenever $a \leq c$ and $b \leq d$ $(a, b, c, d \in [0,1])$.

Definition 2.3. ([5])The $5-$tuple $(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous $t-$norm, $\diamond$ is a continuous $t-$conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions; for all $x, y, z \in X$, such that

(a)$M(x, y, t) > 0$,
(b)$M(x, y, t) = 1 \Longleftrightarrow x = y$,
(c)$M(x, y, t) = M(y, x, t)$,
(d)$M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
(e)$M(x, y, \cdot) : (0, \infty) \rightarrow (0, 1]$ is continuous,
(f)$N(x, y, t) > 0$,
(g)$N(x, y, t) = 0 \Longleftrightarrow x = y$,

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+ Corresponding Author : Jong Seo Park
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(h)$N(x,y,t) = N(y,x,t)$,
(i)$N(x,y,t) \diamond N(y,z,s) \geq N(x,z,t+s)$,
(j)$N(x,y,\cdot) : (0,\infty) \to (0,1]$ is continuous.

Note that (M,N) is called an intuitionistic fuzzy metric on X. The functions $M(x,y,t)$ and $N(x,y,t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Example 2.4. Let (X,d) be a metric space. Denote $a*b = ab$ and $a \diamond b = \min\{1, a+b\}$ for all $a,b \in [0,1]$ and let M_d, N_d be fuzzy sets on $X^2 \times (0,\infty)$ defined as follows :

$$M_d(x,y,t) = \frac{kt^n}{kt^n + md(x,y)},$$
$$N_d(x,y,t) = \frac{d(x,y)}{kt^n + md(x,y)}$$

for $k,m,n \in R^+ (m \geq 1)$. Then $(X, M_d, N_d, *, \diamond)$ is an intuitionistic fuzzy metric space. It is called the intuitionistic fuzzy metric space induced by the metric d.

Lemma 2.5. ([8])In an intuitionistic fuzzy metric space $(X,M,N,*,\diamond) = X$, $M(x,\ y,\ \cdot)$ is nondecreasing and $N(x,\ y,\ \cdot)$ is nonincreasing for all $x,\ y \in X$.

Definition 2.6. ([9]) Let X be an intuitionistic fuzzy metric space.

(a) A sequence $\{x_n\}$ in X is convergent to x in X iff $\lim_{n\to\infty} M(x_n,x,t) = 1$, $\lim_{n\to\infty} N(x_n,x,t) = 0$ for each $t > 0$.

(b) A sequence $\{x_n\}$ in X is called Cauchy sequence iff $\lim_{n\to\infty} M(x_{n+p},x_n,t) = 1$, $\lim_{n\to\infty} N(x_{n+p},x_n,t) = 0$ for all $t > 0$ and $p > 0$.

(c) X is called complete if every Cauchy sequence is convergent in it.

Remark 2.7. Since $*$ and $\diamond$ are continuous, it follows from $(d),(i)$ of Definition 2.3 that the limit of a sequence in an intuitionistic fuzzy metric space is unique, if it exists.

Definition 2.8. Functions M and N are continuous in intuitionistic fuzzy metric space iff whenever $\{x_n\} \to x$ and $\{y_n\} \to y$ then

$$\lim_{n\to\infty} M(x_n,y_n,t) = M(x,y,t),$$
$$\lim_{n\to\infty} N(x_n,y_n,t) = N(x,y,t) \ \text{ for all } \ t > 0.$$

Lemma 2.9. ([8])Let X be an intuitionistic fuzzy metric space. If there exists a number $k \in (0,1)$ such that for all $x,y \in X$ and $t > 0$,

$$M(x,y,kt) \geq M(x,y,t), \ \ N(x,y,kt) \leq N(x,y,t),$$

then $x = y$.

3. Compatible mapping

In this section, we introduce compatible mapping and properties for our main result.

Definition 3.1. Let A,B be mappings from intuitionistic fuzzy metric space X into itself. The mappings are said to be compatible if

$$\lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1,$$
$$\lim_{n\to\infty} N(ABx_n, BAx_n, t) = 0$$

for all $t > 0$, whenever $\{x_n\} \subset X$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$ for some $x \in X$.

