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ON THE MINIMUM LENGTH OF SOME LINEAR CODES OF DIMENSION 6

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ABSTRACT. For $q^5 - q^3 - q^2 - q + 1 \le d \le q^5 - q^3 - q^2$, we prove the non-existence of a $[g_q(6,d), 6, d]_q$ code and we give a $[g_q(6,d) + 1, 6, d]_q$ code by constructing appropriate 0-cycle in the projective space, where $g_q(k,d) = \sum_{i=0}^{k-1} \lceil \frac{d}{q^i} \rceil$. Consequently, we have the minimum length $n_q(6,d) = g_q(6,d) + 1$ for $q^5 - q^3 - q^2 - q + 1 \le d \le q^5 - q^3 - q^2$ and $q \ge 3$.

1. Introduction and preliminaries

One of the interesting problems in coding theory is to determine the value $n_q(k,d)$ which denotes the smallest number n such that an $[n,k,d]_q$ code exists for given k, d and q. We shall deal with this problem for $k = 6$.

An $[n,k,d]_q$ code is a k-dimensional linear subspace of $\mathbb{F}_q^n$ of minimum distance d over the finite field $\mathbb{F}_q$ of order q. The Griesmer bound provides an important lower bound on the length n for an $[n,k,d]_q$ code,

$$n \ge g_q(k,d) := \sum_{i=0}^{k-1} \lceil \frac{d}{q^i} \rceil,$$

where $\lceil x \rceil$ denotes the smallest integer greater than or equal to x. In this paper, we shall prove the following theorems:

Theorem A. *For $q \ge 3$, a $[g_q(6,d), 6, d]_q$ code does not exist for $q^5 - q^3 - q^2 - q + 1 \le d \le q^5 - q^3 - q^2$, which means that $n_q(6,d) \ge g_q(6,d) + 1$.*

Theorem B. *There exists a $[g_q(6,d) + 1, 6, d]_q$ for $q^5 - q^3 - q^2 - q + 1 \le d \le q^5 - q^3 - q^2$.*

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From the above results, for small $q = 3, 4, 5$, we have

$$n_3(6,d) = g_3(6,d) + 1 \text{ for } d = 205, 206, 207,$$
$$n_4(6,d) = g_4(6,d) + 1 \text{ for } d = 941, 942, 943, 944,$$
$$n_5(6,d) = g_5(6,d) + 1 \text{ for } d = 2971, 2972, 2973, 2974, 2975.$$

When $q = 3$, it was already determined and we could find it in [5].

In [3], R. Hill constructed a large class of codes which meet the Griesmer bound and obtained the following theorem:

Theorem 1 ([3]). *Let* $d = sq^{k-1} - \sum_{i=1}^{p} q^{u_i - 1}$ *such that* $k > u_1 \geq u_2 \geq \cdots \geq u_p$ *with* $u_i > u_{i+q-1}$ *for* $1 \leq i \leq p - q + 1$, *where* $s = \lceil \frac{d}{q^{k-1}} \rceil$. *If* $\sum_{i=1}^{\min\{s+1,p\}} u_i \leq sk$, *then* $n_q(k,d) = g_q(k,d)$.

For $k \leq 5$, there are many results for $q \leq 5$ (see [5]). In this paper, we shall treat the problem to find the exact value of $n_q(6,d)$. By Theorem 1, we have $n_q(6,d) = g_q(6,d)$ for $q^5 - q^3 - q^2 + 1 \leq d \leq q^5$ and $q^5 - q^4 - q + 1 \leq d \leq q^5 - q^4$ for any q. For $k = 6$, the results obtained by Hamada-Helleseth [2] and Maruta [5] are restricted to the ternary code. Our results is to determine the exact value of $n_q(6,d)$ in the range $q^5 - q^3 - q^2 - q + 1 \leq d \leq q^5 - q^3 - q^2$ for an arbitrary $q \geq 3$ by using a finite projective geometry.

