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Comparing Operation Cycle Times of Container Yard Cranes under Various Sequencing Rules*

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ABSTRACT

This study compares the cycle times of handling operations of yard cranes under different sequencing rules. An operation cycle was divided into several elementary movements and formulas for the expectation and the variance of each elementary movement were analytically derived. The expected waiting time of trucks was estimated based on the given arrival rate of trucks. The previous studies focused on developing a method to make an efficient schedule of operations for yard cranes. This paper introduces several sequencing rules, such as *first-come-first-served, unidirectional travel*, and *Z pick travel rules*. In addition, a formula for estimating the cycle times of yard cranes under each sequencing rule is derived, and the performance under the different sequencing rules are compared with each other.

Keywords: Yard Cranes, Sequencing Rules, Cycle Times, Container Yards

1. Introduction

The handling operations at container terminals are generally divided into three types: vessel operations for discharging and loading containers from/onto container ships, receiving/delivery operations for road trucks, and container handling and storage operations in the yard.

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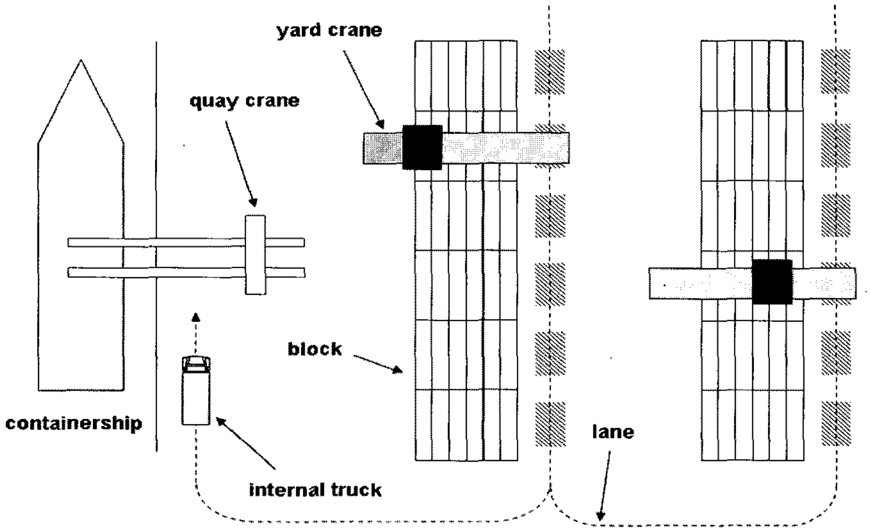

Figure 1. A block with horizontal layout

Figure 1 shows the simplified components of a container terminal. The yard consists of many blocks. A block is the basic unit of storage space of container yards, and the yard cranes (YCs) are positioned at each block. Yard cranes are important handling equipment in container yards. The major role of YCs is to pick up a container from a truck and store it to a bay, or to retrieve a container from a bay and release it onto a truck in a block. Thus, the efficiency of yard operations depends heavily on the operations of YCs.

There have been several studies dealing with sequencing operations for YCs. Linn and Zhang [13] and Zhang *et al.* [19] studied deployment of YCs between blocks over a planning horizon to minimize the total workload among blocks. Kim and Kim [8] studied the problem of routing a YC to support loading operations. Their study was designed to determine the visiting sequence of a YC and the number of containers to be picked up at each visiting bay simultaneously. Some heuristic algorithms using search techniques are suggested by Kim and Kim [10]. Kim *et al.* [9] proposed a method for determining the sequence of tasks for a YC by using a reinforcement learning technique. Their proposed method is compared to other sequencing rules currently in practice such as *first-come-first-served, uni-directional travel, nearest truck first served,* and *shortest processing time rule.* Guo *et al.* [5] proposed a YC dispatching algorithm to solve the problem of YC job sequencing to minimize average vehicle waiting time for each planning window. The proposed algorithm is compared to the existing dispatching rules such as *first-come-first-served, nearest job first* and so on, and by the size of planning window. Li *et al.* [12] proposed a mathematical model to solve the optimal job sequence at a block with two YCs which has the same size. Petering *et al.* [16] studied the rule-based and look-ahead dispatching algorithms, and test them by using simulation. The rule-based dispatching algorithm assigns vehicles to YCs by using specific rules and avoiding interference between YCs. However, the look-ahead dispatching algorithm considers vehicles within 2 hours time horizon in advance. Ng and Mak [14, 15] studied the problem of scheduling operations of a YC to perform handling jobs with different ready times within a movement zone so that the sum of job waiting times is minimized. They derived expressions for lower and upper bounds that are used for finding the optimal solution by applying a branch and bound algorithm. They also suggested a heuristic algorithm to solve the same problem more efficiently.

The previous studies focused on developing a method to make an efficient schedule of operations for YCs. This paper introduces several sequencing rules, such as *first-come-first-served, unidirectional travel,* and *Z pick travel rules*. In addition, a formula for estimating the cycle times of YCs under each sequencing rule is derived, and the performance under the different sequencing rules are compared with each other.

The remainder of this paper is organized as follows. Formulas are derived which describe the cycle times of a YC under each given sequencing rule as well as the expected waiting time of trucks in Section 2. The results of numerical experiments are provided in Section 3, and conclusions are given in Section 4.

2. Estimating the Cycle Times under Different Sequencing Rules and the Expected Waiting Time of Trucks

The sequencing rules for the operations of a YC significantly affect the cycle times of the YC. This section details the analytical derivation of the expressions for the expected cycle time and the variance of the cycle time under different sequencing rules. Section 2.1 introduces the sequencing rules under consideration in this paper and Section 2.2 derives the various formulas for estimating the cycle times of a YC.

2.1 Sequencing Rules of a YC

The sequencing rules under consideration in this paper are first-come-first-served, unidirectional travel, and Z pick travel rules. Each rule is described below.

- **First-come-first-served rules**

Trucks are served in the order of their arrival times at each block. This rule is equivalent to a randomized truck sequencing rule from the perspective of a YC.

- **Unidirectional travel rules**

A YC travels in one direction and serves trucks whenever the YC meets a truck until no more trucks remain in the direction of travel. After serving all of the trucks in the direction of travel, the YC travels in the opposite direction (Kim *et al.* [9]).

