한국환경과학회지 제18권(제1호), 1~7, 2009
Journal of the Environmental Sciences

Comparison of Sediment Yield by IUSG and Tank Model in River Basin

Yeonghwa Lee
Department of Civil Engineering, Daegu Haany University, Kyungsan 712-715, Korea
(Manuscript received 26 September, 2008; revised 16 October, 2008; accepted 26 October, 2008)

Abstract

In this study a sediment yield is compared by IUSG, IUSG with Kalman filter, tank model and tank model with Kalman filter separately. The IUSG is the distribution of sediment from an instantaneous burst of rainfall producing one unit of runoff. The IUSG, defined as a product of the sediment concentration distribution (SCD) and the instantaneous unit hydrograph (IUH), is known to depend on the characteristics of the effective rainfall. In the IUSG with Kalman filter, the state vector of the watershed sediment yield system is constituted by the IUSG. The initial values of the state vector are assumed as the average of the IUSG values and the initial sediment yield estimated from the average IUSG. A tank model consisting of three tanks was developed for prediction of sediment yield. The sediment yield of each tank was computed by multiplying the total sediment yield by the sediment yield coefficients; the yield was obtained by the product of the runoff of each tank and the sediment concentration in the tank. A tank model with Kalman filter is developed for prediction of sediment yield. The state vector of the system model represents the parameters of the tank model. The initial values of the state vector were estimated by trial and error.

Key Words : IUSG, Kalman filter, Sediment yield, Tank model, State vector, System model, Sediment distribution, Sediment concentration distribution, Sediment infiltration

1. Introduction

Estimates of watershed sediment yield are required for design of dams and reservoirs, soil conservation practices, and debris basins; determination of pollutants; depletion of reservoirs, lakes and wetlands; determination of the effects of basin management; and cost evaluation. The sediment concentration of the IUSG was assumed to vary with the effective rainfall amount. A sediment routing function, using the travel time and sediment particle size, was used to determine the SCD. Runoff influencing on sediment yield is, in general, nonlinear and time-variant. The parameters of tank model vary in time and space, and when they are assumed constant, they are so only by assumption. The coefficients of the tank model for runoff and sediment yield are assumed to be the same. Thus the estimation of the parameters by Kalman filter has accomplished for runoff and the sediment yield is calculated by the parameters. The errors in a tank model may arise due to inadequacy of the model itself, parameter uncertainty, errors in the data used for parameter estimation, and inadequate understanding of the rainfall-runoff-sediment yield process due, in part, to randomness. The error in the prediction of sediment yield (runoff) due to the uncertainty caused by the physical process, the model, and the input data can be reduced if Kalman filter is incorporated in a tank model. Lee and Singh[1,2] analyzed sediment yield by coupling Kalman filter with the IUSG and the tank model. And Lee[3] analyzed sediment yield by the tank model with Kalman filter.

Corresponding Author : Yeonghwa Lee, Department of Civil Engineering, Daegu Haany University, Kyungsan 712-715, Korea
Phone: +82-53-819-1419
E-mail: yhlee@dhu.ac.k

The objective of this study is to suggest a suitable model for predicting sediment yield in the river basin, from comparing with IUSG, IUSG with Kalman filter, tank model and tank model with Kalman filter separately using the results analyzed by Lee and Singh[1] and Lee[3].

2. Theory

2.1. Instantaneous Unit Sediment Graph (IUSG)

Following Williams[4] the IUSG[5] can be defined as the distribution of sediment from an instantaneous burst of rainfall producing one unit of runoff and is considered to be the product of the IUH and the sediment concentration distribution:

$$h_s(t) = h(t)C(t) \tag{1}$$

in which $h_s(t)$: the IUSG ordinate
$h(t)$: the IUH ordinate
$C(t)$: the sediment concentration

The sediment concentration distribution can be estimated by considering the sediment-routing equation,

$$Y = Y_0 \exp(-aTd^{0.5}) \tag{2}$$

in which Y : the sediment yield at a particular channel section
Y_0 : the sediment yield at an upstream section
a : the routing coefficient
T : the travel time between the two sections
d : the median sediment particle diameter

