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EXISTENCE OF NONTRIVIAL SOLUTIONS OF A NONLINEAR BIHARMONIC EQUATION

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Abstract. We consider the existence of solutions of a nonlinear biharmonic equation with Dirichlet boundary condition, $\Delta^2 u + c\Delta u = f(x,u)$ in Ω, where Ω is a bounded open set in R^N with smooth boundary $\partial\Omega$. We obtain two new results by linking theorem.

Let us consider the problem:

$$\begin{aligned} &\Delta^2 u + c\Delta u = f(x,u) \qquad \text{in } \Omega, \\ &u = 0, \qquad \Delta u = 0 \qquad\qquad \text{on } \partial\Omega, \end{aligned} \tag{0.1}$$

where Δ^2 denote the biharmonic operator, Δ is the Laplacian on R^N, $u^+ = \max\{u, 0\}$, $\Omega \subset R^N$ is a smooth open bounded set. Here $\lambda_1 < c < \lambda_2$, where $\{\lambda_k\}_{k\geq 1}$ denote the sequence of the eigenvalue of $-\Delta$ in $H_0^1(\Omega)$ and b is not eigenvalue of $\Delta^2 + c\Delta$,f is a differentiable function such that $f(0) = 0$.

The existence of solutions of the biharmonic boundary value problem have been extensively studied by many authors. Lazer and McKenna in [5] point out that this kind of nonlinearity furnishes a good model to study traveling waves in a suspension bridge. In [6] the authors Lazer and McKenna proved the existence of $2k-1$ solutions of (0.1) when $\Omega \subset R$ is an interval and $b > \lambda_k(\lambda_k - c)$, for the assumption of $f(x,u) = b(u+1)^+ - 1$ by global bifurcation method, for the same $f(x,u)$.Tarantello [10] showed by degree theory that if $b \geq \lambda_1(\lambda_1 - c)$, then (0.1) has a solution u such that $u(x) < 0$ in Ω, for $f(x,u) = (u+1)^+ - 1$ when $c < \lambda_1$. Choi and Jung [2] showed that equation (0.1) has only the trivial solution when $\lambda_k < c < \lambda_{k+1}$ and the nonlinear term is $bu^+ (b < \lambda_1(\lambda_1 - c))$. Micheletti

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and Pistoia [7] showed that equation (0.1) has at least two solutions when $c > \lambda_1$ and the nonlinear term is $b[(u+1)^+ - 1](b < \lambda_1(\lambda_1 - c))$. Choi and Jin [1] consider equation (0.1) that the nonlinear term has both bu^+ and $b[(u+1)^+ - 1]$. In this paper we will study the problem (0.1), when the nonlinearity is replaced by a more general function $bu^+ + g(x,u)$, by using a "variation of linking " theorem.

1. Notations and preliminaries

We consider the problem of the existence of solutions of the biharmonic equation:

$$(1.1) \qquad \begin{aligned} &\Delta^2 u + c\Delta u = bu^+ + g(x,u) \qquad \text{in } \Omega, \\ &u = 0, \qquad \Delta u = 0 \qquad \text{on } \partial\Omega, \end{aligned}$$

where Ω is a smooth open boundary set in R^N, $g : \Omega \times R \to R$ is a Caratheodory's function and $c, b \in R$.

In this section we introduce the Sobolev space spanned by the eigenfunctions of the operator $\Delta^2 + c\Delta$ with Dirichlet boundary condition. We define the associated functional I corresponding to (1.1) and show that the functional I satisfies the (PS) condition.

Let λ_k denote the eigenvalues and e_k the corresponding eigenfunctions, suitably normalized with respect to $L^2(\Omega)$ inner product, of the eigenvalue problem $\Delta u + \lambda u = 0$ in Ω, with Dirichlet boundary condition, where each eigenvalue λ_k is repeated as often as its multiplicity. We recall that $0 < \lambda_1 < \lambda_2 \leq \lambda_3 \leq \cdots, \lambda_i \to +\infty$ and that $e_1 > 0$ for all $x \in \Omega$.