- **Z pick travel rules**

The Z pick travel rule is one of most famous heuristic rules in order-picking systems. It is used to determine the operation sequence of a picker who is in charge of picking items from both sides of the racks along the aisle. The aisle is divided into a predetermined length called the Z pattern length. When a picker enters an aisle, he/she picks items along one side of the aisle until he/she reaches the Z pattern length. After arriving at the Z pattern length, he/she moves to the position of the first item at the other side of the aisle and picks items on the side until he/she reaches the position of twice the Z pattern length. After arriving at twice the Z pattern length, he/she moves to the position of the first item which is not picked up at the opposite side of the aisle and picks items on the side until he/she reaches again the position of twice the Z pattern length. This process is repeated until he/she reaches the end of the aisle. The shape of the trajectory of a picker is similar to the character 'Z' (Goetschalckx and Ratliff [3]).

The major advantage of the Z pick is that it is sufficient to determine the pattern only once. By using the determined pattern, slots are visited in a fixed sequence which remains the same for any different set of orders.

The following assumptions are introduced for estimating cycle times of YC under each sequencing rule.

1) A single block is considered and a single YC is assigned at the block. In this study, a block is defined as a group of stacks which are assigned to a YC. Thus, when two YCs are deployed to a physical block in the conventional sense, a block in this study corresponds to half of a physical block. It is a reasonable strategy to segregate the service area into multiple sub-areas and assign one YC to each sub-area, because interference among cranes can be avoided by segregating the service area.
2) Trucks can park on both sides of the block. That is, a rail-mounted gantry crane of the cantilever type is considered. The rail-mounted gantry crane of the cantilever type can handle containers at both sides of it as shown in Figure 3.
3) Trucks are located in random positions in a block. Even though bay positions for YCs to receive (store) or deliver (retrieve) containers are pre-determined, because there is no predetermined pattern, they can be considered to be random positions.
4) The trolley moves simultaneously with the movement of the crane in the gantry direction. This means that a YC travels in the Tchebychev metric.

2.2 Deriving the Expressions for the Expectation and the Variance of Cycle Times

Formulas for estimating the expected cycle time and its variance under each sequencing rule are derived in this sub-section.

• Notations

The following list introduces the various parameters required for deriving the expressions for the expectation and the variance of the cycle times of YCs. Also, the basic parameters are illustrated in Figure 2 and Figure 3.

b = Number of bays, which are assigned to a YC, in a block.

t = Number of tiers of stacks.

r = Number of rows in a bay.

n = Pre-determined number of trucks in a block

k = Parameter of the Z pattern length. The Z pattern length is equal to the block length divided by k.

c_w = Width of a storage slot (m).

c_h = Height of a storage slot (m).

c_l = Length of a storage slot (m).

d_c = Distance between the end of the bay and the center of the chassis (m).

g_b = Empty gap between two consecutive bays (m).

g_r = Empty gap between two consecutive rows (m).

h_c = Height of a chassis (m).

v_g^e = Speed of empty gantry travel of a YC (m/min).

v_g^l = Speed of loaded gantry travel of a YC (m/min).

v_t^e = Speed of empty trolley move of a YC (m/min).

v_t^l = Speed of loaded trolley move of a YC (m/min).

v_h^e = Speed of empty hoisting of a YC (m/min).

v_h^l = Speed of loaded hoisting of a YC (m/min).

s_g = Time required for a spreader to grasp a container (min).

s_r = Time required for a spreader to release a container (min).

h_{max} = Height of the spreader at the top position (m).

$h_{max} = c_h(t+1) + 1.5$. Note that (t + 1) is the height including the distance for rehandling, and 1.5 is the allowed distance.

d_{max}^h = Distance between the top position of the spreader and the position of the spreader when the YC picks up a container from a chassis (m).

$d_{max}^h = h_{max} - (h_c + c_h)$.

b_l = The gantry travel distance between two ends of a block (m).

$b_l = (c_l + g_b)(b-1)$.

b_w = The trolley moving distance between two ends of a bay (m).

$b_w = (c_w + g_r)(r-1)$.

D_t^h = Distance between the top position of the spreader and the pickup (releasing) position of a container from (to) a bay (*m*). (this is a random variable).

$E(D_t^h)$ = The expected hoisting (lowering) distance (*m*).

$$E(D_t^h) = h_{max} - c_h\left(\frac{t+1}{2}\right).$$

$Var(D_t^h)$ = The variance of the hoisting (lowering) distance (*m*).

$$Var(D_t^h) = c_h^{\,2}\left(\frac{t^2-1}{2}\right).$$

R_{rt} = Number of rehandles required to pick up a random container from a bay with t tiers and *r* rows. (this is a random variable).

$E(R_{rt})$ = The expected number of rehandles in a bay (Kim [7]).

$$E(R_{rt}) = \frac{t-1}{4} + \frac{t+2}{16r}.$$

$Var(R_{rt})$ = The variance in the number of rehandles in a bay (Lee and Kim [11]).

