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Existence of Solutions for the Semilinear Fuzzy Integrodifferential Equations using by Successive Iteration

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Abstract

This paper is to investigate the existence theorem for the semilinear fuzzy integrodifferential equation in E_N by using the concept of fuzzy number whose values are normal, convex, upper semicontinuous and compactly supported interval in E_N. Main tool is successive iteration method.

Key words : Semilinear Fuzzy Integrodifferential Equations, fuzzy number, successive iteration

1. Introduction

Many authors have studied several concepts of fuzzy systems. Kaleva [3] studied the existence and uniqueness of solution for the fuzzy differential equation on E^n where E^n is normal, convex, upper semicontinuous and compactly supported fuzzy sets in R^n. Seikkala [7] proved the existence and uniqueness of fuzzy solution for the following equation:

$$\dot{x}(t) = f(t, x(t)) \ , \ \ x(0) = x_0,$$

where f is a continuous mapping from $R^+ \times R$ into R and x_0 is a fuzzy number in E^1. Diamond and Kloeden [2] proved the fuzzy optimal control for the following system:

$$\dot{x}(t) = a(t)x(t) + u(t), \ \ x(0) = x_0,$$

where $x(\cdot), u(\cdot)$ are nonempty compact interval-valued functions on E^1. Kwun and Park [4] proved the existence of fuzzy optimal control for the nonlinear fuzzy differential system with nonlocal initial condition in E_N^1 using by Kuhn-Tucker theorems. Balasubramaniam and Muralisankar [1] proved the existence and uniqueness of fuzzy solutions for the semilinear fuzzy integrodifferential equation with nonlocal initial condition. Recently, Park, Park and Kwun [6] find the sufficient condition of nonlocal controllability for the semilinear fuzzy integrodifferential equation with nonlocal initial condition.

In this paper, we study the existence theorem for the following semilinear fuzzy integrodifferential equation:

$$\frac{dx(t)}{dt} = A\left[x(t) + \int_0^t G(t-s)x(s)ds\right] \quad (1)$$
$$+f(t, x(t)) \ t \in I = [0, T],$$
$$x(0) = \phi_0 \in E_N, \quad (2)$$

where $A : I \to E_N$ is a fuzzy coefficient, E_N is the set of all upper semicontinuous convex normal fuzzy numbers with bounded α-level intervals, $f : I \times E_N \to E_N$ is nonlinear continuous functions, $G(t)$ is $n \times n$ continuous matrix such that $\frac{dG(t)x}{dt}$ is continuous for $x \in E_N$ and $t \in I$ with $\|G(t)\| \leq k$, $k > 0$.

2. Preliminaries

A fuzzy subset of R^n is defined in terms of membership function which assigns to each point $x \in R^n$ a grade of membership in the fuzzy set.

Such a membership function $m : R^n \to [0, 1]$ is used synonymously to denote the corresponding fuzzy set.

Assumption 1. m maps R^n onto $[0, 1]$.

Assumption 2. $[m]^0$ is a bounded subset of R^n.

Assumption 3. m is upper semicontinuous.

Assumption 4. m is fuzzy convex.

We denote by E^n the space of all fuzzy subsets m of R^n which satisfy assumptions 1-4; that is, normal, fuzzy

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convex and upper semicontinuous fuzzy sets with bounded supports.

In particular, we denote by E^1 the space of all fuzzy subsets m of R which satisfy assumptions 1-4.

A fuzzy number a in real line R is a fuzzy set characterized by a membership function m_a as $m_a : R \rightarrow [0, 1]$.

A fuzzy number a is expressed as $a = \int_{x \in R} m_a(x)/x$, with the understanding that $m_a(x) \in [0, 1]$ represent the grade of membership of x in a and $\int$ denotes the union of $m_a(x)/x$'s [5].

Let E_N be the set of all upper semicontinuous convex normal fuzzy number with bounded α-level intervals.

This means that if $a \in E_N$ then the α-level set.

$$[a]^\alpha = \{x \in R : m_a(x) \geq \alpha, \ \ 0 < \alpha \leq 1\}$$

is a closed bounded interval which we denote by

$$[a]^\alpha = [a_l^\alpha, a_r^\alpha]$$

and there exists a $t_0 \in R$ such that $a(t_0) = 1$. (see[4])

The support Γ_a of a fuzzy number a is defined, as a special case of level set by the following

$$\Gamma_a = \{x \in R : m_a(x) > 0\}.$$

Two fuzzy numbers a and b are called equal, $a = b$, if $m_a(x) = m_b(x)$ for all $x \in R$. It follows that

$$a = b \Leftrightarrow [a]^\alpha = [b]^\alpha \quad \text{for} \quad \text{all} \quad \alpha \in (0, 1]$$

A fuzzy number a maybe decomposed into its level sets through the resolution identity

$$a = \int_0^1 \alpha[a]^\alpha$$

where $\alpha[a]^\alpha$'s with α ranging from 0 to 1.