The eigenvalue problem

$$(1.2) \qquad \begin{aligned} &\Delta^2 u + c\Delta u = \lambda u \qquad \text{in } \Omega, \\ &u = 0, \qquad \Delta u = 0 \qquad \text{on } \partial\Omega, \end{aligned}$$

has infinitely many eigenvalues $\Lambda_k(c) = \lambda_k(\lambda_k - c)$, $k = 1, 2, \cdots$ and corresponding eigenfunctions e_k. Set $H_k = span\{e_1, \cdots, e_k\}$, $H_k^\perp = \{w \in H | (w,v)_H = 0, \ \ \forall v \in H_k\}$.

Let $H = H^2(\Omega) \cap H_0^1(\Omega)$ be the Hilbert space equipped with the inner product $(u,v)_H = \int \Delta u \Delta v + \int \nabla u \nabla v$. The functional corresponding to (1.1) given by $I : H \to R$

$$(1.3) \qquad I(u) := \frac{1}{2}\left(\int (\Delta u)^2 - c\int |\nabla u|^2\right) - \frac{b}{2}\int (u^+)^2 + \int g(x,u).$$

Let $C^1(H,R)$ denote the set of all functionals which are Fréchet differentiable and whose Fréchet derivatives are continuous on H. It is easy to prove that I is a C^1 functional and its critical points are weak solutions of problem (1.1).

We will use the following assumptions:

- (g1) $2G(x,u) - g(x,u)u \le a_0(x)s^+ - a_1(x) \quad \forall s \in R$ where $a_0 \in L^\infty(\Omega)$, $a_0(x) > 0$ a.e in Ω and $a_1 \in L^1(\Omega)$;
- (g2) $\frac{G(x,u)}{u^2} \to 0$ as $|u| \to \infty$ uniformly for $x \in \Omega$;
- (g3) $\lim\limits_{\|u\|_H \to 0} \int \frac{G(x,u)}{\|u\|_H^2} = 0$;
- (g4) $\lim\limits_{u \to 0} \frac{g(x,u)}{u} > \Lambda_2$;
- (g5) $\lim\limits_{u \to \infty} \frac{g(x,u)}{u} < \Lambda_2$:

The following is the main result of this paper.

THEOREM 1.1. *Assume that* (g1),(g2),(g3). *Let* $\lambda_1 < c < \lambda_2$, $b < \Lambda_1$ *then problem (1.1) has at least two solutions.*

THEOREM 1.2. *Assume that* (g1),(g4),(g5). *Let* $\lambda_1 < c < \lambda_2$, $b < \max\{0, \frac{1-\beta}{T}\}$ *then problem (1.1) has at least three solutions.*

2. Proof of Theorem 1.1 and Theorem 1.2

DEFINITION 2.1. *We say G satisfies the (PS) condition if any sequence* $\{u_k\} \subset H$ *for which* $G(u_k)$ *is bounded and* $G'(u_k) \to 0$ *as* $k \to \infty$ *possesses a convergent subsequence.*

The (PS) condition is a convenient way to build some "compactness" into the functional G. Indeed observe that (PS) implies that $K_c \equiv \{u \in H \mid G(u) = c$ and $G'(u) = 0\}$, i.e. the set of critical points having critical value c, is compact for any $c \in R$.