$$Var(R_{rt}) = -0.0186r + 0.0585t^2 + 0.2169.$$

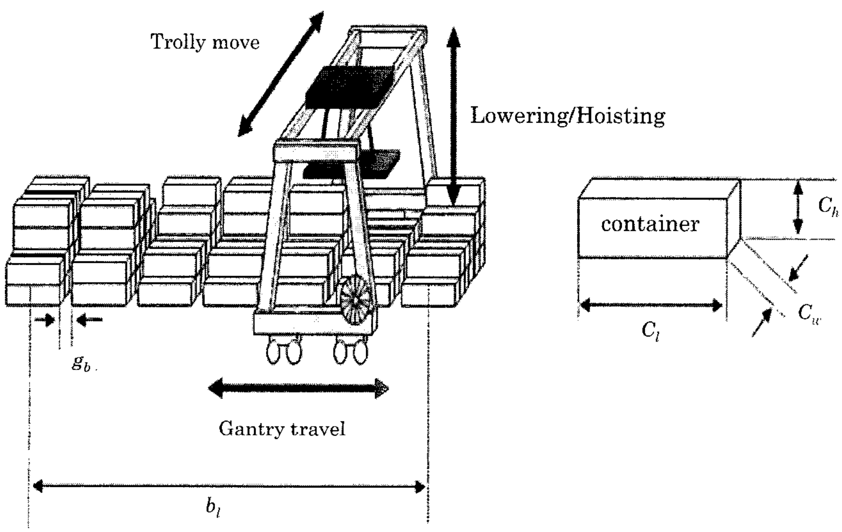

Figure 2. Illustration of notations on blocks

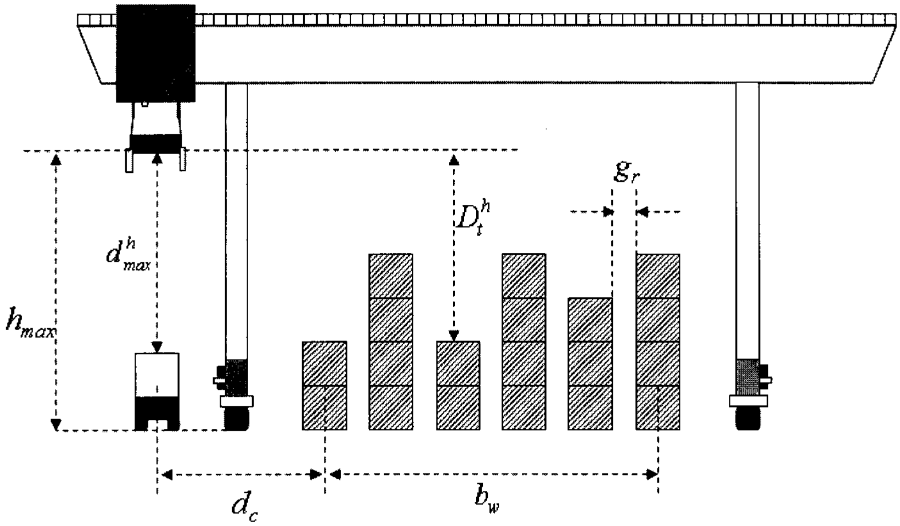

Figure 3. Illustration of notations of a bay

• Notations for handling elements

Each handling element of a YC is represented by "T" with subscripts or superscripts. Subscripts indicate the handling part and its state (loaded or empty). Subscripts *g*, *t*, and *h* indicate the handling components of 'gantry', 'trolley', and 'spreader' of an YC, respectively. However, *s*, instead of *h*, is used to represent the handling component of 'spreader' when the spreader grasps or releases a container. Subscripts *e* and *l* represent the state of a handling component: *e* is for empty state and *l* is for loaded state. Superscripts indicate the starting and ending positions for the movement. Superscripts *r*, *e*, *t*, and *c* represent the starting and ending positions of a handling component for the movement: *r* is a random position within the range in which the handling component can move; *e* is an ending position; *t* is the highest position which the spreader can reach; *c* is the position of a chassis for a truck.

The "t" is used to represent handling time, and indicates that the time has a constant value. The time element represented by "T" is a random variable. Followings are some examples for notations for handling elements.

T_{te}^{er} = Time required for a trolley to move from the end of a bay to a random position while empty.

t_{hl}^{tc} = Time required for hoisting a spreader from the top position on the chassis with a container.

T_{hl}^{tr} = Time required for hoisting a spreader from the top position to a random position with a container.

T_{ge}^{i} = Time required for a YC to travel between two consecutive trucks which are located randomly.

$T^{(re)}$ = Time required for a rehandle.

t_{sg} = Time required for a spreader to grasp a container.

t_{sr} = Time required for a spreader to release a container.

• First-come-first-served rules

Lee and Kim [11] proposed various formulas for estimating the cycle times of a YC. They assumed that all trucks are served randomly. As described earlier, the randomized sequencing rule is the same as the first-come-first-served sequencing rule. This study use the formulas derived by Lee and Kim [11].

• Unidirectional travel rules

Various handling time elements for the receiving and delivery operations are listed in Table 1 and the sequence of task "receiving" in Table 1 and Figure 4.

Table 1. Handling time element of a YC under the unidirectional sequencing rule

Order	Receiving	Delivery
1	$Max\left(T_{te}^{re}, T_{ge}^{i}\right)$	$Max\left(T_{te}^{er}, T_{ge}^{i}\right)$
2	t_{he}^{tc}	$T^{(re)}$
3	t_{sg}	T_{he}^{tr}
4	t_{hl}^{ct}	t_{sg}
5	T_{tl}^{er}	T_{hl}^{rt}
6	T_{hl}^{tr}	T_{tl}^{re}
7	t_{sr}	t_{hl}^{tc}
8	T_{he}^{rt}	t_{sr}
9		t_{he}^{ct}

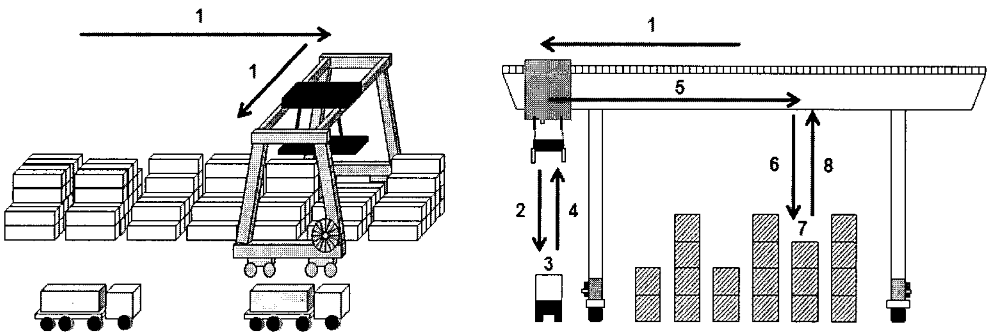

Figure 4. Illustration of ordering notations in Table 1

For example, $Max\left(T_{te}^{re},\ T_{ge}^{i}\right)$ is the Tchebychev travel time of the empty trolley travel and the empty gantry travel of a YC. When a YC serves waiting trucks while moving in a single direction, the gantry travel distance for the YC to move from one truck to the adjacent truck becomes the distance between two consecutive trucks.

$$E\left[Max\left(T_{te}^{re},\ T_{ge}^{i}\right)\right]=\frac{1}{b_f}\left[-\frac{1}{(n+1)(n+2)}\left(1-b_f\right)^{n+2}+\frac{1}{2}b_f^{\,2}+1+\frac{n+1}{n+2}-\frac{2n+1}{n+1}\right] \tag{1}$$

$$\begin{aligned}Var\left[Max\left(T_{te}^{re},\ T_{ge}^{i}\right)\right]=\frac{1}{b_f}&\left[\frac{n+1}{n+3}\left(1-b_f\right)^{n+3}+\frac{\left(b_f+1\right)n^2+\left(b_f-1\right)n-2}{(n+1)(n+2)}\left(1-b_f\right)^{n+2}\right.\\&\left.+\frac{1}{3}b_f^{\,3}+1-\frac{n+1}{n+3}+\frac{3n+2}{n+2}-\frac{3n+1}{n+1}\right]\\&-E^2\left[Max\left(T_{te}^{re},\ T_{ge}^{i}\right)\right]\end{aligned} \tag{2}$$