Definition 3.2. Let A,B be mappings from intuitionistic fuzzy metric space X into itself. The mappings are said to be weak-compatible if they commute at their coincidence points. that is, $Ax = Bx$ implies $ABx = BAx$.

Remark 3.3. Let (A,S) be pair of self mappings of intuitionistic fuzzy metric space X. Then (A,S) is commuting implies (A,S) is compatible. Also, (A,S) is compatible implies (A,S) is weak-compatible but the converse is not true.

Definition 3.4. Let A,B be mappings from intuitionistic fuzzy metric space X into itself. The mappings are said to be semi-compatible if

$$\lim_{n\to\infty} M(ABx_n, Bx, t) = 1,$$
$$\lim_{n\to\infty} N(ABx_n, Bx, t) = 0$$

for all $t > 0$, whenever $\{x_n\} \subset X$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x \in X$.

Lemma 3.5. Let A,B be self mappings on intuitionistic fuzzy metric space X. If B is continuous, then (A,B) is semi-compatible iff (A,B) is compatible.

Proof. Let (A,S) be semi-compatible. Then

$$\lim_{n\to\infty} M(ASx_n, Sx, t) = 1, \ \lim_{n\to\infty} N(ASx_n, Sx, t) = 0$$

for all $t > 0$, whenever $\{x_n\} \subset X$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$ for some $x \in X$. Therefore, since S is continuous,

$$\begin{aligned} &\lim_{n\to\infty} M(ASx_n, Sx, t) \\ &= \lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1, \\ &\lim_{n\to\infty} N(ASx_n, Sx, t) \\ &= \lim_{n\to\infty} N(ASx_n, SAx_n, t) = 0. \end{aligned}$$

Hence (A,S) is compatible. Also, the converse is true. □

4. Main Results

Theorem 4.1. Let A, B, S and T be self mappings of a complete intuitionistic fuzzy metric space satisfying

(a)$A(X) \subset T(X)$, $B(X) \subset S(X)$,

(b)A or S is continuous,

(c)Pair (A, S) is semi-compatible and (B, T) is weak-compatible,

(d)There exists $k \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$,

$$\begin{aligned} &M(Ax, By, kt) \\ &\geq \min\{M(By, Ty, t), M(Sx, Ty, t), M(Ax, Sx, t)\}, \\ &N(Ax, By, kt) \\ &\leq \max\{N(By, Ty, t), N(Sx, Ty, t), N(Ax, Sx, t)\}, \end{aligned}$$

(e)$\lim_{t\to\infty} M(x, y, t) = 1$, $\lim_{t\to\infty} N(x, y, t) = 0$ for all $x, y \in X$ and $t > 0$.

Then A, B, S and T have unique common fixed point in X.

Proof. Let $x_0 \in X$ be any arbitrary point as $A(X) \subset T(X)$ and $B(X) \subset S(X)$, there exist $x_1, x_2 \in X$ such that $Ax_0 = Tx_1$, $Bx_1 = Sx_2$. Inductively construct sequences $\{x_n\}, \{y_n\}$ such that $y_{2n+1} = Ax_{2n} = Tx_{2n+1}$, $y_{2n+2} = Bx_{2n+1} = Sx_{2n+2}$ for $n = 0, 1, 2, \cdots$. Now using (d) with $x = x_{2n}, y = x_{2n+1}$,