We denote the supremum metric d_∞ on E^n and the supremum metric H_1 on $C(I : E^n)$.

Definition 2.1 Let $a, b \in E^n$

$$d_\infty(a, b) = \sup\{d_H([a]^\alpha, [b]^\alpha) : \alpha \in (0, 1]\},$$

where d_H is the Hausdorff distance.

Definition 2.2 Let $x, y \in C(I : E^n)$

$$H_1(x, y) = \sup\{d_\infty(x(t), y(t)) : \ \ t \in I\}.$$

Let I be a real interval. A mapping $x : I \rightarrow E$ is called a fuzzy process.

We denote

$$[x(t)]^\alpha = [(x_l^\alpha)'(t), (x_r^\alpha)'(t)], \ \ t \in I, \ \ 0 < \alpha \leq 1.$$

The derivative $x'(t)$ of a fuzzy process x is defined by

$$[x'(t)]^\alpha = [(x_l^\alpha)'(t), (x_r^\alpha)'(t)], \ \ 0 < \alpha \leq 1$$

provided that is equation defines a fuzzy $x'(t) \in E_N$.

The fuzzy integral

$$\int_a^b x(t)dt, \ \ a, b \in I$$

is defined by

$$\Big[\int_a^b x(t)dt\Big]^\alpha = \Big[\int_a^b x_l^\alpha(t)dt, \int_a^b x_r^\alpha(t)dt\Big]$$

provided that the Lebesgue integral on the right exist.

Definition 2.3 (Puri and Ralescu [4]) The integral of a fuzzy mapping $F : [0, 1] \rightarrow E^n$ is defined levelwise by

$$\begin{aligned} \Big[\int_T F(t)dt\Big]^\alpha &= \int_T F_\alpha(t)dt \\ &= \Big\{\int_T f(t)dt : f : T \rightarrow R^n \\ &\qquad \text{is a measurable selection for } F_\alpha\Big\} \end{aligned}$$

for all $\alpha \in [0, 1]$.

Definition 2.4 A mapping $F : T \rightarrow E^n$ is bounded if there exists a constant $M > 0$ such that $d_\infty(F(t), \{0\}) \leq M$ for all $t \in T$.

It was proved by Puri and Ralescu [4] that a strongly measurable and integrably bounded mapping $F : T \rightarrow E^n$ is integrable (i.e, $\int_T F(t)dt \in E^n$).

Theorem 2.1 If $F : T \rightarrow E^n$ is continuous then it is integrable.

Theorem 2.2 Let $F, G : T \rightarrow E^n$ be integrable and $\lambda \in R$. Then

(1) $\int_T (F(t) + G(t))dt = \int_T F(t)dt + \int_T G(t)dt$
(2) $\int_T \lambda F(t)dt = \lambda \int_T F(t)dt$
(3) $d_\infty(F, G)$ is integrable.
(4) $d_\infty(\int_T F(t)dt, \int_T G(t)dt) \leq \int_T d_\infty(F(t), G(t))dt$.

Definition 2.5 [1] The fuzzy process $x : I \rightarrow E_N$ is a solution of equations (1)-(2) without the inhomogeneous term if and only if

$$\begin{aligned} (\dot{x}_l^\alpha)(t) &= \min\Big\{A_l^\alpha(t)\big[x_j^\alpha(t) \\ &\quad + \int_0^t G(t-s)x_j^\alpha(s)ds\big], \ i, j = l, r\Big\}, \\ (\dot{x}_r^\alpha)(t) &= \max\Big\{A_r^\alpha(t)\big[x_j^\alpha(t) \\ &\quad + \int_0^t G(t-s)x_j^\alpha(s)ds\big], \ i, j = l, r\Big\}, \end{aligned}$$

and

$$(x_l^\alpha)(0) = x_{0l}^\alpha, \qquad (x_r^\alpha)(0) = x_{0r}^\alpha.$$

Now we assume the following:

(H1) $S(t)$ is a fuzzy number satisfying, for $y \in E_N$ and $S'(t)y \in C^1(I : E_N) \cap C(I : E_N)$, the equation

$$\begin{aligned}\frac{d}{dt}S(t)y &= A\left[S(t)y + \int_0^t G(t-s)S(s)yds\right] \\ &= S(t)Ay + \int_0^t S(t-s)AG(s)yds, \ \ t \in I,\end{aligned}$$

such that

$$[S(t)]^\alpha = [S_l^\alpha(t), S_r^\alpha(t)],$$

and $S_i^\alpha(t)$ $(i = l, r)$ is continuous. That is, there exists a constant $c > 0$ such that $|S_i^\alpha(t)| \le c$ for all $t \in I$ and $S(0) = I$.