Let $\mathbb{P}^{k-1}$ be the $(k-1)$-dimensional projective space over $\mathbb{F}_q$. As a notational convention, in this paper, P, P_i, Q, R, (resp. l, l_i, δ, δ_i, Δ, Δ_i, Π, Π_i) etc. stand for points (resp. lines, planes, solids, 4-flats) in $\mathbb{P}^{k-1}$. We denote by $\mathcal{F}_j$ the set of all j-flats in $\mathbb{P}^{k-1}$ and θ_j the number of all points in $\mathbb{P}^j$, i.e., $\theta_j = q^j + \cdots + q + 1$.

Let C be an $[n,k,d]_q$ code with a generator matrix G. C is said to be non-degenerate if every column of G is nonzero. Thus if C is a non-degenerate code, each column of G can be regarded as a point in $\mathbb{P}^{k-1}$. The formal sum of columns of G as points in $\mathbb{P}^{k-1}$ is called a 0-cycle of the code C, denoted by $\mathcal{X}$. Denoting $m(P) \geq 0$ the number of times the point P occurring as a column of G, we have $\mathcal{X} = \sum_{P \in \mathbb{P}^{k-1}} m(P)P$. Then we have the parameters of C in terms of the coefficients in the 0-cycle $\mathcal{X}$ as follows:

$$n = \deg \mathcal{X} := \sum_{P \in \mathbb{P}^{k-1}} m(P),$$
$$d = n - \max\{\sum_{P \in H} m(P) \mid H \in \mathcal{F}_{k-2}\}.$$

For a 0-cycle $\mathcal{Z} = \sum_{P \in \mathbb{P}^{k-1}} m(P)P$, and a subset $S \subset \mathbb{P}^{k-1}$, we denote the restriction $\mathcal{Z}$ to S by $\mathcal{Z}(S) = \sum_{P \in S} m(P)P$. For simplicity's sake, we denote the 0-cycle $[S] := \sum_{P \in S} P$ which can be identified with the set S.

For a 0-cycle $\mathcal{X}_C = \sum_{P \in \mathbb{P}^{k-1}} m(P)P$ corresponding to a given code C, let $\gamma_0 = \max\{m(P) \mid P \in \mathbb{P}^{k-1}\}$. C is said to be projective provided that $\gamma_0 = 1$. In this paper, we only concern the projective code. Let $\mathcal{Y}_C = [\mathbb{P}^{k-1}] - \mathcal{X}_C =$

$\sum_{P\in\mathbb{P}^{k-1}}(1-m(P))P$, which is called the complement of $\mathcal{X}_C$. We use the following notations:

$$c(S) := \deg \mathcal{X}_C(S) \quad \text{and} \quad c_0(S) := \deg \mathcal{Y}_C(S).$$

We need the following theorems to prove our main results:

Theorem 2 ([4]). *Let C be a $[g_q(k,d),k,d]_q$ code. Then we have*

$$\gamma_j = \sum_{i=0}^{j} \left\lceil \frac{d}{q^{k-1-i}} \right\rceil \quad \textit{for} \quad 0 \le j \le k-1,$$

where $\gamma_j := \max\{c(L) | L \in \mathcal{F}_j\}$ *for* $1 \le j \le k-1$.

Theorem 3 ([4]). *Let C be a $[g_q(k,d),k,d]_q$ code. Then there exist j-dimensional subspaces L_j in $\mathbb{P}^{k-1}$ with $c(L_j)=\gamma_j$ for $j=0,1,\dots,k-2$ such that $L_0\subset L_1\subset\cdots\subset L_{k-2}$ and that L_j gives a $[\gamma_j, j+1, \gamma_j-\gamma_{j-1}]_q$ code which attains the Giesmer bound for $1\le j\le k-2$.*

To know the structure of minihypers is important to prove our results. A subset F with f points in $\mathbb{P}^t$ is called an $\{f,m;t,q\}$-minihyper if $^{\#}(F\cap H)\ge m$ for any hyperplane H in $\mathbb{P}^t$ and $^{\#}(F\cap H)=m$ for some hyperplane H in $\mathbb{P}^t$, where $m\ge 0$.