$$\begin{aligned} &M(Ax_{2n}, Bx_{2n+1}, kt) \\ &= M(y_{2n+1}, y_{2n+2}, kt) \\ &\geq \min\{M(Bx_{2n+1}, Tx_{2n+1}, t), \\ &\qquad M(Sx_{2n}, Tx_{2n+1}, t), M(Ax_{2n}, Sx_{2n}, t)\} \\ &= \min\{M(y_{2n+2}, y_{2n+1}, t), M(y_{2n}, y_{2n+1}, t), \\ &\qquad M(y_{2n+1}, y_{2n}, t)\} \\ &= \min\{M(y_{2n+1}, y_{2n+2}, t), M(y_{2n}, y_{2n+1}, t)\}, \\ &N(Ax_{2n}, Bx_{2n+1}, kt) \\ &= N(y_{2n+1}, y_{2n+2}, kt) \\ &\leq \max\{N(Bx_{2n+1}, Tx_{2n+1}, t), \\ &\qquad N(Sx_{2n}, Tx_{2n+1}, t), N(Ax_{2n}, Sx_{2n}, t)\} \\ &= \max\{N(y_{2n+2}, y_{2n+1}, t), N(y_{2n}, y_{2n+1}, t), \\ &\qquad N(y_{2n+1}, y_{2n}, t)\} \\ &= \max\{N(y_{2n+1}, y_{2n+2}, t), N(y_{2n}, y_{2n+1}, t)\}. \end{aligned}$$

Therefore

$$\begin{aligned} &M(y_{2n+1}, y_{2n+2}, kt) \\ &\geq \min\{M(y_{2n+2}, y_{2n+1}, \frac{t}{k}), M(y_{2n}, y_{2n+1}, \frac{t}{k}), \\ &\qquad M(y_{2n+1}, yx_{2n}, t)\} \\ &= \min\{M(y_{2n+1}, y_{2n+2}, \frac{t}{k}), M(y_{2n}, y_{2n+1}, t)\} \\ &\geq \min\{M(y_{2n+2}, y_{2n+1}, \frac{t}{k^2}), M(y_{2n}, y_{2n+1}, \frac{t}{k^2}), \\ &\qquad M(y_{2n+1}, yx_{2n}, t)\} \\ &\geq \cdots\cdots\cdots \\ &\geq \min\{M(y_{2n+1}, y_{2n+2}, \frac{t}{k^m}), M(y_{2n}, y_{2n+1}, t)\}, \end{aligned}$$

$$\begin{aligned} &N(y_{2n+1}, y_{2n+2}, kt) \\ &\leq \max\{N(y_{2n+2}, y_{2n+1}, \frac{t}{k}), N(y_{2n}, y_{2n+1}, \frac{t}{k}), \\ &\qquad N(y_{2n+1}, yx_{2n}, t)\} \\ &= \max\{N(y_{2n+1}, y_{2n+2}, \frac{t}{k}), N(y_{2n}, y_{2n+1}, t)\} \\ &\leq \max\{N(y_{2n+2}, y_{2n+1}, \frac{t}{k^2}), N(y_{2n}, y_{2n+1}, \frac{t}{k^2}), \\ &\qquad N(y_{2n+1}, yx_{2n}, t)\} \\ &\leq \cdots\cdots\cdots \\ &\leq \max\{N(y_{2n+1}, y_{2n+2}, \frac{t}{k^m}), N(y_{2n}, y_{2n+1}, t)\}. \end{aligned}$$

Taking limit as $m \to \infty$, we get

$$\begin{aligned} M(y_{2n+1}, y_{2n+2}, kt) &\geq M(y_{2n}, y_{2n+1}, t) \\ N(y_{2n+1}, y_{2n+2}, kt) &\leq N(y_{2n}, y_{2n+1}, t) \end{aligned}$$

for all $t > 0$.

Similarly, we get

$$\begin{aligned} M(y_{2n+2}, y_{2n+3}, kt) &\geq M(y_{2n+1}, y_{2n+2}, t) \\ N(y_{2n+2}, y_{2n+3}, kt) &\leq N(y_{2n+1}, y_{2n+2}, t). \end{aligned}$$

Thus for all n and $t > 0$,

$$\begin{aligned} M(y_n, y_{n+1}, kt) &\geq M(y_{n-1}, y_n, t) \\ N(y_n, y_{n+1}, kt) &\leq N(y_{n-1}, y_n, t). \end{aligned}$$