(H2) The inhomogeneous term $f : I \times E_N \to E_N$ is continuous function and satisfies a global Lipschitz condition.

$$\begin{aligned}d_H([f(s, x(s))]^\alpha, [f(y, y(s))]^\alpha) \\ \le c_2 d_H([x(s)]^\alpha, [y(s)]^\alpha)\end{aligned} \tag{3}$$

for all $x(\cdot), y(\cdot) \in E_N$ and a finite constant $c_2 > 0, cc_2 < 1$ and $f(s, \{0\})$ is bounded on $C(I : E_N)$.

3. Existence theorems

In this section, we prove the existence and uniqueness of fuzzy solutions for the equation (1)-(2).

The (1)-(2) is related to the following fuzzy integral equation :

$$x(t) = S(t)\phi_0 + \int_0^t S(t-s)f(s, x(s))ds, \tag{4}$$

$$x(0) = \phi_0 \in E_N, \tag{5}$$

where $S(t)$ is satisfy (H1).

Theorem 3.1 Suppose that hypotheses (H1)-(H2) are satisfied. Then there exists a unique solution $x(t)$ of the equation (1)-(2) on $C(I : E_N)$ and the successive iteration

$$x_0(t) = S(t)\phi_0 \tag{6}$$

$$x_{n+1}(t) = x_0(t) + \int_0^t S(t-s)f(s, x_n(s))ds, \tag{7}$$

are uniformly convergent to $x(t)$ on $C(I : E_N)$ $(n = 0, 1, 2, \ldots)$.

Proof. It is easy to see that all $x_n(t)$ are bounded on $C(I : E_N)$. Indeed $x_0(t) = S(t)\phi_0$ is bounded by hypothesis. Assume that $x_{n-1}(t)$ is bounded. From (6) and (7) we have,

$$\begin{aligned}&d_H([x_n(t)]^\alpha, \{0\}) \\ &= d_H\Big([x_0(t) + \int_0^t S(t-s)f(s, x_{n-1}(s))ds]^\alpha, \{0\}\Big) \\ &\le d_H([x_0(t)]^\alpha, \{0\}) \\ &\quad + d_H\Big(\Big[\int_0^t S(t-s)f(s, x_{n-1}(s))ds\Big]^\alpha, \{0\}\Big) \\ &\le d_H([x_0(t)]^\alpha, \{0\}) \\ &\quad + c\int_0^t d_H([f(s, x_{n-1}(s))]^\alpha, \{0\})ds\end{aligned}$$

Taking into account that

$$\begin{aligned}&d_H([f(s, x_{n-1}(s))]^\alpha, \{0\}) \\ &\le d_H([f(s, x_{n-1}(s))]^\alpha, [f(s, \{0\})]^\alpha) \\ &\quad + d_H([f(s, \{0\})]^\alpha, \{0\}) \\ &\le c_2 d_H([x_{n-1}(s)]^\alpha, \{0\}) + d_H([f(s, \{0\})]^\alpha, \{0\})\end{aligned}$$

We obtain that $x_n(t)$ is bounded. Thus, $\{x_n(t)\}$ is a sequence of bounded functions on $C(I : E_N)$.

Next we prove that $\{x_n(t)\}$ are continuous in $C(I : E_N)$. For $0 \le t \le t' \le T$,