The initial concentration for one unit of runoff,

$$C_{01} = \frac{Yv_1}{\left[H\sum_{i=1}^{m} v_i^2\right]^{-1}} \tag{3}$$

The routing coefficient a can be determined by equation (4),

$$a = -\frac{\ln(q_p/Q_p)^{0.56}}{T_p d^{0.5}} \tag{4}$$

in which Q_p : the peak source runoff rate
T_p : the watershed time to peak

2.2. IUSG with Kalman filter

2.2.1. System model

The state space model using Kalman filter is constituted by the IUSG, which is then allowed to vary in time. The state vector X(k) : (2×1) is as follows:

$$X(k) = \begin{bmatrix} ESY(k) \\ U(k) \end{bmatrix} \tag{5}$$

in which ESY(k) : the estimated sediment yield at time k
U(k) : the ordinate of IUSG at time k

and the state transition matrix , Φ(k) : (2×2), is

$$\Phi(k) = I = \left[\delta_{ij}\right] \tag{6}$$

in which δ_{ij} is the Kronecker delta, defined as

$\delta_{ij} = 1$ if i = j
$\delta_{ij} = 0$ if i ≠ j

Then the system model is described by

$$X(k) = X(k-1) + w(k);\ w(k) \sim N\ (0,\ Q(k)) \tag{7}$$

2.2.2. Measurement model

The observation variable applicable to the IUSG is sediment yield, Y. Therefore the measurement model can be described as

$$Q(k) = Z(k) = H(k)\cdot X(k) + v(k);\ v(k) \sim N\ (0,\ R(k)) \tag{8}$$

in which H(k), (1×2), is the observation transition matrix assumed by

$$H(k) = [1,0] \tag{9}$$

Sediment yield Y(k) can be estimated by multiplying H(k) by the state vector X(k) : (2×1) in equation (7).

2.3. Tank model

The tank model (Sugawara[6]) considered in this study is represented by a cascade of conceptual tanks. For determining the sediment yield by the tank model, the SCD of the first tank is produced by the incremental source runoff (or the effective rainfall) and sediment concentration of the next lower tank is computed from the sediment infiltration of the upper tank. The sediment concentration of the first tank is computed from its storage and the SCD; the sediment concentration of the next lower tank is obtained by its storage and the sediment infiltration of the upper tank; and so on. The sediment yield through the side outlet is obtained by multiplying the total sediment yield, obtained by the product of runoff and the sediment concentration, by the sediment yield coefficient. The sediment infiltration through the bottom outlet is obtained by multiplying the total sediment infiltration, obtained by the product of infiltration and the sediment concentration, by the sediment infiltration coefficient.

2.4. Tank model with Kalman filter

2.4.1. System model

The state vector of system model is used the parameters of tank model. Therefore the state vector X(k) : (13×1) is as follows:

$$X(k) = [A1, A2, A3, B1, C1, A0, B0, C0, HA1, HA2, HA3, HB, HC]^T \quad (10)$$

and the state transition matrix , Φ(k) : (13×13) and the system error transition matrix(n×n), $\Gamma(k|k-1)$: (13×13) are assumed as unit matrix I, then the system model is described by

$$X(k) = X(k-1) + w(k) ; w(k) \sim N(0, Q(k)) \quad (11)$$

2.4.2. Measurement model

The observation variable applicable to the tank model is runoff, Q. Therefore the measurement model can be described as

$$Q(k) = Z(k) = H(k) \cdot X(k) + v(k) ; v(k) \sim N(0, R(k)) \quad (12)$$

where H(k) : (1×13) is the observation transition matrix expressed by

$$H(k) = [h_1, h_2, h_3, h_4, h_5, 0,0,0,0,0,0,0,0] \quad (13)$$

Where h_1, h_2, h_3, h_4, h_5 are, respectively, the head of water at the outlets of each tank.

2.5. Kalman filter

Kalman filter is a state estimation algorithm of a state-space model and optimally represents the system state of a deterministic or a stochastic model which has uncertainties in observed data, initial and boundary conditions, and parameters. The Kalman filter algorithm is constituted by three components: system model, measurement model, and Kalman filtering.

2.5.1 System model

A system which has a discrete dynamic behavior can be described in terms of the state vector as (Todini[7], Wood[8])

$$X(k) = \Phi(k|k-1) \cdot X(k-1) + \Gamma(k|k-1) \cdot w(k-1) \quad (14)$$

where $X(k)$ = state vector (n×1), $\Phi(k|k-1)$ = state transition matrix (n×n) for time k at time(k-1), $\Gamma(k|k-1)$=system error transition matrix(n×n), $w(k-1)$ = system error vector (n×1).