Therefore

$$\begin{aligned} M(y_n, y_{n+1}, t) &\geq M(y_{n-1}, y_n, \frac{t}{k}) \\ &\geq \cdots\cdots \geq M(y_0, y_1, \frac{t}{k^n}) \end{aligned}$$

$$\begin{aligned} N(y_n, y_{n+1}, t) &\leq N(y_{n-1}, y_n, \frac{t}{k}) \\ &\leq \cdots\cdots \leq N(y_0, y_1, \frac{t}{k^n}). \end{aligned}$$

Hence

$$\begin{aligned} \lim_{n\to\infty} M(y_n, y_{n+1}, t) = 1, \\ \lim_{n\to\infty} N(y_n, y_{n+1}, t) = 0 \end{aligned}$$

for all $t > 0$.

Now, for any integer p,

$$M(y_n, y_{n+p}, t) \geq M(y_n, y_{n+1}, \frac{t}{p}) * M(y_{n+1}, y_{n+2}, \frac{t}{p}) \cdots * M(y_{n+p-1}, y_{n+p}, \frac{t}{p}),$$

$$N(y_n, y_{n+p}, t) \leq N(y_n, y_{n+1}, \frac{t}{p}) \diamond N(y_{n+1}, y_{n+2}, \frac{t}{p}) \cdots \diamond N(y_{n+p-1}, y_{n+p}, \frac{t}{p}).$$

Therefore

$$\lim_{n\to\infty} M(y_n, y_{n+p}, t) \geq 1 * 1 * \cdots\cdots * 1 = 1,$$
$$\lim_{n\to\infty} N(y_n, y_{n+p}, t) \leq 0 \diamond 0 \diamond \cdots\cdots \diamond 0 = 0.$$

Hence $\{y_n\}$ is a Cauchy sequence in X which is complete. Therefore $\{y_n\}$ converges to $z \in X$. Its subsequences $\{Ax_{2n}\}$, $\{Bx_{2n+1}\}$, $\{Sx_{2n}\}$, $\{Tx_{2n+1}\}$ also converges to z. i.e.,

$$\{Ax_{2n}\} \to z, \quad \{Bx_{2n+1}\} \to z \qquad (1)$$
$$\{Sx_{2n}\} \to z, \quad \{Tx_{2n+1}\} \to z.$$

Let A be continuous function. Then

$$AAx_{2n} \to Az, \quad ASx_{2n} \to Az$$

Since pair (A, S) is semi-compatibility,

$$\lim_{n\to\infty} ASx_{2n} = Sz.$$

As limit of a sequence in intuitionistic fuzzy metric space is unique, we have

$$Az = Sz \qquad (2)$$

First, putting $x = z, y = x_{2n+1}$ in (d), we get

$$M(Az, Bx_{2n+1}, kt) \geq \min\{M(Bx_{2n+1}, Tx_{2n+1}, t), M(Sz, Tx_{2n+1}, t), M(Az, Sz, t)\},$$
$$N(Az, Bx_{2n+1}, kt) \leq \max\{N(Bx_{2n+1}, Tx_{2n+1}t), N(Sz, Tx_{2n+1}, t), N(Az, Sz, t)\}.$$

Taking limit as $n \to \infty$,

$$M(Az, z, kt) \geq \min\{M(z, z, t), M(Az, z, t), M(Az, Az, t)\} = M(Az, z, t),$$
$$N(Az, z, kt) \leq \max\{N(z, z, t), N(Az, z, t), N(Az, Az, t)\} = N(Az, z, t).$$

By Lemma 2.9, $Az = z$. Thus $Az = Sz = z$.