$$\begin{aligned}&d_H([x_n(t)]^\alpha, [x_n(t')]^\alpha) \\ &= d_H\Big(\Big[x_0(t) + \int_0^t S(t-s)f(s, x_{n-1}(s))ds\Big]^\alpha, \\ &\qquad \Big[x_0(t') + \int_0^{t'} S(t'-s)f(s, x_{n-1}(s))ds\Big]^\alpha\Big) \\ &\le d_H([x_0(t)]^\alpha, [x_0(t')]^\alpha) \\ &\quad + d_H\Big(\Big[\int_0^t S(t-s)f(s, x_{n-1}(s))ds\Big]^\alpha, \\ &\qquad \Big[\int_0^{t'} S(t'-s)f(s, x_{n-1}(s))ds\Big]^\alpha\Big) \\ &= d_H([x_0(t)]^\alpha, [x_0(t')]^\alpha) \\ &\quad + d_H\Big(\int_0^t [S(t-s)f(s, x_{n-1}(s))]^\alpha ds, \\ &\qquad \int_0^{t'} [S(t'-s)f(s, x_{n-1}(s))]^\alpha ds\Big)\end{aligned}$$

$$
\begin{aligned}
&= d_H([x_0(t)]^\alpha, [x_0(t')]^\alpha) \\
&\quad + d_H\Big(\int_0^t [S(t-s)f(s, x_{n-1}(s))]^\alpha ds, \\
&\quad\quad \Big[\int_0^t S(t'-s)f(s, x_{n-1}(s))ds \\
&\quad\quad + \int_t^{t'} S(t'-s)f(s, x_{n-1}(s))ds \Big]^\alpha \Big) \\
&\le d_H([x_0(t)]^\alpha, [x_0(t')]^\alpha) \\
&\quad + \int_t^{t'} d_H([S(t'-s)f(s, x_{n-1}(s))]^\alpha, \{0\})ds \\
&\quad + \int_0^t d_H\Big([S(t-s)f(s, x_{n-1}(s))]^\alpha, \\
&\quad\quad [S(t'-s)f(s, x_{n-1}(s))]^\alpha\Big) ds \\
&= d_H([x_0(t)]^\alpha, [x_0(t')]^\alpha) \\
&\quad + \int_t^{t'} d_H([S(t'-s)f(s, x_{n-1}(s))]^\alpha, \{0\})ds \\
&\quad + \int_0^t d_H\Big(\Big[S_l^\alpha(t-s)f_l^\alpha(s, x_{n-1}(s)), \\
&\quad\quad S_r^\alpha(t-s)f_r^\alpha(s, x_{n-1}(s)) \Big], \\
&\quad\quad \Big[S_l^\alpha(t'-s)f_l^\alpha(s, x_{n-1}(s)), \\
&\quad\quad S_r^\alpha(t'-s)f_r^\alpha(s, x_{n-1}(s)) \Big] \Big) ds \\
&= d_H([x_0(t)]^\alpha, [x_0(t')]^\alpha) \\
&\quad + \int_t^{t'} d_H([S(t'-s)f(s, x_{n-1}(s))]^\alpha, \{0\})ds \\
&\quad + \int_0^t \max\Big(\Big| S_l^\alpha(t'-s)f_l^\alpha(s, x_{n-1}(s)) \\
&\quad\quad - S_l^\alpha(t-s)f_l^\alpha(s, x_{n-1}(s)) \Big|, \\
&\quad\quad \Big| S_r^\alpha(t'-s)f_r^\alpha(s, x_{n-1}(s)) \\
&\quad\quad - S_r^\alpha(t-s)f_r^\alpha(s, x_{n-1}(s)) \Big| \Big) ds \\
&= d_H([x_0(t)]^\alpha, [x_0(t')]^\alpha) \\
&\quad + \int_t^{t'} d_H([S(t'-s)f(s, x_{n-1}(s))]^\alpha, \{0\})ds \\
&\quad + \int_0^t \max\Big(\Big| S_l^\alpha(t'-s) - S_l^\alpha(t-s) \Big| \\
&\quad\quad \times f_l^\alpha(s, x_{n-1}(s)), \\
&\quad\quad \Big| S_r^\alpha(t'-s) - S_r^\alpha(t-s) \Big| \\
&\quad\quad \times f_r^\alpha(s, x_{n-1}(s)) \Big) ds.
\end{aligned}
$$

We have

$$d_\infty(x_n(t), x_n(t')) \to 0 \ \text{ as } t \to t'.$$

Thus the sequence $\{x_n(t)\}$ is continuous on $C(I : E_N)$.