Theorem 4 ([1]). (1) *Let F be a $\{\theta_\alpha,\theta_{\alpha-1};t,q\}$-minihyper with $t\ge 2$. Then F is an α-flat in $\mathbb{P}^t$.*

(2) *Let F be a $\{\theta_2+\theta_1,\theta_1+\theta_0;t,q\}$-minihyper with $t\ge 4$. Then F consists of a plane and a line which are disjoint.*

2. Proofs of Theorems A and B

A proof of Theorem A. Since the existence of an $[n,k,d]_q$ code with $d\ge 2$ implies the existence of an $[n-1,k,d-1]_q$ code, it is sufficient to show that there does not exist a $[q^5+q^4-q^2-2q,6,q^5-q^3-q^2-q+1]_q$ code for $q\ge 3$.

Assume that there exists a $[q^5+q^4-q^2-2q,6,q^5-q^3-q^2-q+1]_q$ code C. Then by Theorem 2, we have

$$\gamma_0=1,\ \gamma_1=q+1,\ \gamma_2=q^2+q,\ \gamma_3=q^3+q^2-1, \text{ and } \gamma_4=q^4+q^3-q-1.$$

Since $\gamma_0=1$, C is a projective code. Let C_0 be the set of complement of 0-cycle of C in $\mathbb{P}^5$. For a set $S\subset\mathbb{P}^5$, we note $c_0(S)=\,^{\#}(S\cap C_0)$. Then, we have

$$c_0(l)\ge 0 \text{ for any line } l\subset\mathbb{P}^5,$$
$$c_0(\delta)\ge 1 \text{ for any plane } \delta\subset\mathbb{P}^5,$$
$$c_0(\Delta)\ge q+2 \text{ for any solid } \Delta\subset\mathbb{P}^5,$$
$$c_0(\Pi)\ge q^2+2q+2 \text{ for any 4-flat } \Pi\subset\mathbb{P}^5 \text{ and}$$
$$c_0=c_0(\mathbb{P}^5)=q^3+2q^2+3q+1.$$

Let Π_0 be a 4-flat with $c_0(\Pi_0) = q^2 + 2q + 2 = \theta_2 + \theta_1$. Then $\Pi_0 \cap C_0$ consists of a plane δ_0 and a line l_0 with $\delta_0 \cap l_0 = \emptyset$ by Theorem 4(2). Thus we have

$$c_0(\Delta) = q + 2, 2q + 2 \text{ or } q^2 + q + 2 \tag{1}$$

for any solid Δ contained in Π_0. For an arbitrary 4-flat Π, we have $c_0(\Pi_0 \cap \Pi) \leq q^2 + q + 2$. Letting $\Delta_0 = \Pi_0 \cap \Pi$, we have

$$\begin{aligned} q^3 + 2q^2 + 3q + 1 = {}^{\#}C_0 &= c_0(\Pi) + c_0(\Pi_0) - qc_0(\Delta_0) + \sum_{\Pi' \supset \Delta_0, \Pi' \neq \Pi_0, \Pi} c_0(\Pi') \\ &\geq c_0(\Pi) + q(q^2 + 2q + 2) - q(q^2 + q + 2), \end{aligned}$$

whence $c_0(\Pi) \leq q^3 + q^2 + 3q + 1$.

Next, we shall prove that there does not exist 4-flat Π with $2q^2 + 3q + 2 \leq c_0(\Pi) \leq q^3 + q^2 + 3q + 1$ in the following two claims:

Claim 1. *There exists no 4-flat Π with $2q^2+3q+2 \leq c_0(\Pi) \leq q^3+q^2+2q+1$.*

Let Π_1 be a 4-flat with $c_0(\Pi_1) = q^3 + q^2 + 2q + 1 - eq - f$ for some integers $0 \leq e \leq q^2 - q - 2$, $0 \leq f \leq q - 1$. For any solid $\Delta \subset \Pi_1$, we have

$$\begin{aligned} q^3 + 2q^2 + 3q + 1 = {}^{\#}C_0 &= c_0(\Pi_1) + \sum_{\Pi \supset \Delta, \Pi \neq \Pi_1} c_0(\Pi) - qc_0(\Delta) \\ &\geq q^3 + q^2 + 2q + 1 - eq - f + q(q^2 + 2q + 2) - qc_0(\Delta). \end{aligned}$$

Hence, $c_0(\Delta) \geq q^2 + q + 1 - e$.