Second, since $A(X) \subset T(X)$, there exist $u \in X$ such that $z = Az = Tu$. Putting $x = x_{2n},\ y = u$ in (d), we get

$$M(Ax_{2n}, Bu, kt) \geq \min\{M(Bu, Tu, t), M(Sx_{2n}, Tu, t), M(Ax_{2n}, Sx_{2n}, t)\},$$
$$N(Ax_{2n}, Bu, kt) \leq \max\{N(Bu, Tu, t), N(Sx_{2n}, Tu, t), N(Ax_{2n}, Sx_{2n}, t)\}.$$

Taking limit as $n \to \infty$, we get

$$M(z, Bu, kt) \geq \min\{M(Bu, z, t), M(z, z, t), M(z, z, t)\} = M(Bu, z, t),$$
$$N(z, Bu, kt) \leq \max\{N(Bu, z, t), N(z, z, t), N(z, z, t)\} = N(Bu, z, t).$$

for all $t > 0$. By Lemma 2.9, $z = Bu = Tu$ and weak compatibility of (B, T) gives $TBu = BTu$. i.e., $Tz = Bz$.

Third, putting $x = z,\ y = z$ in (d), we have

$$M(z, Bz, kt) \geq \min\{M(Bz, Tz, t), M(Sz, Tz, t), M(Az, Sz, t)\},$$
$$N(z, Bz, kt) \leq \max\{N(Bz, Tz, t), N(Sz, Tz, t), N(Az, Sz, t)\}.$$

Using the results from Second, we get

$$M(z, Bz, kt) \geq \min\{M(Bz, Bz, t), M(z, Bz, t), M(z, z, t)\} = M(z, Bz, t),$$
$$N(z, Bz, kt) \leq \max\{N(Bz, Bz, t), N(z, Bz, t), N(z, z, t)\} = N(z, Bz, t).$$

for all $t > 0$, which gives $Bz = z$. Hence we get $z = Az = Bz = Sz = Tz$. that is, z is common fixed point of A, B, S and T.

Now, let S be continuous function. Then $SAx_{2n} \to Sz$, $S^2x_{2n} \to Sz$. Since pair (A, S) is semi-compatibility, $\lim_{n\to\infty} ASx_{2n} = Sz$.

Fourth, putting $x = Sx_{2n}$, $y = x_{2n+1}$ in (d), we get

$$\begin{aligned}&M(ASx_{2n}, Bx_{2n+1}, kt)\\ &\geq \min\{M(Bx_{2n+1}, Tx_{2n+1}, t),\\ &\quad M(SSx_{2n}, Tx_{2n+1}, t), M(ASx_{2n}, SSx_{2n}, t)\},\\ &N(ASx_{2n}, Bx_{2n+1}, kt)\\ &\leq \max\{N(Bx_{2n+1}, Tx_{2n+1}, t),\\ &\quad N(SSx_{2n}, Tx_{2n+1}, t), N(ASx_{2n}, SSx_{2n}, t)\}.\end{aligned}$$

Taking limit as $n \to \infty$,

$$\begin{aligned}&M(Sz, z, kt)\\ &\geq \min\{M(z, z, t), M(Sz, z, t), M(Sz, Sz, t)\}\\ &= M(Sz, z, t)\\ &N(Sz, z, kt)\\ &\leq \max\{N(z, z, t), N(Sz, z, t), N(Sz, Sz, t)\}\\ &= N(Sz, z, t).\end{aligned}$$

for all $t > 0$. By above Lemma 2.9, $Sz = z$.

Fifth, putting $x = z$, $y = x_{2n+1}$ in (d),

$$\begin{aligned}&M(Az, Bx_{2n+1}, kt)\\ &\geq \min\{M(Bx_{2n+1}, Tx_{2n+1}, t),\\ &\quad M(Sz, Tx_{2n+1}, t), M(Az, Sz, t)\},\\ &N(Az, Bx_{2n+1}, kt)\\ &\leq \max\{N(Bx_{2n+1}, Tx_{2n+1}, t),\\ &\quad N(Sz, Tx_{2n+1}, t), N(Az, Sz, t)\}.\end{aligned}$$

Taking limit as $n \to \infty$,

$$\begin{aligned}&M(Az, z, kt)\\ &\geq \min\{M(z, z, t), M(Sz, z, t), M(Az, z, t)\},\\ &= M(Az, z, t)\\ &N(Az, z, kt)\\ &\leq \max\{N(z, z, t), N(z, z, t), N(Az, z, t)\},\\ &= N(Az, z, t).\end{aligned}$$

By Lemma 2.9, $Az = z$. Therefore $Az = Sz = z$. Applying Second and Third, $Tz = Bz = z$. Thus $z = Az = Bz = Sz = Tz$. That is, z is unique common fixed point of A, B, S and T.