LEMMA 2.2. *Assume* $b \neq \Lambda_i, b \neq 0$ *and* g *satisfies the condition* (g1).*Then For any* $c \in R$ *the functional* I *satisfies the* (PS) *condition.*

Proof. We compute

$$\begin{aligned} I(u) &= \frac{1}{2}\left(\int (\Delta u)^2 - c\int |\nabla u|^2\right) - \frac{b}{2}\int (u^+)^2 - \frac{1}{2}\int G(x,u) \\ &= \frac{1}{2}\left(\int (\Delta u)^2 + \int |\nabla u|^2\right) - \frac{1+c}{2}\int |\nabla u|^2 \\ &\quad - \frac{b}{2}\int (u^+)^2 - \frac{1}{2}\int G(x,u) \\ &= \frac{1}{2}\|u\|_H^2 - \int \left(\frac{1+c}{2}|\nabla u|^2 + \frac{b}{2}(u^+)^2 + \frac{1}{2}\int G(x,u)\right). \end{aligned}$$

We observe that

$$\nabla I(u) = u - i^*[(1+c)\Delta u + bu^+ + g(x,u)]. \tag{2.1}$$

Here $i^* : L^2(\Omega) \to H$ is a compact operator. (i^* is the adjoint of the immersion $i : H \hookrightarrow L^2(\Omega)$). Suppose $\{u_k\}_{k=1}^{\infty} \subset H$ is the (PS) sequence, i.e $\{I(u_k)\}_{k=1}^{\infty}$ bounded and $\nabla I(u_k) \to 0$ in H. It is enough to prove that $\{u_k\}_{k=1}^{\infty}$ is bounded(because $i^* : L^2(\Omega) \to H$ is a compact operator.). By contradiction we suppose that $\lim_k \|u_k\|_H = +\infty$. Up to a subsequence we can assume that $\lim_k \frac{u_k}{\|u_k\|_H} = u$ weakly in H, strongly in $L^2(\Omega)$ and pointwise in Ω. By (2.1) we deduce

$$\begin{aligned} \left(\nabla I(u_k), \frac{u_k}{\|u_k\|_H}\right)_H &= \frac{1}{\|u_k\|_H}\left(\int |\Delta u_k|^2 - c\int |\nabla u_k|^2\right) \\ &\quad - b\int \frac{{u^+_k}^2 - g(x,u_k)u_k}{\|u_k\|_H} \\ &= \frac{2I(u_k)}{\|u_k\|_H} + \frac{1}{\|u_k\|_H}\int 2G(x,u_k) - g(x,u_k)u_k \end{aligned}$$

and, passing to the limit, we get

$$\lim \frac{1}{\|u_k\|_H}\int 2G(x,u_k) - g(x,u_k)u_k = 0$$

Moreover by (g1) we get

$$\frac{1}{\|u_k\|_H}\int 2G(x,u_k) - g(x,u_k)u_k \le \int a_0\frac{(u_k)^+}{\|u_k\|_H} - \int \frac{a_1}{\|u_k\|_H}$$

and so passing to the limit , we get $\int a_0(u_k)^+ \geq 0$,since $a_0 \neq 0$. Hence $u \geq 0$ a.e. in Ω and $u \not\equiv 0$. From $\nabla I(u_k) \to 0 \quad in \quad H$, we get

$$\begin{aligned}\lim_{k\to\infty} \frac{\nabla I(u_k)}{\|u_k\|_H} &= \lim_{k\to\infty} \Big\{ \frac{u_k}{\|u_k\|_H} - i^*[(1+c)\frac{\Delta u_k}{\|u_k\|_H} \\ &- b\frac{u_k}{\|u_k\|_H} - \frac{g(x,u_k)}{\|u_k\|_H} \Big\} \\ &= 0\end{aligned}$$

strongly in H. Here $i^* : L^2(\Omega) \to H$ is a compact operator. So the bounded sequence $\lim \frac{u_k}{\|u_k\|_H}\}_{k\in N}$ converges strongly in H. Hence $u - i^*[(1+c)\Delta u - bu] = 0$. This implies that $u \geq 0$ is a nontrivial solution of

$$\Delta^2 u + c\Delta u = bu, \tag{2.2}$$

which contradicts to the equation (2.2) $(b \neq \Lambda_i(c), b \neq 0)$ that has only the trivial solution. So we discovered that $\{u_k\}_{k=1}^{\infty}$ is bounded in H, hence there exists a subsequence $\{u_{kj}\}_{kj=1}^{\infty}$ and $u \in H$ with $u_{kj} \to u$ in H. □

To prove Theorem 1.1 we need the following two lemmas.