Here, b_f is the shape factor which is determined by the ratio between the time for the trolley to move from one end of a bay to the other end and the time for the YC to perform the gantry travel from one end of a block to the other end of the block. Detailed derivations are given in Appendix A.

t_{he}^{tc} is the handling time it takes for the YC to lower its empty spreader from its maximum height to a container on a truck chassis, which we consider to be a constant as follows.

$$t_{he}^{tc}=d_{max}^{h}\,\frac{1}{v_h^e} \tag{3}$$

t_{sg} is the time it takes the YC spreader to grasp a target container. We also assume that this time value is a constant.

$$t_{sg} = s_g \tag{4}$$

t_{hl}^{ct} is the time it takes the YC to hoist its loaded spreader from the position of a container on a truck chassis to its maximum height.

$$t_{hl}^{ct} = d_{max}^{h} \frac{1}{v_h^l} \tag{5}$$

T_{tl}^{er} is the time for the YC to move its trolley with a container from the end position of the bay to a random position in the bay (designated for container storage).

$$E\left(T_{tl}^{er}\right) = \left(\frac{b_w}{2} + d_c\right)\frac{1}{v_t^l} \tag{6}$$

$$Var\left(T_{tl}^{er}\right) = \frac{b_w^{\ 2}}{12}\left(\frac{1}{v_t^l}\right)^2 \tag{7}$$

T_{hl}^{tr} is the time it takes for the YC to lower its spreader with a container from the top position to a random tier of a stack designated for container storage.

$$E\left(T_{hl}^{tr}\right) = E\left(D_t^h\right)\frac{1}{v_h^l} \quad where\ E\left(D_t^h\right) = h_{max} - c_h\left(\frac{t+1}{2}\right) \tag{8}$$

$$Var\left(T_{hl}^{tr}\right) = Var\left(D_t^h\right)\left(\frac{1}{v_h^l}\right)^2 \quad where\ Var\left(D_t^h\right) = c_h^{\ 2}\left(\frac{t^2-1}{12}\right) \tag{9}$$

t_{sr} is the time it takes the YC spreader to release a container onto a stack, which is assumed to be constant.

$$t_{sr} = s_r \tag{10}$$

T_{he}^{rt} is the time it takes the YC to hoist its empty spreader from the position where the container was released to its maximum height.

$$E\left(T_{he}^{rt}\right)=E\left(D_t^h\right)\frac{1}{v_h^e} \quad where\ E\left(D_t^h\right)=h_{max}-c_h\left(\frac{t+1}{2}\right) \tag{11}$$

$$Var\left(T_{he}^{rt}\right)=Var\left(D_t^h\right)\left(\frac{1}{v_h^e}\right)^2 \quad where\ Var\left(D_t^h\right)=c_h{}^2\left(\frac{t^2-1}{12}\right) \tag{12}$$

By summing up all the handling elements in the 'Receiving' column in Table 1, the expectation and variance of the cycle time for a receiving operation can be formulated as follows.

$$\begin{aligned} E\left[T_{Uni}\left(R \mid b,t,r\right)\right] &= E\left[Max\left(T_{te}^{re},T_{ge}^{i}\right)\right]+t_{he}^{tc}+t_{sg}+t_{hl}^{ct}+E\left(T_{tl}^{er}\right) \\ &\quad +E\left(T_{hl}^{tr}\right)+t_{sr}+E\left(T_{he}^{rt}\right) \end{aligned} \tag{13}$$

$$Var\left[T_{Uni}\left(R \mid b,t,r\right)\right]=Var\left[Max\left(T_{te}^{re},T_{ge}^{i}\right)\right]+Var\left(T_{tl}^{er}\right)+Var\left(T_{hl}^{tr}\right)+Var\left(T_{he}^{rt}\right) \tag{14}$$

In a similar way, the expectation and the variance of a delivery operation, $E\left[T_{Uni}\left(D \mid b,\ t,\ r\right)\right]$ and $Var\left[T_{Uni}\left(D \mid b,\ t,\ r\right)\right]$, can be obtained by using the handling time elements in the 'Delivery' column in Table 1. A detailed derivation of the formula for the variance of rehandles is given in Lee and Kim [11]. Therefore, formulas for estimating the cycle time for a delivery operation can be represented in the following equations.

$$\begin{aligned} E\left[T_{Uni}\left(D \mid b,\ t,\ r\right)\right] &= E\left[Max\left(T_{te}^{er},\ T_{ge}^{i}\right)\right]+E\left(T^{(re)}\right)+E\left(T_{he}^{tr}\right)+t_{sg}+E\left(T_{hl}^{rt}\right) \\ &\quad +E\left(T_{tl}^{re}\right)+t_{hl}^{tc}+t_{sr}+t_{he}^{ct} \end{aligned} \tag{15}$$

$$\begin{aligned} Var\left[T_{Uni}\left(D \mid b,\ t,\ r\right)\right] &= Var\left[Max\left(T_{te}^{er},\ T_{ge}^{i}\right)\right]+E\left(R_{rt}\right)Var\left(T^{(re)}\right)+E^2\left(T^{(re)}\right)Var\left(R_{rt}\right) \\ &\quad +Var\left(T_{he}^{tr}\right)+Var\left(T_{hl}^{rt}\right)+Var\left(T_{tl}^{re}\right) \end{aligned} \tag{16}$$

• Z pick travel rules

Figure 5 shows a conceptual drawing of the trolley trajectory when applying the Z pick travel rule. The Z pattern length is determined by dividing the length of a block by the number of Z patterns.

