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# NONBINARY INCIDENCE CODES OF $(n, n-1, j)$-POSET

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Abstract. Let $P$ be a $(n, n-1, j)$-poset, which is a partially ordered set of cardinality $n$ with $n-1$ maximal elements and $j$ $(1 \leq j \leq n-1)$ minimal elements, and $P^*$ the dual poset of $P$. In this paper, we obtain two types of incidence codes of nonempty proper subset $S$ of $P$ and $P^*$, respectively, by using Bogart's method [1] (see Theorem 3.3).

## 1. Introduction

Let $P$ be a partially ordered set (or poset, for short) labelled as $\{x_1, \cdots, x_n\}$ and ordered by a relation $\leq$. Let $N$ be an integer $> 2$ and let $A_N = \{0, 1, 2, \cdots, N-1\}$ denote the commutative ring of integers modulo $N$. (If $N$ is a prime number, $A_N$ is a finite field.) Let $A_N^n$ be the set of $n$-tuples over $A_N$. The set $A_N^n$ has $N^n$ members and it is an abelian group under addition modulo $N$. Also, the set $A_N^n$ forms a free module over $A_N$ with the standard basis $e_x$, $x \in P$, where

$$e_x = (e_x(1), \cdots, e_x(n))\text{and} \quad e_x(j) = \delta(x, x_j) = \begin{cases} 1 & \text{if } x = x_j, \\ 0 & \text{otherwise.} \end{cases}$$

We define a code over $A_N$ as a submodule of $A_N^n$.

In [1], Bogart introduced a method for constructing incidence codes over a finite field from a poset. When above method is applied to the subsets of a set, ordered by set inclusion, it yields the well known Reed-Muller codes. Applying to a larger class of posets (Eulerian posets with the least upper bound property), it yields majority logic decodable codes quite analogous to Reed-Muller codes.

Recently, in [3], authors investigated binary linear $(n, n-1, j)$-poset codes was investigated (the reference [2] gave a full detail of the poset

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codes), where $(n, n-1, j)$-poset is a partially ordered set of cardinality $n$ with $n-1$ maximal elements and $j$ $(1 \leq j \leq n-1)$ minimal elements. The Hass diagram of $(n, n-1, j)$-poset is given by:

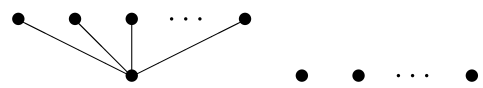


Figure 1

In this paper, we obtain, respectively, nonbinary incidence codes over $A_N$ from $(n, n-1, j)$-poset and its dual poset:

**Theorem 3.3** Let $P$ be a $(n, n-1, j)$-poset and $P^*$ the dual poset of $P$. For each nonempty proper subset $S$ of $P$ and $P^*$, we have

$$RM(P, S) = \begin{cases} [n, |S|, 1]\text{-code} & \text{if } S \neq \{x_1\}, \\ [n, 1, n-j+1]\text{-code} & \text{if } S = \{x_1\} \end{cases}$$

and

$$RM(P^*, S) = \begin{cases} [n, |S|, 1]\text{-code} & \text{if } S \not\subseteq Q^-, \\ [n, |S|, 2]\text{-code} & \text{if } S \subseteq Q^-. \end{cases}$$

## 2. Quick review of the incidence codes of poset over $A_N$

Let $P$, $A_N$ and $A_N^n$ be as in section 1. The incidence algebra $I(P, A_N)$ of $P$ over $A_N$ is the $A_N$-algebra of all functions $f : Int(P) \to A_N$, where $Int(P)$ is the set of intervals $[x_i, x_j] = \{x_k \in P \mid x_i \leq x_k \leq x_j\}$ of $P$ when $x_i$ is less than or equal to $x_j$, write $f(x_i, x_j)$ for $f([x_i, x_j])$ and multiplication (or convolution) is defined by

$$fg(x_i, x_j) = \sum_{x_i \leq x_k \leq x_j} f(x_i, x_k) g(x_k, x_j).$$

It is easy to see that $I(P, A_N)$ is an associative $A_N$-algebra with (two-side) identity $\delta$. If $Z$ is the matrix given by

$$Z_{ij} = \zeta(x_i, x_j) = \begin{cases} 1 & \text{if } x_i \leq x_j, \\ 0 & \text{otherwise}, \end{cases}$$

then $Z$ has an inverse $M = (M_{ij})$ (over the integers) and the Möbius function of $P$ is given by $M_{ij} = \mu(x_i, x_j)$, that is, the relation $\mu\zeta = \delta$ is equivalent to

$$\mu(x_i, x_j) = \begin{cases} 1 & \text{if } x_i = x_j, \\ -\sum_{x_i \leq x_k < x_j} \mu(x_i, x_k) & \text{if } x_i < x_j. \end{cases} \tag{2.1}$$