Lemma 2.3. *If g satisfies* (g2), *then we have* $\lim_{r\to+\infty} I(-re_1) = -\infty$.

Proof. From definition of I and condition (g2), since $\Lambda_1(c) < 0$ we get

$$\begin{aligned}I(-re_1) &= \frac{1}{2}\Lambda_1(c)r^2 \int e_1^2 - \frac{b}{2}\int [(-re_1)^2 - \frac{1}{2}\int G(-re_1) \\ &\leq \frac{1}{2}(\Lambda_1(c) - \epsilon)r^2 \int e_1^2 \to -\infty\end{aligned}$$

with $r \to \infty$, which proves our claim. □

Consider the values of I in the set $\Gamma_\rho(H)$, where

$$\Gamma_\rho(H) = \Big\{ u_1+u_2 \in span\{e_1\}\oplus H_1^\perp \ \Big| \ \int u_1^2 + \int (\Delta u_2)^2 - c\int |\nabla u_2|^2 \leq \rho^2 \Big\}.$$

The set $\Gamma_\rho(H)$ is homeomorphic to a ball in H, whose boundary is the set

$$\gamma_\rho(H) = \Big\{ u_1+u_2 \in span\{e_1\}\oplus H_1^\perp \ \Big| \ \int u_1^2 + \int (\Delta u_2)^2 - c\int |\nabla u_2|^2 = \rho^2 \Big\}.$$

Lemma 2.4. *Assume* $b < \Lambda_1(c)$ *and* (g3). *Then there exists a small* $\rho > 0$ *such that*

$$\inf_{u \in \gamma_\rho(H)} I(u) > 0.$$

Proof. For any $u \in H$, we can write u as $u = u_1 + u_2$, where $u_1 \in H_1, u_2 \in H_1^\perp$. By (g3), for sufficiently small $\rho > 0$, we get

$$\begin{aligned} I(u) \geq\ & \frac{1}{2}\left(\int (\Delta u_1)^2 - c\int |\nabla u_1|^2\right) + \frac{1}{2}\left(\int (\Delta u_2)^2 - c\int |\nabla u_2|^2\right) \\ & - \frac{b}{2}\int u_1^2 - \frac{b}{2}\int u_2^2 \\ & - \frac{1}{2}(\|u_1\|_H^2 + \|u_2\|_H^2) \cdot o(\|u\|_H) \\ \geq\ & \frac{1}{2}(\Lambda_1(c) - b - c_1 \cdot o(\|u\|_H)) \int u_1^2 \\ & + \frac{1}{2}(1 - c_2 \cdot o(\|u\|_H)) \left(\int (\Delta u_2)^2 - c\int |\nabla u_2|^2\right) > 0 \end{aligned}$$

for some positive constants c_1, c_2. This proves the lemma. □

Proof of Theorem 1.1

Since $\lambda_1 < c < \lambda_2$, $b < \Lambda_1(c)$, by Lemma 2.4, there is a small $\rho > 0$ such that $\inf_{u \in \gamma_\rho(H)} I(u) > 0$. From the definition of I, we have $I(0) = 0$ with $0 \in \Gamma_\rho(H)$. Set $A = \{-re_1, 0\}, B = \gamma_\rho(H)$. Then A links B. By Lemma 2.3 there is sufficiently large $r > 0$ such that $-re_1 \notin \Gamma_\rho(H)$ and $I(-re_1) < 0$. And thus $\sup_A I(u) < \inf_B I(u)$. By the Mountain Pass Theorem I possesses a critical value $c_1 \geq \inf_{u \in \gamma_\rho(H)} I(u) > 0$ and $0 = \min_{u \in \Gamma_\rho(H)} I(u)$. So I has two critical values. Hence the problem (1.1) has at least two solutions, one of which is nontrivial.