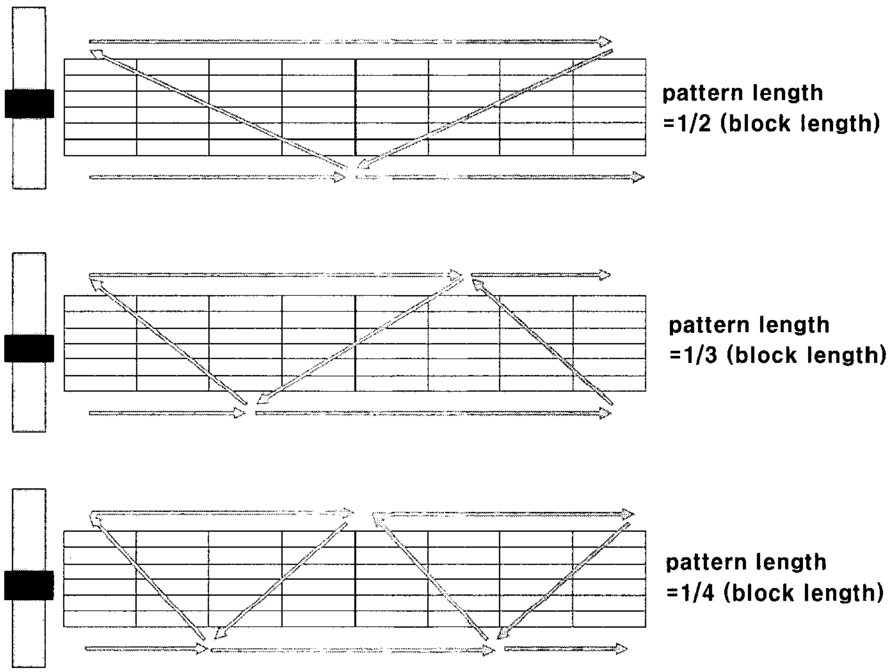

Figure 5. Pattern length for the Z pick travel

The actual travel for this example is depicted in Figure 6. The Z pattern length is set to a quarter of a block length.

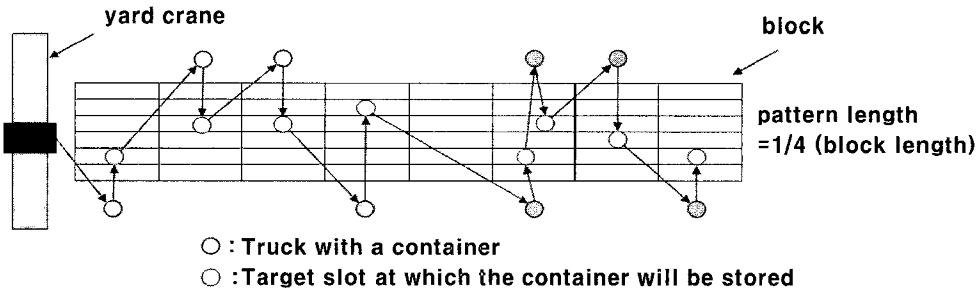

Figure 6. An example of the sequence of receiving operations using the Z pick travel

To estimate the cycle time, the movements of a YC are separated into two parts. One is the movement from one truck to another on the same side within a Z pattern length (movement within a Z pattern length). The other is the movement from one truck on a certain side to another truck on the opposite side of the block (movement between adjacent patterns). Within a Z pattern length, the unidirectional travel rule can be used. However, the movement between adjacent patterns, $Max\left(T_{te}^{re},\ T_{ge}^{p}\right)$, should include movement between a storage position at a bay for the last truck on one side of a pattern and the position of the first truck on the opposite side of the pattern. The derivation of this movement is described in Appendix B. The expected travel distance of the movement between adjacent patterns can be shown as follows.

$$E\left[Max\left(T_{te}^{re},\ T_{ge}^{p}\right)\right]=\frac{n+1}{n+2}\frac{1}{b_f}\left(1-\frac{1}{k}+b_f\right)^{n+2}-\frac{1}{n+1}\left[(2n+1)\frac{1}{b_f}\left(1-\frac{1}{k}\right)+n\right]\left(1-\frac{1}{k}+b_f\right)^{n+1}$$
$$+\left(1+\frac{1}{b_f}\right)\left(1-\frac{1}{k}\right)\left(1-\frac{1}{k}+b_f\right)^{n}-\frac{1}{(n+1)(n+2)}\frac{1}{b_f}\left(1-\frac{1}{k}\right)^{n+2}$$
$$-\left(1-\frac{1}{k}\right)+\frac{n}{n+1}\qquad where\ b_f\leq\frac{1}{k}\tag{17}$$

$$E\left[Max\left(T_{te}^{re},\ T_{ge}^{p}\right)\right]=-\frac{1}{(n+1)(n+2)}\frac{1}{b_f}\left(1-\frac{1}{k}\right)^{n+2}+\frac{1}{b_f}\left(1-\frac{1}{k}\right)^{2}-\frac{2n+1}{n+1}\frac{1}{b_f}\left(1-\frac{1}{k}\right)$$
$$+\frac{n+1}{n+2}\frac{1}{b_f}+\frac{1}{2}b_f-\frac{1}{2}\frac{1}{b_f}\frac{1}{k^2}\qquad where\ b_f>\frac{1}{k}\tag{18}$$

In the above expressions, the block length is assumed to be 1, *bf* is the shape factor, and k is a constant for the Z pattern length. The Z pattern length is the block length divided by k.

2.3 Estimating Waiting Times of Trucks

To represent the expected waiting time of trucks, the well-known Pollaczek-Khintchine (P-K) formula was used for the average waiting time in the queue for the M/G/1 queuing system. The phrase 'truck waiting time' represents the time that elapses from when a truck first parks beside a TP in the block until a YC begins gantry travel towards the TP to serve that truck. Using the expected cycle times derived in this study, we can estimate the expected waiting time of trucks at the block for various truck arrival rates, which is one of the important performance measures of container terminals. The average time a truck spends waiting for the completion of the service for other trucks before being served is given by the well-known Pollaczek-Khintchine (P-K) formula for an M/G/1 queuing system (Gross and Harris [4]).

$$W_q = \frac{\rho E(S)}{2(1-\rho)}\left(1+\frac{Var(S)}{E^2(S)}\right) \tag{19}$$

Here, W_q is the expected number of trucks and ρ is the traffic intensity (the average arrival rate of trucks multiplied by the expected cycle time of a YC). Note that $E(S)$ is the expected time for an operation of a YC and $Var(S)$ is the variance.

3. Numerical Experiments

A numerical experiment was conducted to compare the cycle times among the different sequencing rules. It was assumed that $b = 34$, $t = 6$, $r = 9$, $g_b = g_r = 0.4\ m$, $d_c = 6\ m$, $h_c = 1.5\ m$, $v_g^e = v_g^l$ =180 *m/min*, v_t^e =140 *m/min*, v_t^l =100 *m/min*, v_h^e =120 *m/min*, v_h^l = 80 *m/min*, and $s_g = s_r = 2\ sec$. The size of a 20 ft container is $c_l = 6.058\ m$, $c_w = 2.438\ m$, and $c_h = 2.591\ m$.