The fundamental theorem of Möbius inversion(denoted by **MIF** hereafter) is:

(**MIF**) If $f$ and $g$ are functions defined on $P$ with values in an abelian group, then

$$f(x) = \sum_{y \geq x} g(y) \iff g(x) = \sum_{y \geq x} \mu(x, y) f(y).$$

For each $x \in P$, the vectors $v_x = (v_x(1), \cdots, v_x(n))$ defined by

$$v_x(j) = \zeta(x, x_j) \tag{2.2}$$

form a basis for $A_N^n$ since, by **MIF**, each $e_x$ is a linear combination of the vectors $v_x$, i.e.,

$$e_x = \sum_{y \in P:\ y \geq x} \mu(x, y) v_y.$$

Thus, if $v = \sum_{x \in P} a_x v_x = \sum_{x \in P} b_x e_x \in A_N^n$, then the relation between $a_x$ and $b_x$ is given by

$$a_x = \sum_{z \leq x} \mu(z, x) b_z. \tag{2.3}$$

For each nonempty subset $S$ of $P$, let $IC_N(P, S)$ be the submodule of $A_N^n$ spanned by the elements $v_s$ for $s \in S$. This is an $|S|$-dimensional linear nonbinary code over $A_N$. The codes $IC_N(P, S)$ is called the *incidence codes* of $P$ over $A_N$.

## 3. Nonbinary incidence codes of $(n, n-1, j)$-poset over $A_N$

For the fixed integers $j$ and $n$ with $1 \leq j \leq n-1$ let $P = \{x_1, x_2, \cdots, x_n\}$ be a $(n, n-1, j)$-poset (see Figure 1). By rearranging the elements of $P$ we can separate $P$ into two disjoint subposets, say $Q$

and $R$, where without loss of generality we may assume that (see Figure 2)

(3.1)
$$\begin{cases} Q=\{x_1,x_2,\cdots,x_k\} \text{ is a } (k,k-1,1)\text{-poset, where } k=n-j+1, \\ \qquad\qquad \text{in which } x_1<x_i \text{ for each } i=2,3,\cdots,k; \\ R=\{x_{k+1},\cdots,x_n\} \text{ is an anti-chain with } (n-k) \text{ elements.} \end{cases}$$

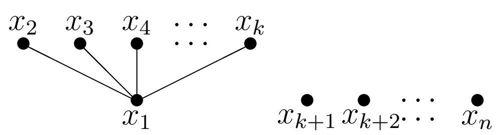



Figure 2

Let $P^*$ denote the dual poset of $P$, i.e., $P^*$ is a poset on the same set as $P$, but such that $x \le y$ in $P^*$ if and only if $y \le x$ in $P$. Note that if $j=1$, then $Q=P$ and $R=\emptyset$. For convenience, put $P^- = P-\{x_1\}$, $Q^- = Q-\{x_1\}$ and $R^+ = R\cup\{x_1\}$ as set, i.e.,

$$P = P^* = \{x_1\}\cup R\cup Q^- = R^+\cup Q^- \text{ (as set, disjoint union).} \tag{3.2}$$

By (2.1) we have the Möbius function of $P$ (resp., of $P^*$)

$$\mu(x,y) = \begin{cases} 1, & \text{if } x=y, \\ -1, & \text{if } x=x_1 \text{ and } y\in Q^- \text{ in } P \\ & \quad \text{(resp., if } x\in Q^- \text{ and } y=x_1 \text{ in } P^*\text{)}, \\ 0, & \text{otherwise.} \end{cases} \tag{3.3}$$

Proposition 3.1. *The Möbius inversion formula on $P$ and $P^*$ is, respectively, given by*

(3.4$a$)
$$f(x)=\sum_{y\ge x} g(y) \iff \begin{cases} f(x)=g(x), \textit{for all } x\in P^- \textit{ in } P, \\ f(x_1)=\sum_{y\in Q} g(y) \textit{ in } P, \end{cases}$$