2.5.2. Measurement model

The state vector X(k) of the system is observed through a measurement system, that inherently contains error. Therefore the measurement vector Z(k) can be described as a linear combination of a state vector X(k) and a measurement error vector v(k):

$$Z(k) = H(k) \cdot X(k) + v(k) \quad (15)$$

where $Z(k)$ = measurement vector (m×1), $H(k)$= measurement transition matrix (m×n), $v(k)$= measurement error vector (m×1).

The error vectors w(k) of (14) and v(k) of (15) are assumed independent Gaussian processes.

2.5.3. Kalman filtering

By (11) of the system model the state prediction value $\overline{X}(k|k-1)$ at time k, given its value at time (k-1), is (Wu[9])

$$\overline{X}(k|k-1) = \Phi(k|k-1) \cdot \hat{X}(k-1|k-1) \quad (16)$$

Knowing the state prediction value $\overline{X}(k|k-1)$ and the measurement vector Z(k) of the measurement model, the state estimation value $\hat{X}(k|k)$ is obtained by filtering the measurement error with use of the Kalman gain K(k) as

$$\hat{X}(k|k) = \overline{X}(k|k-1) + K(k)[Z(k) - H(k) \cdot \overline{X}(k|k-1)] \quad (17)$$

where

$$K(k) = P(k|k-1)\cdot H(k)^T\left[H(k)\cdot P(k|k-1)\cdot H(k)^T + R(k)\right]^{-1} \quad (18)$$

$\left[Z(k) - H(k)\cdot \overline{X}(k|k-1)\right]$ = measurement error.

The covariance P(k|k) of the state estimation error is

$$P(k|k) = \left[I - K(k)\cdot H(k)\right]\cdot P(k|k-1) \quad (19)$$

3. Application and Analysis

3.1. Study basin

A small upland watershed, W-5, a part of the Pigeon Roost basin located near Oxford in Marshall County, Mississippi, was selected for testing IUSG, IUSG with Kalman filter, a tank model and a tank model with Kalman filter. The watershed has an area of approximately 4.04 km^2, is 1288 m long and 128.8 m wide. The watershed consists of a rather flat flood plain with natural channels and rolling, severely dissected interfluvial areas. The channels have a few straight reaches, and most have banks that scour easily. The average channel width-depth ratio is approximately 2:1 at the gaging station. A detailed description of this watershed is given by Bowie and Bolton[10].

3.2. IUSG

The IUSG was determined by equation (1) and the IUH was determined by Nash model for each event. The parameters for the sediment yield estimated by MUSLE for watershed W-5 are as follows: The soils factor, K, is 0.26, the crop management factor, C, is 0.07, the erosion control practice factor, P, is 0.47 and the slope length and gradient factor, LS, is 0.34. The routing coefficient *a*, for estimating H is estimated for each event by equation (4) and is given in Table 1.

The initial concentration for one unit of runoff, C_{01}, the sediment yield Y, estimated by MUSLE and H for each event are given in Table 1.

3.3. IUSG with Kalman filter

The initial parameter values for the state vector of the IUSG using Kalman filter, X(0) = $[ESY(1)\ U(0)]^T$, were defined by the mean value of IUSG which was taken as a function of the IUSG values. Here ESY(1) is the sediment yield estimated from the mean value of the IUSG ,U(0). The initial values P(0|0) of the covariance matrix P(k) of the state estimation error were assumed such that the diagonal elements were P(0|0) = [(OBSY(1)-ESY(1) 3.0]. Other initial values were assumed as follows: the measurement error covariance matrix R(k) of the measurement error vector V(k), R(0) = 0; the system error covariance matrix Q(k) of the system error vector W(k), Q(0) = 0; the state transition matrix Φ(0) and the system error transition matrix Γ(0) were assumed as unit matrix I. The sediment yield graph of the IUSG with Kalman filter is in closer agreement with the observed sediment yield graph than is the IUSG alone. These error indices for the sediment yield by the IUSG and the IUSG with Kalman filter are given in Table 2.

3.4. Tank model

The tank model parameters are the runoff and the sediment yield coefficients (*A1, A2, A3, B1, C1*), the infiltration and the sediment infiltration coefficients (*A0, B0, C0*) and the heights of the runoff orifices (*HA1, HA2, HA3, HA4, HA5*). These parameters were estimated by minimizing the error between observed and computed sediment yield and the estimated parameters and shown in Table 3.