Relation (7) and its analogue corresponding to $n+1$ will give, for $n \ge 1$,

$$
\begin{aligned}
& d_H([x_{n+1}(t)]^\alpha, [x_n(t)]^\alpha) \\
&= d_H\Big(\Big[x_0(t) + \int_0^t S(t-s)f(s, x_n(s))ds \Big]^\alpha, \\
&\quad\quad \Big[x_0(t) + \int_0^t S(t-s)f(s, x_{n-1}(s))ds \Big]^\alpha \Big) \\
&\le d_H\Big(\Big[\int_0^t S(t-s)f(s, x_n(s))ds \Big]^\alpha, \\
&\quad\quad \Big[\int_0^t S(t-s)f(s, x_{n-1}(s))ds \Big]^\alpha \Big) \\
&\le \int_0^t d_H([S(t-s)f(s, x_n(s))]^\alpha, \\
&\quad\quad [S(t-s)f(s, x_{n-1}(s))]^\alpha) ds \\
&= \int_0^t d_H([S_l^\alpha(t-s)f_l^\alpha(s, x_n(s)), \\
&\quad\quad S_r^\alpha(t-s)f_r^\alpha(s, x_n(s))], \\
&\quad\quad [S_l^\alpha(t-s)f_l^\alpha(s, x_{n-1}(s)), \\
&\quad\quad S_r^\alpha(t-s)f_r^\alpha(s, x_{n-1}(s))]) ds \\
&\le c \int_0^t d_H([f_l^\alpha(s, x_n(s)), f_r^\alpha(s, x_n(s))], \\
&\quad\quad [f_l^\alpha(s, x_{n-1}(s)), f_r^\alpha(s, x_{n-1}(s))]) ds \\
&= c \int_0^t d_H([f(s, x_n(s))]^\alpha, [f(s, x_{n-1}(s))]^\alpha) ds \\
&\le c \int_0^t c_2 d_H([x_n(s)]^\alpha, [x_{n-1}(s)]^\alpha) ds.
\end{aligned}
$$

For $n = 0$, we have

$$
\begin{aligned}
& d_H([x_1(t)]^\alpha, [x_0(t)]^\alpha) \\
&= d_H\Big(\Big[x_0(t) + \int_0^t S(t-s)f(s, x_n(s))ds \Big]^\alpha, \\
&\quad\quad [x_0(t)]^\alpha \Big) \\
&= d_H\Big(\Big[\int_0^t S(t-s)f(s, x_0(s))ds \Big]^\alpha, \{0\} \Big) \\
&\le \int_0^t d_H([S(t-s)f(s, x_0(s))]^\alpha, \{0\}) ds \\
&= c \int_0^t d_H([f(s, x_0(s))]^\alpha, \{0\}) ds \\
&= c \cdot N,
\end{aligned}
$$

where

$$N = \int_0^t d_H([f(s, x_0(s))]^\alpha, \{0\})ds.$$

We obtain

$$\begin{aligned} & d_H([x_{n+1}(t)]^\alpha, [x_n(t)]^\alpha) \\ & \leq cc_2 \int_0^t d_H([x_n(s)]^\alpha, [x_{n-1}(s)]^\alpha)ds \\ & \leq (cc_2)^2 \int_0^t \int_0^t d_H([x_{n-1}(s)]^\alpha, [x_{n-2}(s)]^\alpha)dsds \\ & \vdots \\ & \leq c^{n+1}c_2^n \cdot N \cdot \frac{t^n}{n!}, \end{aligned}$$

where $N = \int_0^t d_H([f(s, x_0(s))]^\alpha, \{0\})ds$. Thus we get

$$\begin{aligned} & d_\infty(x_{n+1}(t), x_n(t)) \\ = & \sup_{0 \leq \alpha \leq 1} d_H([x_{n+1}(t)]^\alpha, [x_n(t)]^\alpha) \\ \leq & c^{n+1}c_2^n \cdot N \cdot \frac{t^n}{n!} \end{aligned}$$

where

$$N = \int_0^t d_H([f(s, x_0(s))]^\alpha, \{0\})ds.$$

Therefore, it implies the uniform convergence of the sequence $\{x_n(t)\}$. If we denote $x(t) = \lim_{n \to \infty} x_n(t)$ satisfies (4)-(5). It is obviously continuous on $C(I : E_N)$ and bounded.
To prove the uniqueness, let $y(t)$ be a continuous solution of (4)-(5) on $C(I : E_N)$. Then