*and*

(3.4$b$)
$$f(x)=\sum_{y\ge x} g(y) \iff \begin{cases} f(x)=g(x) \quad \textit{for all } x\in R^+ \textit{ in } P^*, \\ f(x)=g(x)+g(x_1) \quad \textit{for all } x\in Q^- \textit{ in } P^*. \end{cases}$$

*Proof.* By **MIF** we have

$$g(x) = \sum_{y \geq x} \mu(x, y) f(y) \quad \text{for all } x \text{ in } P \text{ (resp., } P^*).$$

Thus by (3.3), $g(x) = f(x)$ if $x \in P^-$ in $P$ (resp., $x \in R^+$ in $P^*$). Also, in $P$

$$\begin{aligned} g(x_1) &= \sum_{y \geq x_1} \mu(x_1, y) f(y) = f(x_1) + \sum_{y > x_1} \mu(x_1, y) f(y) \\ &= f(x_1) - \sum_{y \in Q^-} f(y) \\ &= f(x_1) - \sum_{y \in Q^-} \sum_{z \geq y} g(z) = f(x_1) - \sum_{y \in Q^-} g(y). \end{aligned}$$

Thus

$$f(x_1) = g(x_1) + \sum_{y \in Q^-} g(y) = \sum_{y \in Q} g(y).$$

Similarly, for $x \in Q^-$ in $P^*$

$$\begin{aligned} g(x) &= \sum_{y \geq x} \mu(x, y) f(y) = f(x) + \sum_{y > x} \mu(x, y) f(y) = f(x) - f(x_1) \\ &= f(x) - \sum_{y \geq x_1} g(y) = f(x) - g(x_1). \end{aligned}$$

Thus $f(x) = g(x_1) + g(x)$. □

Proposition 3.2. *If $v = \sum_{x \in P} a_x v_x = \sum_{x \in P} b_x e_x \in A_N^n$, then*

(3.5*a*)

$$\begin{cases} v_x = e_x, & \text{for all } x \in P^- \text{ in } P \text{ (or, for all } x \in R^+ \text{ in } P^*) \\ v_{x_1} = \sum_{x \in Q} e_x & \text{in } P \\ v_x = e_x + e_{x_1} & \text{for all } x \in Q^- \text{ in } P^*; \end{cases}$$

$$\text{(3.5}b\text{)} \qquad \begin{cases} a_x = b_x, \text{ for all } x \in R^+ \text{ in } P \text{ (or, for all } x \in P^- \text{ in } P^*) \\ a_x = b_x - b_{x_1}, \text{for all } x \in Q^- \text{ in } P \\ a_{x_1} = b_{x_1} - \sum_{x \in Q^-} b_x \text{ in } P^*. \end{cases}$$

*Proof.* Since

$$e_x(i) = \delta(x, x_i) = (\mu\zeta)(x, x_i) = \sum_{x \leq y \leq x_i} \mu(x, y) \zeta(y, x_i) = \sum_{x \leq y \leq x_i} \mu(x, y) v_y(i),$$

each $e_x$ is a linear combination of the vectors $v_x$, that is,

$$
\begin{aligned}
& e_x = \sum_{y \geq x} \mu(x,y) v_y \quad \text{for all } x \text{ in } P \text{ (resp., } P^*) \\
\Longleftrightarrow \quad & v_x = \sum_{y \geq x} e_x \quad (\text{by } \mathbf{MIF}) \\
\Longleftrightarrow \quad & \begin{cases} v_x = e_x & \text{for all } x \in P^- \text{ in } P \text{ (resp., } x \in R^+ \text{ in } P^*), \\ v_{x_1} = \sum_{x \in Q} e_y & \text{in } P, \\ v_x = e_x + e_{x_1} & \text{for all } x \in Q^- \text{ in } P^*. \quad (\text{by } (3.4)) \end{cases}
\end{aligned}
$$

On the other hand, we have

$$
\sum_{x \in P} b_x e_x = \sum_{x \in P} b_x \left( \sum_{y \geq x} \mu(x,y) v_y \right) = \sum_{y \in P} \left( \sum_{z \leq y} \mu(z,y) b_z \right) v_y.
$$