Finally, we show the uniqueness of common fixed point. Let w be another common fixed point of A, B, S and T. Then $w = Aw = Bw = Sw = Tw$. Putting $x = z$ and $y = w$ in (d),

$$\begin{aligned}&M(Az, Bw, kt)\\ &\geq \min\{M(Bw, Tw, t), M(Sz, Tw, t), M(Az, Sz, t)\},\\ &N(Az, Bw, kt)\\ &\leq \max\{N(Bw, Tw, t), N(Sz, Tw, t), N(Az, Sz, t)\}.\end{aligned}$$

Therefore

$$M(z, w, kt) \geq M(z, w, t), \quad N(z, w, kt) \leq N(z, w, t).$$

Thus $z = w$. Hence z is common fixed point of the four self maps A, B, S and T. □

Corollary 4.2. Let A be a self mappings of a complete intuitionistic fuzzy metric space X satisfying $\lim_{t\to\infty} M(x, y, t) = 1$, $\lim_{t\to\infty} N(x, y, t) = 0$ such that, for all $x, y \in X$, $k \in (0, 1)$ and $t > 0$

$$\begin{aligned}&M(Ax, Ay, kt) \geq M(x, y, t),\\ &N(Ax, Ay, kt) \leq N(x, y, t).\end{aligned}$$

Then A has a unique fixed point in X.

Example 4.3. Let (X, d) be the metric space with $X = [0, 1]$. Denote $a*b = ab$ and $a\diamond b = \min\{1, a+b\}$ for all $a, b \in [0, 1]$ and let M, N be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows :

$$M(x, y, t) = \frac{t}{t + d(x, y)}, \quad N(x, y, t) = \frac{d(x, y)}{t + d(x, y)}.$$

Then (M, N) is an intuitionistic fuzzy metric on X and $(X, M, N, *, \diamond)$ is an intuitionistic fuzzy metric space. Define self mappings A, B, S and T by

$$A(X) = \begin{cases} 0 & \text{if } 0 \leq x \leq \frac{1}{2} \\ \frac{1}{3} & \text{otherwise} \end{cases},$$

$$S(X) = \begin{cases} 2x & \text{if } 0 \leq x \leq \frac{1}{2} \\ 1 & \text{otherwise} \end{cases},$$

$$B(X) = \begin{cases} 0 & \text{if } 0 \leq x \leq \frac{1}{4} \\ \frac{1}{3} & \text{otherwise} \end{cases},$$

$$T(X) = \begin{cases} 0 & \text{if } x = 0 \\ \frac{1}{3} & \text{if } 0 < x \leq \frac{1}{4} \\ 1 & \text{if } \frac{1}{4} < x \leq 1. \end{cases}$$

Then S is continuous, the pair (A, S) is semi-compatible and (B, T) is weak-compatible. Also, $A(X) = B(X) = \{0, \frac{1}{3}\}$, $S(X) = [0, 1]$ and $T(X) = \{0, \frac{1}{3}, 1\}$ satisfy the containment conditions of Theorem 4.1. For $k = \frac{1}{2}$, (d) of Theorem 4.1 is satisfied and we obtain that 0 is the unique common fixed point of four mappings.

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저 자 소 개

Jong Seo Park
Professor of Chinju National University of Education
Research Area: Fuzzy mathematics, Fuzzy fixed point theory, Fuzzy differential equation
E-mail : parkjs@cue.ac.kr
Fax : +82-55-740-1230
Phone : +82-55-740-1238

Seon Yu Kim
Professor of Chinju National University of Education
Research Area: Fuzzy mathematics, Fuzzy differential equation
E-mail : sykim@cue.ac.kr