Assume there exists a solid $\Delta_1 \subset \Pi_1$ with $c_0(\Delta_1) = q^2 + q + 1 - e$. If there were 4-flat $\Pi \supset \Delta_1$ with $c_0(\Pi) = q^2 + 2q + 2$, then by (1), $c_0(\Delta_1) = c_0(\Pi \cap \Pi_1) = q^2+q+2$, $2q+2$ or $q+2$ which would not be equal to $q^2+q+1-e$. Thus, $c_0(\Pi) \geq q^2 + 2q + 3$ for any 4-flat $\Pi \supset \Delta_1$. Hence, we have

$$\begin{aligned} q^3 + 2q^2 + 3q + 1 = {}^{\#}C_0 &= c_0(\Pi_1) + \sum_{\Pi \supset \Delta_1, \Pi \neq \Pi_1} c_0(\Pi) - qc_0(\Delta_1) \\ &\geq q^3 + q^2 + 2q + 1 - eq - f + q(q^2 + 2q + 3) \\ &\quad - q(q^2 + q + 1 - e) \\ &= q^3 + 2q^2 + 4q + 1 - f, \end{aligned}$$

which is a contradiction. Therefore, $c_0(\Delta) \geq q^2+q+2-e$ for any solid $\Delta \subset \Pi_1$. Since $c_0(\Pi_1) = q^3+q^2+2q+1-eq-f$ and $c_0(\Delta) \geq q^2+q+2-e$ for any solid $\Delta \subset \Pi_1$, Π_1 gives a $[q^4-q+eq+f, 5, d']_q$ code with $d' \geq q^4-q^3+(e-1)(q-1)+f$.

By the Griesmer bound, we have

$$\begin{aligned} q^4 - q + eq + f &\geq d' + \left\lceil \frac{d'}{q} \right\rceil + \left\lceil \frac{d'}{q^2} \right\rceil + \left\lceil \frac{d'}{q^3} \right\rceil + \left\lceil \frac{d'}{q^4} \right\rceil \\ &\geq q^4 + (e-1)q + f + \left\lceil \frac{f-e+1}{q} \right\rceil + \left\lceil \frac{(e-1)(q-1)+f}{q^2} \right\rceil \\ &\quad + \left\lceil \frac{(e-1)(q-1)+f}{q^3} \right\rceil + \left\lceil \frac{(e-1)(q-1)+f}{q^4} \right\rceil . \end{aligned}$$

Hence, $f = 0$ and $e = 1$, i.e., $c_0(\Pi_1) = q^3 + q^2 + q + 1 = \theta_3$ and $c_0(\Delta) \geq \theta_2$ for any solid $\Delta \subset \Pi_1$. By Theorem 4(1), we have $\Pi_1 \cap C_0 = \Delta_0$ for some solid Δ_0 in Π_1. Therefore, $c_0(\Delta) = \theta_2$ or θ_3 for any solid $\Delta \subset \Pi_1$. On the other hand, $c_0(\Pi_0 \cap \Pi_1) = q^2 + q + 2$ for a given 4-flat Π_0 with $c_0(\Pi_0) = q^2 + 2q + 2$. This is a contradiction.

Claim 2. *There exists no 4-flat Π with $q^3+q^2+2q+2 \leq c_0(\Pi) \leq q^3+q^2+3q+1$.*

Let Π_1 be a 4-flat with $c_0(\Pi_1) = q^3 + q^2 + 3q + 1 - f$ for some integer $0 \leq f \leq q-1$. For any solid $\Delta \subset \Pi_1$, we have

$$\begin{aligned} q^3 + 2q^2 + 3q + 1 &= {}^{\#}C_0 = c_0(\Pi_1) + \sum_{\Pi \supset \Delta, \Pi \neq \Pi_1} c_0(\Pi) - qc_0(\Delta) \\ &\geq q^3 + q^2 + 3q + 1 - f + q(q^2 + 2q + 2) - qc_0(\Delta). \end{aligned}$$