Thus we obtain

$$
\begin{aligned}
\sum_{x \in P} a_x v_x &= \sum_{x \in P} b_x e_x = \sum_{x \in P} \left( \sum_{z \leq x} \mu(z,x) b_z \right) v_x \\
&= \sum_{x \in R^+} \left( \sum_{z \leq x} \mu(z,x) b_z \right) v_x + \sum_{x \in Q^-} \left( \sum_{z \leq x} \mu(z,x) b_z \right) v_x \quad (\text{by } (3.2)) \\
&= \sum_{x \in R^+} b_x v_x + \sum_{x \in Q^-} (b_x - b_{x_1}) v_x \quad (\text{by } (3.3))
\end{aligned}
$$

and

$$
\begin{aligned}
\sum_{x \in P^*} a_x v_x &= \sum_{x \in P^*} b_x e_x = \sum_{x \in P^*} \left( \sum_{z \leq x} \mu(z,x) b_z \right) v_x \\
&= \sum_{x \in R^+} \left( \sum_{z \leq x} \mu(z,x) b_z \right) v_x + \sum_{x \in Q^-} \left( \sum_{z \leq x} \mu(z,x) b_z \right) v_x \quad (\text{by } (3.2)) \\
&= \sum_{z \leq x_1} \mu(z,x_1) b_z v_{x_1} + \sum_{x \in R} b_x v_x + \sum_{x \in Q^-} b_x v_x \quad (\text{by } (3.3)) \\
&= \left( b_{x_1} - \sum_{x \in Q^-} b_x \right) v_{x_1} + \sum_{x \in P^-} b_x v_x.
\end{aligned}
$$

□

THEOREM 3.3. *Let $P$ be a $(n, n-1, j)$-poset given by (3.1) and $P^*$ the dual poset of $P$. For each nonempty proper subset $S$ of $P$ and $P^*$, we have*

$$RM(P,S) = \begin{cases} [n, |S|, 1]\text{-code} & \text{if } S \neq \{x_1\}, \\ [n, 1, n-j+1]\text{-code} & \text{if } S = \{x_1\} \end{cases}$$

*and*

$$RM(P^*,S) = \begin{cases} [n, |S|, 1]\text{-code} & \text{if } S \not\subseteq Q^-, \\ [n, |S|, 2]\text{-code} & \text{if } S \subseteq Q^-. \end{cases}$$

*Proof.* Let $S$ be a nonempty proper subset of $P$. Using (3.5a) we can obtain incidence codes $RM(P,S) = \left\{\sum_{x\in S} a_x v_x \mid a_x \in A_N\right\}$ as follows: If $S = \{x_1\}$, then

$$RM(P,S) = \left\{ a \sum_{x\in Q} e_x \mid a \in A_N \right\}. \tag{3.6}$$

It is the $[n, 1, |Q|]$-code, where $|Q| = k = n - j + 1$. If $x_1 \notin S$, then $S = S \cap P^-$ and

$$RM(P,S) = \left\{ \sum_{x\in S\cap P^-} a_x e_x \mid a_x \in A_N \right\}. \tag{3.7}$$

If $x_1 \in S$, then $S = \{x_1\} \cup (S \cap P^-)$ and

$$RM(P,S) = \left\{ a_{x_1} \sum_{x\in Q} e_x + \sum_{x\in S\cap P^-} a_x e_x \mid a_{x_1}, a_x \in A_N \right\}. \tag{3.8}$$

From (3.7) and (3.8), if $S \neq \{x_1\}$, then each $RM(P,S)$ is a $[n, |S|, 1]$-code. Similarly, from (3.5a), for any subset $S$ of $Q^-$ in $P^*$,

$$RM(P^*,S) = \left\{ \sum_{x\in S\cap Q^-} a_x e_{x_1} + \sum_{x\in S\cap Q^-} a_x e_x \mid a_x \in A_N \right\}. \tag{3.9}$$

It is a $[n, |S|, 2]$-code. If $S \not\subseteq Q^-$ in $P^*$, then let $\sum_{x\in S} a_x v_x = v \in RM(P^*, S)$. Then by (3.5b) we have $a_x = b_x$ for each $x \in S \cap P^-$ in $P^*$. If $a_x \neq 0$ for some $x \in S \cap P^-$ in $P^*$, then $b_x \neq 0$ for each $x$. On the other hand, if $a_x$ is zero for each $x \in S \cap P^-$ in $P$ (resp., $P^*$), then simply remove all elements $x \in S \cap P^-$ in $P$ (resp., $P^*$) from $S$ to get $S'$, note that $v \in RM(P, S')$ (resp., $RM(P^*, S')$). Since $S' = \{x_1\}$ or $S' = \emptyset$, by repeating the argument, we complete the proof. □

Note that $RM(P,P) = RM(P^*,P^*)$ is the entire space $A_N^n$ which is the trivial perfect code.