Lemma 2.5. *Suppose* $b > 0$ *and* g *satisfies* (g4), *then there exists a small* $\rho > 0$ *such that* $\sup_{\|u\|=\rho, u \in H_2} I(u) < 0$.

Proof. From the condition of (g4) that there exist constant $\alpha > 0$ such that $\frac{g(x,u)}{u} \leq \alpha < \Lambda_1(c)$ for $x \in \Omega$. Since g is subcritical growth, we get $G(x,u) \leq \frac{1}{2}\alpha u^2 - a|u|^s$, and $a > 0$, $s \in (2, 2^*)$. So for sufficiently small $\|u\|$ we have,

$$\begin{aligned} I(u) &\le \frac{1}{2}\left(\int (\Delta u)^2 - c|\nabla u|^2\right) - \frac{b}{2}\int u^{+2} - \frac{1}{2}\int \alpha u^2 + a\int |u|^s \\ &\le \frac{1}{2}\left(\int (\Delta u)^2 - c|\nabla u|^2\right) - \frac{1}{2}\int \alpha u^2 + \int a|u|^s \\ &\le \frac{1}{2}(\Lambda_2(c)u^2 - \alpha)\int u^2 + a\int |u|^s \end{aligned}$$

for some positive constant $\alpha > \Lambda_2(c)$. The norms $\|\cdot\|_H$ and $\|\cdot\|_{L^2(\Omega)}$ in H_2 are equivalent, since dim $H_2 = 2$. Condition $\alpha > \Lambda_2(c)$ implies that $\Lambda_2(c)u^2 - \alpha < 0$. So, for small $\rho > 0$ we have $\sup_{\|u\|=\rho, u\in H_2} I(u) < 0$. □

LEMMA 2.6. *Let* $\lambda_1 < c < \lambda_2$ *and*

$$T = \max\left\{\int (u^+)^2 \;\middle|\; u \in H_1^\perp, \int (\Delta u)^2 - c|\nabla u|^2 = 1\right\}.$$

Then we have $T < \frac{1}{\Lambda_2(c)}$.

Proof. We know that $\int(\Delta u)^2 - c|\nabla u|^2 \ge \Lambda_2(c)\int u^2 \ge \Lambda_2(c)\int(u^+)^2$ for $u \in H_1^\perp$. Hence $T \le \frac{1}{\Lambda_2(c)}$. Suppose $T = \frac{1}{\Lambda_2(c)}$. Then there exists a sequence $\{u_k\}_{k\in N}$ in $H_1^\perp$ such that $\int(\Delta u)^2 - c|\nabla u|^2 = 1$ and $\lim \int(u_k^+)^2 = \frac{1}{\Lambda_2(c)}$. We have $\lim u_k = u$ in $L^2(\Omega)$ and $\int(u^+)^2 = \frac{1}{\Lambda_2(c)}$. Since $0 \le \int u^2 = \int(u^+)^2 + \int(u^-)^2 \le \frac{1}{\Lambda_2(c)}$, we have $u^- = 0$. This is a contradiction. □

From the condition of (g5) that there exist constant $\beta > 0$ such that $\frac{g(x,u)}{u} \le \beta < \Lambda_2(c)$ for $x \in \Omega$, and exist $M > 0$ and for $|u| \ge M$ we have $G(x,u) \le \frac{1}{2}\beta u^2 - b$, and $b > 0$.

LEMMA 2.7. *Suppose* g *satisfies* (g5) *and* $b > \frac{1-\beta}{T}$. *Then there exists a large* $R > 0$ *such that*

$$\inf\left\{I(u) \;\middle|\; u = \sigma e_2 + v, \sigma \ge 0, v \in {H_2}^\perp, \int (\Delta u)^2 - c\int |\nabla u|^2 = R^2\right\} > 0.$$