Table 1. Characteristic values for the determination of the IUSG

Storm	Rainfall(mm)	*a*	H	C_{01}(mg/l)	Y(t/h)
No.1(72.12.9)	259.08	1.735	577.88	257880.3	110.08
No.2(73.3.14)	48.84	7.333	833.13	404593.1	18.57
No.3(75.1.10)	173.52	7.536	878.87	239309.7	57.74
No.4(75.3.12)	297.24	1.695	690.28	239137.4	124.06

Table 2. Comparison of error indices for IUSG and IUSG with Kalman filter

Criteria	ME		MSE		Bias		VER(%)		PER(%)		TER(%)	
Storm	IUSG	IUSGKF	IUSG	IUSGKF	IUSG	IUSGKF	IUSG	IUSGKF	IUSG	IUSGKF	IUSG	IUSGKF
No.1	0.71	0.94	143.8	66.4	62.33	-4.06	31.64	-1.88	20.49	-1.68	20	0
No.2	0.85	0.95	11.2	6.3	2.99	-0.04	12.24	-0.17	16.91	-7.43	0	-5
No.3	0.71	0.95	35.6	15.6	1.35	0.27	2.54	0.38	41.13	4.96	-15	-5
No.4	0.76	0.96	112.3	48.8	56.15	-1.87	26.33	-0.86	22.58	6.28	-5	-10

3.5. Tank model with Kalman filter

In order not to bias the physical constraints the numerical values of the tank model parameters shown in Table 3 were used as the mean values to define the initial parameter values for the state vector of the tank model using Kalman filter, X(k) = [0.085 0.085 0.085 0.043 0.009 0.063 0.043 0.009 8 4 1 1 1 $]^T$.

The initial storage values S1, S2, and S3 of each tank for h1~h5 in the matrix H(k) were assumed zero. The initial diagonal elements values P(0|0) for the covariance matrix P(k) of the state estimation error were assumed P(0|0) = [0.06 0.06 0.01 0.06 0.06 10 10 10 10 10 10 10 10]. These error indices for the sediment yield by the tank model and the tank model with Kalman filter are given in Table 4. As shown in the table, the model efficiencies for each event are shown between 0.80 and 0.90 for tank model and between 0.87 and 0.93 for the tank model with Kalman filter. The sediment yield of the tank model with Kalman filter is in closer agreement with the observed sediment yield than that of the tank model alone.

4. Results and Discussion

The IUSG with Kalman filter and a tank model with Kalman filter more accurately predicted sediment yield from a watershed W-5, Mississippi, than did the IUSG and tank model alone. The Kalman filter allowed the state vector to vary very well in time and reduced the physical uncertainty of the rainfall-runoff-sediment yield process in the river basin. The use of Kalman filter for sediment yield modeling is appropriate.

In order to make a quantitative comparison of the

Table 3. Parameters of the tank model

Storm	A1	A2	A3	B1	C1	A0	B0	C0	HA1	HA2	HA3	HB	HC
No.1	.09	.09	.09	.05	.01	.09	.05	.01	8	4	1	1	1
No.2	.07	.07	.07	.02	.005	.07	.02	.005	8	4	1	1	1
No.3	.10	.10	.10	.05	.01	.01	.05	.01	8	4	1	1	1
No.4	.08	.08	.08	.05	.01	.08	.05	.01	8	4	1	1	1
Mean	.085	.085	.085	.043	.009	.063	.043	.009	8	4	1	1	1

Table 4. Error indices for sediment yield by tank model and tank model with Kalman filter

Criteria	ME		MSE		Bias		VER(%)		PER(%)		TER(%)	
Storm	Tank	Tankkf	Tank	Tankkf	Tank	Tankkf	Tank	Tankkf	Tank	Tankkf	Tank	Tankkf
No.1	0.80	0.87	30.69	21.83	10.16	5.21	15.36	-0.68	5.47	2.64	10	5
No.2	0.90	0.93	83.47	35.21	39.53	2.57	20.06	3.26	1.03	-1.12	10	5
No.3	0.85	0.92	50.22	26.87	20.87	3.26	20.22	2.18	20.55	9.24	5	0
No.4	0.83	0.89	56.18	23.56	-4.47	-1.03	-5.15	1.57	-2.28	1.68	5	5

Table 5. Error indices for IUSG with Kalman filter and tank model with Kalman filter