Corollary 3.4. *$RM(P,\{x_1\})$ is a perfect code if and only if $q = 2$, $n \geq 3$ is odd and $j = 1$. In this case, $RM(P,\{x_1\})$ is the binary repetition code. Also, $RM(P^*,S)$, where $S \subseteq Q^-$, is not a perfect code.*

*Proof.* $RM(P,\{x_1\})$ is perfect if and only if

$$q\sum_{l=0}^{\left[\frac{(n-j+1)-1}{2}\right]}\binom{n}{l}(q-1)^l = q^n \iff \sum_{l=0}^{\left[\frac{n-j}{2}\right]}\binom{n}{l}(q-1)^l = q^{n-1}$$
$$\iff q = 2,\ j = 1\text{and } n \geq 3 \text{ is odd number.}$$

Similarly, $RM(P^*,S)$ is perfect if and only if

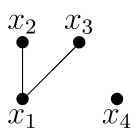
$$q\sum_{l=0}^{\left[\frac{2-1}{2}\right]}\binom{n}{l}(q-1)^l = q^n \iff q^{n-1} = 1 \iff n = 1.$$

Thus $RM(P^*,Q^-)$ is not perfect since $n \geq 2$. □

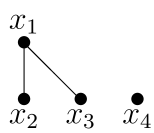
**Example.**

Let $P = \{x_1, x_2, x_3, x_4\}$ be a $(4,3,2)$-poset and $P^*$ the dual of $P$ which are given by:

$P$ :




$P^*$ :




Then

$$RM(P,P) = RM(P^*,P^*) = \mathbf{Z}_2^4 \quad \text{(it is the trivial perfect code).}$$

$$RM(P,\{x_1,x_2,x_3\}) = \left\{\begin{matrix} (1,1,1,0) & (1,1,0,0) & (1,0,1,0) & (1,0,0,0) \\ (0,0,0,0) & (0,0,1,0) & (0,1,0,0) & (0,1,1,0)\end{matrix}\right\} : \text{[4, 3, 1]-code,}$$

$$RM(P,\{x_1,x_2,x_4\}) = \left\{\begin{matrix} (1,1,1,0) & (1,1,1,1) & (1,0,1,0) & (1,0,1,1) \\ (0,0,0,0) & (0,0,0,1) & (0,1,0,0) & (0,1,0,1)\end{matrix}\right\} : \text{[4, 3, 1]-code,}$$

$$RM(P, \{x_1, x_3, x_4\}) = \left\{ \begin{array}{cccc} (1,1,1,0) & (1,1,1,1) & (1,1,0,0) & (1,1,0,1) \\ (0,0,0,0) & (0,0,0,1) & (0,0,1,0) & (0,0,1,1) \end{array} \right\}$$
$$: [4,3,1]\text{-code},$$

$$RM(P, \{x_2, x_3, x_4\}) = \left\{ \begin{array}{cccc} (0,1,1,0) & (0,1,1,1) & (0,1,0,0) & (0,1,0,1) \\ (0,0,1,0) & (0,0,1,1) & (0,0,0,0) & (0,0,0,1) \end{array} \right\}$$
$$: [4,3,1]\text{-code}.$$

$$RM(P, \{x_1, x_2\}) = \{(0,0,0,0), (1,1,1,0), (1,0,1,0), (0,1,0,0)\}$$
$$: [4,2,1]\text{-code},$$

$$RM(P, \{x_1, x_3\}) = \{(0,0,0,0), (1,1,1,0), (1,1,0,0), (0,0,1,0)\}$$
$$: [4,2,1]\text{-code},$$

$$RM(P, \{x_1, x_4\}) = \{(0,0,0,0), (1,1,1,0), (1,1,1,1), (0,0,0,1)\}$$
$$: [4,2,1]\text{-code},$$

$$RM(P, \{x_2, x_3\}) = \{(0,0,0,0), (0,1,1,0), (0,1,0,0), (0,0,1,0)\}$$
$$: [4,2,1]\text{-code},$$