Hence, $c_0(\Delta) \geq q^2+q+2$. Let $\Delta_1 = \Pi_0 \cap \Pi_1$ for a given 4-flat Π_0 with $c_0(\Pi_0) = q^2+2q+2$. Since $c_0(\Delta_1) = q^2+q+2$, Π_1 gives a $[q^4-2q+f, 5, q^4-q^3-2q+f+1]_q$ code which attains the Griesmer bound. By Theorem 2, we have $c_0(l) \geq 1$, $c_0(\delta) \geq \theta_1$ and $c_0(\Delta) \geq \theta_2 + 1$ for any line l, plane δ and solid Δ in Π_1. Since $\Pi_0 \cap C_0 = \delta_0 \cup l_0$ and $\delta_0 \cap l_0 = \emptyset$, we have $\delta_0 \subset \Delta_1 \cap C_0$. Then, we have

$$\begin{aligned} q^3 + q^2 + 3q + 1 - f &= c_0(\Pi_1) = c_0(\Delta_1) + \sum_{\Pi_1 \supset \Delta \supset \delta_0, \Delta \neq \Delta_1} c_0(\Delta) - qc_0(\delta_0) \\ &= q^2 + q + 2 + \sum_{\Pi_1 \supset \Delta \supset \delta_0, \Delta \neq \Delta_1} c_0(\Delta) - q(q^2 + q + 1). \end{aligned}$$

Therefore, $\sum_{\Pi_1 \supset \Delta \supset \delta_0, \Delta \neq \Delta_1} c_0(\Delta) = 2q^3+q^2+3q-1-f \geq 2q^3+q^2+2q$, whence there exists a solid $\Delta_0 \subset \Pi_1$ containing δ_0 with $c_0(\Delta_0) \geq 2q^2 + q + 2$. Since $c_0(l) \geq 1$ for every line $l \subset \Pi_1$, the fact that $c_0(\Pi_1 \setminus \Delta_0) \leq q^3-q^2+2q-1-f < q^3$ implies that $\Delta_0 \subset C_0$, i.e., $c_0(\Delta_0) = q^3 + q^2 + q + 1$. Then, we have

$$\begin{aligned} q^3 + 2q^2 + 3q + 1 = {}^{\#}C_0 &= c_0(\Pi_1) + \sum_{\Pi \supset \Delta_0, \Pi \neq \Pi_1} c_0(\Pi) - qc_0(\Delta_0) \\ &\geq q^3 + q^2 + 3q + 1 - f + \sum_{\Pi \supset \Delta_0, \Pi \neq \Pi_1} c_0(\Pi) \\ &\quad - q(q^3 + q^2 + q + 1). \end{aligned}$$

Hence,

$$\sum_{\Pi \supset \Delta_0, \Pi \neq \Pi_1} c_0(\Pi) = q(q^3+q^2+2q+1)+f < q(q^3+q^2+2q+2),$$

whence there exists a 4-flat Π with $q^3+q^2+q+1 \leq c_0(\Pi) \leq q^3+q^2+2q+1$. This contradicts Claim 1. Thus, we have Claim 2.

Let Δ_0 be a solid contained in Π_0 with $c_0(\Delta_0) = q^2+q+2$. Then, we have

$$\begin{aligned} q^3+2q^2+3q+1 &= c_0(\Pi_0) + \sum_{\Pi \supset \Delta_0, \Pi \neq \Pi_0} c_0(\Pi) - qc_0(\Delta_0) \\ &= (q^2+2q+2) + \sum_{\Pi \supset \Delta_0, \Pi \neq \Pi_0} c_0(\Pi) - q(q^2+q+2), \end{aligned}$$

whence $\sum_{\Pi \supset \Delta_0, \Pi \neq \Pi_0} c_0(\Pi) = 2q^3+2q^2+3q-1$. Thus, there exists a 4-flat Π_1 containing Δ_0 with $c_0(\Pi_1) \geq 2q^2+2q+3$. By Claims 1 and 2, we have $c_0(\Pi_1) \leq 2q^2+3q+1$. Note that $\Delta_0 \cap C_0 = \delta_0 \cup \{P_0\}$ for a point $P_0 \in l_0$. For a line $l_1 \subset \delta_0$, there exists a solid $\Delta' \subset \Pi_0$ with $c_0(\Delta') = q+2$ which contains l_1 and P_0. Then, we have

$$\begin{aligned} q^3+2q^2+3q+1 &= c_0(\Pi_0) + \sum_{\Pi \supset \Delta', \Pi \neq \Pi_0} c_0(\Pi) - qc_0(\Delta') \\ &= (q^2+2q+2) + \sum_{\Pi \supset \Delta', \Pi \neq \Pi_0} c_0(\Pi) - q(q+2). \end{aligned}$$