The expected cycle times for two operation types under the three sequencing rules were compared with each other. Note that trucks are allowed to be located on both sides of each bay. The results of the numerical experiment are shown in Figure 7. The number of trucks was fixed at 20, and the Z pattern length was set to one-sixth of the block length. From the results, the best performance among the three sequencing rules was obtained when a YC uses the unidirectional travel rules. Kim *et al.* [9] showed the unidirectional travel rule is the best among 6 different rules when the position of a truck arrival follows the uniform distribution in a block and the ratio of the processing time per container to the travel time per bay is significantly large. Thus, the result of Figure 7 is consistent with the results in Kim *et al.* [9].

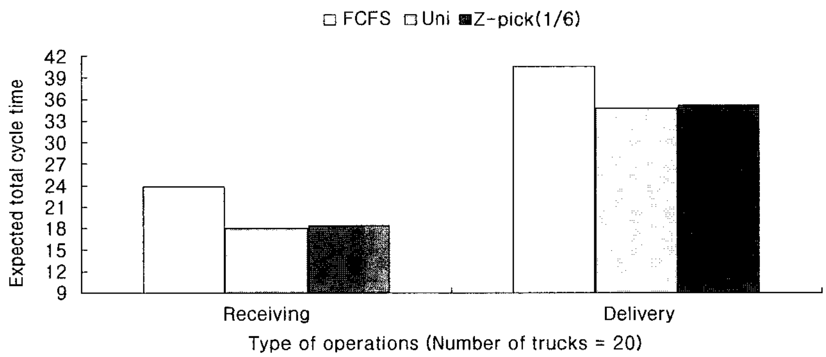

Figure 7. Expected cycle times under different sequencing rules

The performance of the Z pick travel rule highly depends on the pattern length. Thus, another experiment was conducted to observe the expected cycle times for different Z pattern lengths. The number of trucks was also set at 20 and the delivery operation was considered for the comparison. As shown in Figure 8, the minimum value was observed when the Z pattern length is one-sixth of the block length.

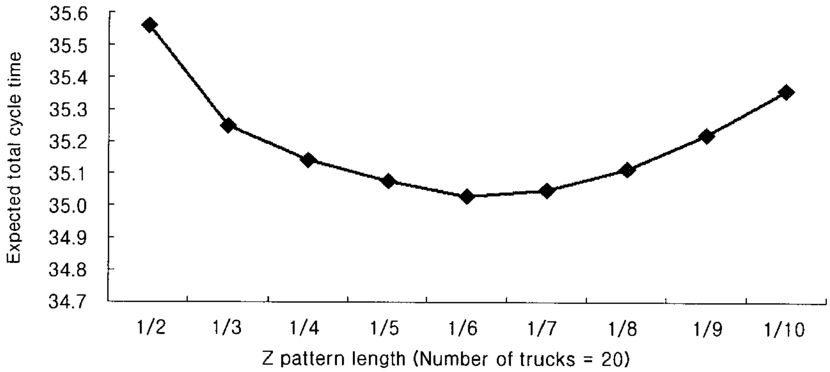

Figure 8. Expected cycle times for a delivery operation under the Z pick travel rule along different pattern lengths

The biggest advantage when the YC follows the unidirectional travel rule is that truck drivers can recognize their service order. Figure 9 shows the expected waiting times of trucks according to their arrival rates. The number of trucks was set at 10 and the delivery operation was considered for the experiment. The expected waiting times of the trucks increased exponentially when the inter-arrival time of trucks decreased.

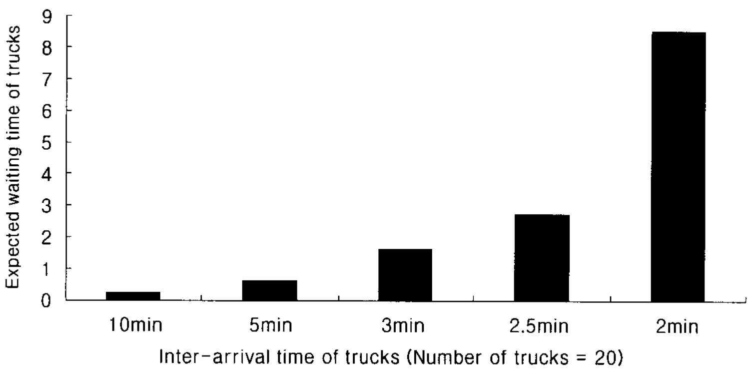

Figure 9. Expected waiting times of trucks for various inter-arrival times for a delivery operation under the unidirectional travel rule

Until now, it has been assumed that a container can be stored at any position in a bay. However, if every truck parks on the side near to the storage location assigned to the container corresponding to the truck, the Z-pick is expected to further reduce the handling time. This can be accomplished by applying the bisectional storage strategy for storage space. Under this storage strategy, a bay is divided into two partitions evenly. Thus, half rows are for outbound containers and other half rows are for inbound containers. The expectation and the variance of cycle times are described in Appendix C.

Figure 10 shows the comparison of the expected total cycle times under different sequencing rules. The pre-determined number of trucks was set at 20 and the Z pattern length was set at one-third of the block length. Unlike the result of Figure 7, the cycle time using the Z pick travel rule is lower than with the unidirectional travel rule. Thus, under the bisectional storage strategy, it is better to use the Z pick travel rule than the other sequencing rules. Goetschalckx and Ratliff [3] showed that the Z pick travel rule is useful in warehouses with low order densities and aisles of narrow widths. The "order density" is the percentage of total number of slots visited for a picking order among all the slots in an aisle. In addition, the unidirectional travel rule performed better in warehouses with a high order density and the larger number of aisles. This seems to be the reason why the Z pick travel rule outperformed the unidirectional travel rule under the bisectional storage strategy.

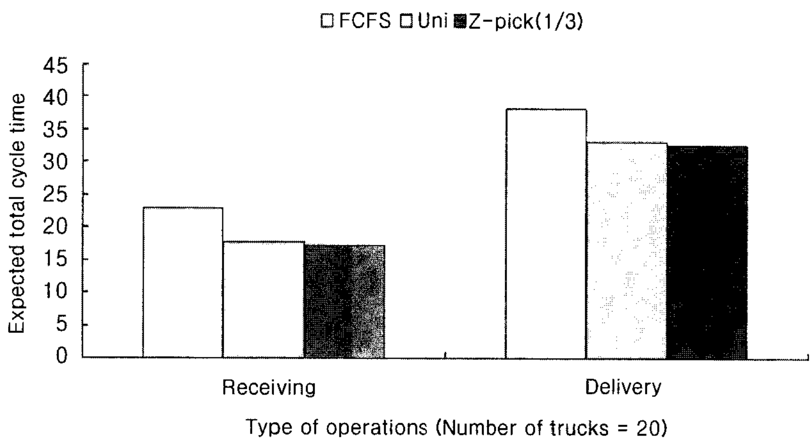

Figure 10. Expected cycle times under different sequencing rules using the bisectional strategy for storage space

The result in Figure 11 shows the expected cycle time for different pattern lengths under the bisectional storage strategy. Note that the best pattern length is longer than under the random storage strategy.