$$RM(P, \{x_2, x_4\}) = \{(0,0,0,0), (0,1,0,0), (0,1,0,1), (0,0,0,1)\}$$
$$: [4,2,1]\text{-code},$$

$$RM(P, \{x_3, x_4\}) = \{(0,0,0,0), (0,0,1,0), (0,0,1,1), (0,0,0,1)\}$$
$$: [4,2,1]\text{-code},$$

$$RM(P, \{x_1\}) = \{(0,0,0,0), (1,1,1,0)\}$$
$$: [4,1,3]\text{-code},$$

$$RM(P, \{x_2\}) = \{(0,0,0,0), (0,1,0,0)\}$$
$$: [4,1,1]\text{-code},$$

$$RM(P,\{x_3\}) = \{(0,0,0,0),(0,0,1,0)\} \qquad : [4,1,1]\text{-code},$$

$$RM(P,\{x_4\}) = \{(0,0,0,0),(0,0,0,1)\} \qquad : [4,1,1]\text{-code},$$

and

$$RM(P^*,\{x_1,x_2,x_3\}) = \left\{ \begin{array}{cccc} (0,1,0,0) & (1,1,1,0) & (1,0,0,0) & (0,0,1,0) \\ (1,1,0,0) & (0,1,1,0) & (0,0,0,0) & (1,0,1,0) \end{array} \right\} \qquad : [4,3,1]\text{-code},$$

$$RM(P^*,\{x_1,x_2,x_4\}) = \left\{ \begin{array}{cccc} (0,1,0,0) & (0,1,0,1) & (1,0,0,0) & (1,0,0,1) \\ (1,1,0,0) & (1,1,0,1) & (0,0,0,0) & (0,0,0,1) \end{array} \right\} \qquad : [4,3,1]\text{-code},$$

$$RM(P^*,\{x_1,x_3,x_4\}) = \left\{ \begin{array}{cccc} (0,0,1,0) & (0,0,1,1) & (1,0,0,0) & (1,0,0,1) \\ (1,0,1,0) & (1,0,1,1) & (0,0,0,0) & (0,0,0,1) \end{array} \right\} \qquad : [4,3,1]\text{-code},$$

$$RM(P^*,\{x_2,x_3,x_4\}) = \left\{ \begin{array}{cccc} (0,1,1,0) & (0,1,1,1) & (1,1,0,0) & (1,1,0,1) \\ (1,0,1,0) & (1,0,1,1) & (0,0,0,0) & (0,0,0,1) \end{array} \right\} \qquad : [4,3,1]\text{-code}.$$

$$RM(P^*,\{x_1,x_2\}) = \{(0,1,0,0),(1,0,0,0),(1,1,0,0),(0,0,0,0)\} \qquad : [4,2,1]\text{-code},$$

$$RM(P^*,\{x_1,x_3\}) = \{(0,0,1,0),(1,0,0,0),(1,0,1,0),(0,0,0,0)\} \qquad : [4,2,1]\text{-code},$$

$$RM(P^*,\{x_1,x_4\}) = \{(1,0,0,0),(1,0,0,1),(0,0,0,0),(0,0,0,1)\} \qquad : [4,2,1]\text{-code},$$

$$RM(P^*,\{x_2,x_3\}) = \{(1,1,0,0),(0,1,1,0),(0,0,0,0),(1,0,1,0)\} \qquad : [4,2,2]\text{-code},$$

$$RM(P^*, \{x_2, x_4\}) = \{(1,1,0,0), (1,1,0,1), (0,0,0,0), (0,0,0,1)\} \\ : [4,2,1]\text{-code},$$

$$RM(P^*, \{x_3, x_4\}) = \{(1,0,1,0), (1,0,1,1), (0,0,0,0), (0,0,0,1)\} \\ : [4,2,1]\text{-code},$$

$$RM(P^*, \{x_1\}) = \{(0,0,0,0), (1,0,0,0)\} : [4,1,1]\text{-code},$$

$$RM(P^*, \{x_2\}) = \{(0,0,0,0), (1,1,0,0)\} : [4,1,2]\text{-code},$$

$$RM(P^*, \{x_3\}) = \{(0,0,0,0), (1,0,1,0)\} : [4,1,2]\text{-code},$$

$$RM(P^*, \{x_4\}) = \{(0,0,0,0), (0,0,0,1)\} : [4,1,1]\text{-code}.$$

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