Hence, there exists a 4-flat Π_2 such that $\Pi_2 \supset \Delta'$, $c_0(\Pi_2) = q^2+2q+2$ and $\Pi_2 \neq \Pi_0$. Let $\Pi_2 \cap C_0 = \delta_2 \cup l_2$ with $\delta_2 \cap l_2 = \emptyset$. Since $\Delta' = \Pi_0 \cap \Pi_2$, we have $\delta_0 \neq \delta_2$. Letting $\Delta_1 = \Pi_1 \cap \Pi_2$, we have

$$\begin{aligned} q^3+2q^2+3q+1 &= {}^{\#}C_0 = c_0(\Pi_1) + \sum_{\Pi \supset \Delta_1, \Pi \neq \Pi_1} c_0(\Pi) - qc_0(\Delta_1) \\ &\geq (2q^2+2q+3) + q(q^2+2q+2) - qc_0(\Delta_1), \end{aligned}$$

whence $c_0(\Delta_1) \geq 2q+2$. By (1), we have $c_0(\Delta_1) = 2q+2$ or q^2+q+2. In case $c_0(\Delta_1) = 2q+2$, either $l_1 \subset \delta_2$ and $P_0 \in l_2$ or $l_1 = l_2$ and $P_0 \in \delta_2$ holds. In case $l_1 \subset \delta_2$ and $P_0 \in l_2$, since $\Delta_3 = \langle \delta_0, \delta_2 \rangle$ is a solid with $c_0(\Delta_3) \geq 2q^2+q+1$, we have

$$\begin{aligned} q^3+2q^2+3q+1 &= {}^{\#}C_0 = \sum_{\Pi \supset \Delta_3} c_0(\Pi) - qc_0(\Delta_3) \\ &\leq \sum_{\Pi \supset \Delta_3} c_0(\Pi) - q(2q^2+q+1), \end{aligned}$$

whence there exists a 4-flat Π containing Δ_3 with $c_0(\Pi) \geq 3q^2+4$ for $q \geq 3$, which contradicts Claims 1 and 2. In case $l_1 = l_2$ and $P_0 \in \delta_2$, since $l_0 \cap \delta_0 = \emptyset$, $\delta_0 \cap \delta_2$ consists of one point, say P_1. Then, $\langle P_0, P_1 \rangle \subset \Pi_0$, because $P_0, P_1 \in \Pi_0$.

This is a contradiction. In case $c_0(\Delta_1) = q^2 + q + 2$, we have $l_1 \subset \delta_2$. This implies a contradiction as in the preceding case. This completes the proof. □

Remark 5. Theorem A implies that there does not exist a $\{\theta_3 + \theta_2 + s, \theta_2 + \theta_1; 5, q\}$-minihyper for $1 \leq s \leq q - 1$.

Proof of Theorem B. To prove this theorem, it suffices to show the existence of $[g_q(6, d) + 1, 6, q^5 - q^3 - q^2]_q$ code. Let C be a code with the 0-cycle

$$\mathcal{X}_C = [\mathbb{P}^5] - [\Delta_0] - [\delta_1] + [P_0],$$

where Δ_0 is a solid and δ_1 is a plane in $\mathbb{P}^5$ and $P_0 = \Delta_0 \cap \delta_1$. Then we have the length $n = \theta_5 - \theta_3 - \theta_2 + 1$ of C and the minimum distance $d = q^5 - q^3 - q^2$. Therefore there exist a $[g_q(6, d) + 1, 6, d]_q$ code for $q^5 - q^3 - q^2 - q + 1 \leq d \leq q^5 - q^3 - q^2$. □

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