Criteria	ME		MSE		Bias		VER(%)		PER(%)		TER(%)	
Storm	IUSG	Tank	IUSG	Tank	IUSG	Tank	IUSG	Tank	IUSG	Tank	IUSG	Tank
No.1	0.94	0.87	66.40	21.83	-4.06	5.21	-1.88	-0.68	-1.68	2.64	0	5
No.2	0.95	0.93	6.30	35.21	-0.04	2.57	-0.17	3.26	-7.43	-1.12	-5	5
No.3	0.95	0.92	15.60	26.87	0.27	3.26	0.38	2.18	4.96	9.24	-5	0
No.4	0.96	0.89	48.80	23.56	-1.87	-1.03	-0.86	1.57	6.28	1.68	-10	5

*IUSG : IUSG with Kalman filter, **Tank : tank model with Kalman filter

IUSG with Kalman filter and a tank model with Kalman filter, the predicted results were evaluated based on : (1) model efficiency, ME; (2) mean square error, MSE; (3) Bias; (4) volume error, VER; (5) peak sediment yield error, PER; and (6) peak time error, TER. The calculated error indices for both models are given in Table 5.

As shown in Table 5, the IUSG with Kalman filter and a tank model with Kalman filter show, respectively, the model efficiency of 0.94～0.96, 0.87~0.93, the mean square error of 6.3～66.4, 21.83~35.21, the bias of -4.06～0.27, -1.03~5.21, the volume error of -1.88%～0.38%, -0.68%~3.26%, the peak sediment yield error of -7.43%～6.28%, -1.12%~9.24% and the peak time error of -10min～0min, 0min~5m. The model efficiency of the IUSG with Kalman filter show more accuracy as 2.11%～7.45% than that of a tank model with Kalman. The above results show that the results of the IUSG with Kalman filter are superior to those of a tank model with Kalman filter, and the IUSG with Kalman filter is suitable model for predicting sediment yield in the river basin.

5. Conclusions

As comparing the sediment yield by IUSG, IUSG with Kalman filter, a tank model and a tank model with Kalman filter seperately, the following conclusions can be drawn from this study.

(1) The IUSG with Kalman filter yields better results than the IUSG. (2) A tank model with Kalman filter yields better results than a tank model. (3) The use of Kalman filter for sediment yield modeling is appropriate. (4) The sediment yield computed by the IUSG with Kalman filter is in good agreement with the observed sediment yield and is more accurate than that by the IUSG, the tank model and the tank model with Kalman filter. (5) The IUSG with Kalman filter is suitable model for predicting sediment yield in the river basin.

References

1) Lee Y. H., Singh V. P., 1999, Prediction of sediment yield by coupling Kalman filter with instantaneous unit sediment graph, Hydrological Processes, 2861-2875.
2) Lee Y. H., Singh V. P., 2005, Tank model for sediment yield, Water Resources Management, 19, 349-362.
3) Lee Y. H., 2007, Tank model using Kalman filter for sediment yield, Journal of the Environmental Sciences, 16(12), 1319-1324.
4) Williams J. R., 1975a, Sediment yield prediction with universal equation using runoff energy factor, In Present and Prospective technology for Predicting Sediment Yields and Sources, 244-52, ARS-S-40, Agricultural Research Service, U.S. department of Agriculture, Washington, D.C.
5) Singh V. P., Baniukiwicz A., Chen V. J., 1982, 'An instantaneous unit sediment graph study for small upland watersheds,' Littieton, Colo. : Water Resources Publications.
6) Sugawara M., 1979, Automatic calibration of the tank model, Hydrological Science Bulletin, 24, 375-388.
7) Todini E., 1978, Mutually interactive state parameter (MISP) estimation, Application of Kalman filter, Proc. of AGU Chapman Conference, Univ. of Pittsburgh, 135-151.
8) Wood E. F., Szollosi-Nagy A., 1978, An adaptive

algorithm for analyzing short-term structural and parameter changes in hydrologic prediction models, Water Resources Research, 14(4), 577-581.

9) Wu C. M., Huang W. C., 1990, Effect of observability in Kalman filtering on rainfall-runoff modeling, Taiwan Water Conservancy Quarterly, 38, 37-47.

10) Bowie A. J., Bolton G. C., 1972, Variations in runoff and sediment yields of two adjacent watersheds as influenced by hydrologic and physical characteristics, Proceeding, Mississippi Water Resources Conference, 37-55, water Resources Research Institute, Mississippi State University, Mississippi State, Mississippi.