Proof. Under the assumptions of (g5) there exists $\beta > 0$ and $b > 0$ such that for $\|u\| \geq R$, we have

$$\begin{aligned} I(v+\sigma e_2) &= \frac{1}{2}\left(\int (\Delta(v+\sigma e_2))^2 - c\int |\nabla(v+\sigma e_2)|^2\right) \\ &\quad - \frac{b}{2}\int (v+\sigma e_2)^{+2} + \frac{b}{2}\int G(v+\sigma e_2) \\ &\geq \frac{1}{2}\left(\int (\Delta u)^2 - c\int |\nabla u|^2\right) - \frac{b}{2}\int (u^+)^2 \\ &\quad - \int \frac{1}{2}(\beta u^2 - b|u|^s) \\ &\geq \frac{1}{2}(1 - bT - \beta)\left(\int (\Delta u)^2 - c\int |\nabla u|^2\right) + b|\Omega|. \end{aligned}$$

Since $b > \frac{1-\beta}{T}$, the argument holds for large $R > 0$. □

Proof of Theorem 1.2

Since $b = max\{0, \frac{1-\beta}{T}\}$ and g satisfies (g4),(g5) by Lemma 2.5 and 2.6 there exist $R > \rho > 0$ such that

$$\sup_{\|u\|=\rho, u\in H_2} I(u) < 0 < \inf_{v\in\Sigma_R(e_2, {H_2}^{\perp})} I(v),$$

where $\Sigma_R(e_2, {H_2}^{\perp})$ is the boundary of the set

$$\left\{I(u) | u = \sigma e_2 + v \mid \sigma \geq 0, v \in H_2, \int (\Delta u)^2 - c\int |\nabla u|^2 \leq R^2\right\}.$$

By the Variational Linking Theorem $I(u)$ has at least two nonzero critical values c_1, c_2 such as

$$c_1 \leq \sup_{\|u\|=\rho, u\in H_2} I(u) < 0 < \inf_{v\in\Sigma_R(e_2, {H_2}^{\perp})} I(v) \leq c_2.$$

Therefore, (1.1) has at least two nontrivial solutions. This implies that (1.1) has at least three solutions.

3. Variational setting

To introduce a Variational Linking Theorem, we define the following sets.

DEFINITION 3.1. *Let X be a Hilbert space, $Y \subset X$, $\rho > 0$, and $e \in X \setminus Y$, $e \neq 0$. Set*

- $B_\rho(Y) = \{x \in Y \mid \|x\|_X \leq \rho\}$,
- $S_\rho(Y) = \{x \in Y \mid \|x\|_X = \rho\}$,
- $\Delta_\rho(e, Y) = \{\sigma e + v \mid \sigma \geq 0, v \in Y, \|\sigma e + v\|_X \leq \rho\}$,
- $\Sigma_\rho(e, Y) = \{\sigma e + v \mid \sigma \geq 0, v \in Y, \|\sigma e + v\|_X = \rho\} \cup \{v \mid v \in Y,\ \|v\|_X \leq \rho\}$.

We recall a theorem of existence of two critical levels for a functional which is a variation of linking theorem.

THEOREM 3.2. *(A variation of Linking.) Let X be a Hilbert space which is topological direct sum of the subspaces X_1, X_2. Let $f \in C^1(X, R)$. Moreover assume*

(a) $dim X_1 < +\infty$,

(b) there exist $\rho > 0$, $R > 0$ and $e \in X_1$, $e \neq 0$ such that $\rho < R$ and $\sup_{S_\rho(X_1)} f < \inf_{\Sigma_R(e,X_2)} f$,

(c)$-\infty < a = \inf_{\Delta_R(e,X_2)} f$,

(d) $(PS)_c$ condition holds for any $c \in [a, b]$ where $b = \sup_{B_\rho(X_1)} f$. Then there exist at least two critical levels c_1 and c_2 for the functional f such that

$$\inf_{\Delta_R(e,X_2)} f \leq c_1 \leq \sup_{S_\rho(X_1)} f < \inf_{\Sigma_R(e,X_2)} f \leq c_2 \leq \sup_{B_\rho(X_1)} f.$$

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