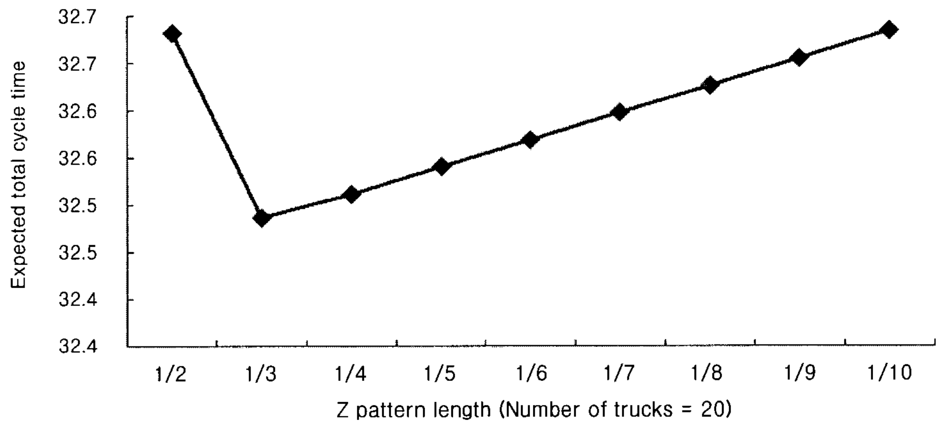

Figure 11. Expected cycle times for a delivery operation under the Z pick travel rule along different pattern lengths using the bisectional strategy for storage space

From the results of Figure 7 and Figure 10, it is seen that the unidirectional travel rule outperforms the Z pick travel rule under the mixed storage strategy, while the Z pick travel rule outperforms the unidirectional travel rule under the bisectional storage strategy. The best combination of sequencing rules and storage strategies is depicted in Figure 12.

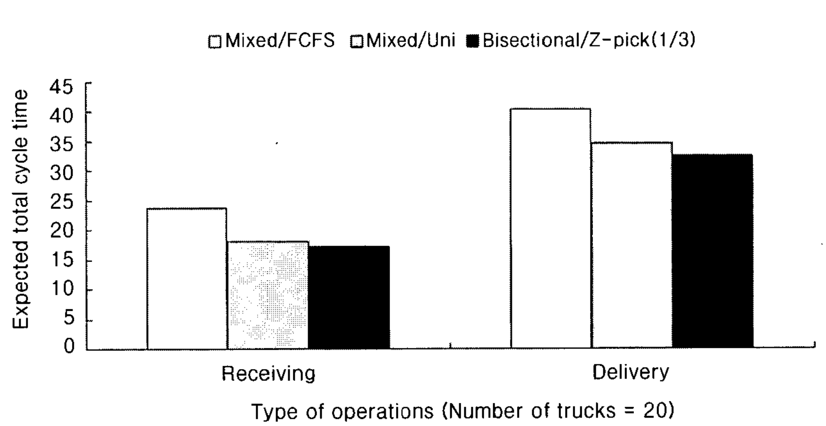

Figure 12. Expected cycle times using the best combination of sequencing rules and storage strategies

4. Conclusions

This paper compared the performance of a YC for the different service sequencing rules for waiting trucks, such as *first-come-first-served, unidirectional travel*, and Z *pick travel rules*. The trucks can be located on both sides of a block and were assumed to be located randomly. Detailed formulas for estimating the expectation and variance of cycle times of a YC were derived under different sequencing rules.

It was found that the unidirectional travel rule outperforms the others in terms of expected cycle times. However, when the bisectional storage strategy was used, the Z pick travel rule resulted in the best performance. The expected waiting times of trucks increased exponentially as the inter-arrival time decreased.

This study addressed only receiving and delivery operations and assumed that all the trucks waiting at a block have the same tasks of receiving or delivery. It is necessary to find good sequencing rules for trucks loading and unloading containers, and to address the cases when different types of operations are mixed in the same block.

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Appendices

Appendix A: Handling Time Models of a YC with the Tchebychev Metric Under the Unidirectional Travel Rule

Bozer and White [1] derived mathematical models to estimate the cycle time of a S/R machine in the AS/RS system using the Tchebychev metric. Based on their study, we estimate the expected cycle time and the variance of the cycle time. The notations are as follows.

t_h = The time required for the trolley to move from one end to the other end of a bay (along the x-axis).

t_v = The time required for the YC to travel from one end to the other end (along the y-axis).

T = $Max\ (t_h, t_v)$.

b_f = Shape factor ($0 \le b_f \le 1$). (this inequality holds because we assume that $t_h < t_v$, $b_f = t_h / t_v$).

t_{xy} = $Max(t_x, t_y)$, where t_x and t_y are the travel times in the x- and y-directions, respectively.

$H(\cdot)$ = Cumulative distribution function for the random variable t_{xy}, $P(t_{xy} \le z)$.

$h(\cdot)$ = Probability density function of the random variable t_{xy}.

Assuming t_x and t_y are mutually independent, $H(z)$ can be represented as follows.

$$H(z) = P(t_{xy} \le z) = P(t_x \le z) P(t_y \le z) \tag{A-1}$$

The trolley movement time, t_x, follows a uniform distribution $U(0, b_f)$.

$$P(t_x \le z) = \begin{cases} \dfrac{z}{b_f} & ,\ 0 \le z \le b_f \\ 1 & ,\ b_f < z \le 1 \end{cases} \tag{A-2}$$

It is already assumed that the number of trucks in a block is given and those trucks are waiting to be served. Thus, order statistics can be used to represent the distance distribution between adjacent trucks (Kendall and Moran [6], David and Nagaraja [2]). Suppose that trucks are parked in an interval $(0, b_l)$ and that trucks are positioned at n random points, $X_1, X_2, \cdots, X_n$. Let $X_{(1)}, X_{(2)}, \cdots, X_{(n)}$ be the positions of trucks which are rearranged in an increasing order.

$$0 \le X_{(1)} \le X_{(2)} \le \cdots \le X_{(n)} \le b_l \tag{A-3}$$

Figure A-1 shows the layout of trucks in increasing order on a block. The bays in the block are numbered in increasing order from left to right, and the position of the YC is represented by Y.