$$y(t) = x_0(t) + \int_0^t S(t-s)f(s, y(s))ds, \quad (t \geq 0) \tag{8}$$

For $n \geq 1$,

$$\begin{aligned} & d_H([y(t)]^\alpha, [x_n(t)]^\alpha) \\ & \leq d_H\Big(\Big[x_0(t) + \int_0^t S(t-s)f(s, y(s))ds\Big]^\alpha, \\ & \qquad \Big[x_0(t) + \int_0^t S(t-s)f(s, x_{n-1}(s))ds\Big]^\alpha\Big) \\ & \leq d_H\Big(\Big[\int_0^t S(t-s)f(s, y(s))ds\Big]^\alpha, \\ & \qquad \Big[\int_0^t S(t-s)f(s, x_{n-1}(s))ds\Big]^\alpha\Big) \\ & \leq \int_0^t d_H([S(t-s)f(s, y(s))]^\alpha, \\ & \qquad [S(t-s)f(s, x_{n-1}(s))]^\alpha)ds \\ & = \int_0^t d_H\Big(\Big[S_l^\alpha(t-s)f_l^\alpha(s, y(s)), \\ & \qquad S_r^\alpha(t-s)f_r^\alpha(s, y(s))\Big], \\ & \qquad \Big[S_l^\alpha(t-s)f_l^\alpha(s, x_{n-1}(s)), \\ & \qquad S_r^\alpha(t-s)f_r^\alpha(s, x_{n-1}(s))\Big]\Big)ds \\ & \leq \int_0^t d_H([f_l^\alpha(s, y(s)), f_r^\alpha(s, y(s))], \\ & \qquad [f_l^\alpha(s, x_{n-1}(s)), f_r^\alpha(s, x_{n-1}(s))])ds \\ & = c\int_0^t d_H(f(s, y(s))]^\alpha, [f(s, x_{n-1}(s))]^\alpha)ds \\ & \leq cc_2 \int_0^t d_H([y(s)]^\alpha, [x_{n-1}(s)]^\alpha)ds \\ & \leq (cc_2)^2 \int_0^t \int_0^t d_H([y(s)]^\alpha, [x_{n-2}(s)]^\alpha)dsds \\ & \vdots \\ & \leq (cc_2)^n \int_0^t \cdots \int_0^t d_H([y(s)]^\alpha, [x_0(s)]^\alpha)ds \cdots ds. \end{aligned}$$

Since $c \cdot c_2 < 1$,

$$lim_{n \to \infty} x_n(t) = y(t) = x(t), \quad 0 \leq t \leq T.$$

4. Examples

Consider the semilinear one dimensional heat equation on a connected domain (0,1) for a material with memory, boundary condition $x(t, 0) = x(t, 1) = 0$ and with initial condition $x(0, z) = \phi_0(z)$, where $\phi_0(z) \in E_N$. Let $x(t, z)$ be the internal energy and $f(t, x(t, z)) = \tilde{2}tx(t, z)^2$ be the external heat.

Let $A = \tilde{2}\frac{\partial^2}{\partial z^2}$,and $G(t-s) = e^{-(t-s)}$, then the balance equation becomes

$$\frac{dx(t)}{dt} = \tilde{2}[x(t) - \int_0^t e^{-(t-s)}x(s)ds] + \tilde{2}tx(t)^2 \tag{9}$$

$$x(0) = \phi_0 \in E_N. \tag{10}$$

The α-level set of fuzzy number $\tilde{2}$ is $[2]^\alpha = [\alpha + 1, 3 - \alpha]$ for all $\alpha \in [0, 1]$. Then α-level sets of $f(t, x(t))$ is

$$[f(t, x(t))]^\alpha = t[(\alpha + 1)(x_l^\alpha(t))^2, (3 - \alpha)(x_r^\alpha(t))^2].$$

Further, we have

$$\begin{aligned}
&d_H([f(t,x(t))]^\alpha,[f(t,y(t))]^\alpha)\\
&= d_H(t[(\alpha+1)(x_l^\alpha(t))^2,(3-\alpha)(x_r^\alpha(t))^2],\\
&\quad t[(\alpha+1)(y_l^\alpha(t))^2,(3-\alpha)(y_r^\alpha(t))^2])\\
&= t\max\{(\alpha+1)|(x_l^\alpha(t))^2-(y_l^\alpha(t))^2|,\\
&\quad (3-\alpha)|(x_r^\alpha(t))^2-(y_r^\alpha(t))^2|\}\\
&\le 3T|x_r^\alpha(t)+y_r^\alpha(t)|\\
&\qquad \times\max\{|x_l^\alpha(t)-y_l^\alpha(t)|,|x_r^\alpha(t)-y_r^\alpha(t)|\}\\
&= c_2 d_H([x(t)]^\alpha,[y(t)]^\alpha),
\end{aligned}$$

where c_2 satisfies the inequality in hypothesis (H2). Then all the conditions stated in Theorem 3.1 are satisfied, so the problem (9)-(10) has a unique fuzzy solution.

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