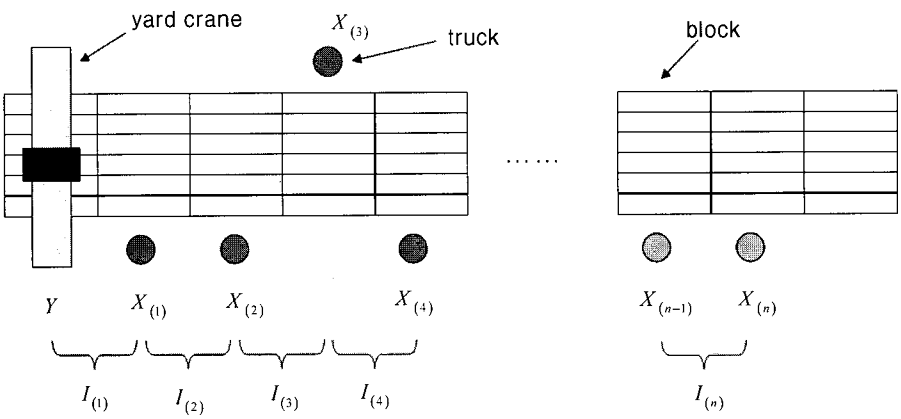

Figure A-1. Order statistics for the unidirectional sequencing rule

Let $I = I_{(k+1)} = X_{(k+1)} - X_{(k)}$. The probability density distribution, $f(i)$, and the cumulative distribution function, $F(i)$, of a single interval I can be derived as follows (Kendall and Moran [6]). The detailed expressions for the probability density distribution and the cumulative distribution function are described in (A-4) and (A-5), respectively.

$$f(i) = nb_l(1-i)^{n-1} \tag{A-4}$$

$$F(i)=b_l\left[1-(1-i)^n\right] \tag{A-5}$$

Thus, the travel time of a YC in the gantry direction, t_y, can be normalized, and its cumulative distribution probability is expressed as follows.

$$P(t_y \le z)=1-(1-z)^n \tag{A-6}$$

Hence, $H(z)$ and $h(z)$ can be derived as follows.

$$H(z)=\begin{cases}\dfrac{z}{b_f}\left[1-(1-z)^n\right] & , 0\le z\le b_f \\ 1-(1-z)^n & , b_f<z\le 1\end{cases} \tag{A-7}$$

$$h(z)=\begin{cases}\dfrac{1}{b_f}\left[1-(1-z)^n+nz(1-z)^{n-1}\right] & , 0\le z\le b_f \\ n(1-z)^{n-1} & , b_f<z\le 1\end{cases} \tag{A-8}$$

Thus, the expected handling time and the variance of the cycle time are obtained as follows.

$$\begin{aligned}E(z)&=\int_0^1 zh(z)\,dz \\ &=\frac{1}{b_f}\left[-\frac{1}{(n+1)(n+2)}(1-b_f)^{n+2}+\frac{1}{2}b_f^{\,2}+1+\frac{n+1}{n+2}-\frac{2n+1}{n+1}\right]\end{aligned} \tag{A-9}$$

$$\begin{aligned}Var(z)&=\int_0^1 z^2h(z)\,dz-E^2(z) \\ &=\frac{1}{b_f}\left[\begin{array}{l}\dfrac{n+1}{n+3}(1-b_f)^{n+3}+\dfrac{(b_f+1)n^2+(b_f-1)n-2}{(n+1)(n+2)}(1-b_f)^{n+2}+\dfrac{1}{3}b_f^{\,3} \\ +1-\dfrac{n+1}{n+3}+\dfrac{3n+2}{n+2}-\dfrac{3n+1}{n+1}\end{array}\right] \\ &\quad -E^2(z)\end{aligned} \tag{A-10}$$

Appendix B: Handling Time Models of a YC with the Tchebychev Metric Under the Z Pick Travel Rule

The trolley movement can be treated as the uniform distribution, $U(0, b_f)$ or $U(0, 1)$, depending on the Z pattern length. If the Z pattern length is reduced, the time for trolley movement becomes longer than the time it takes for the YC traveling. This means that both cases of $t_h < t_v$ and $t_h \geq t_v$ should be considered. However, only the case of $t_h < t_v$ is described in this section.

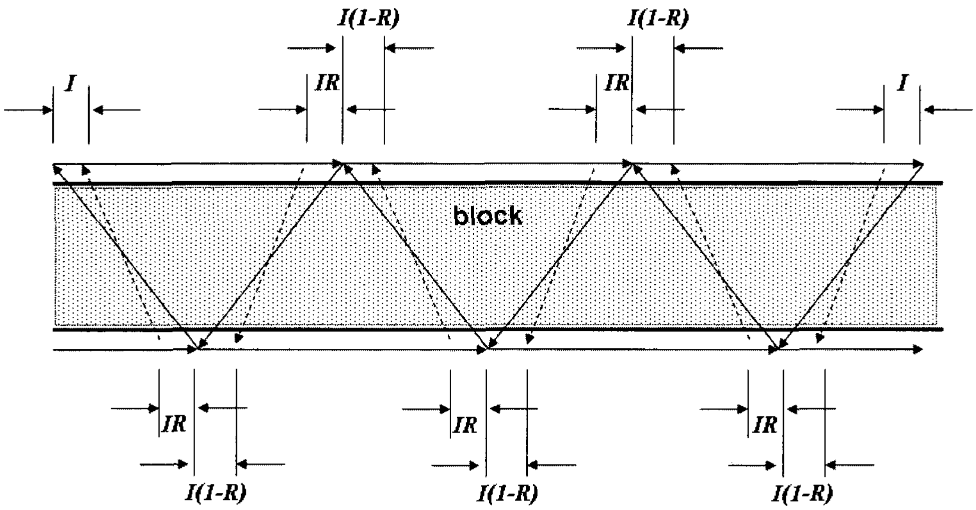

Figure B−1. Extra distances between two Z patterns

In Figure B-1, I is a random variable representing a single interval of the distance between two consecutive positions of trucks. R is also a random variable following $U(0, 1)$. We can call the distance indicated IR or $I(1-R)$ as extra distance, because it should be excluded when the YC travels in the gantry direction. Bold arrows represent the conceptual Z pick travels of the trolley while dotted arrows are the actual Z pick travel considering the extra distances. When the YC moves to the other side of the block within a pattern, this extra distance should be considered.

Here, for convenience and simplification, we assume that R is constant and equal to 0.5, and the extra distances at the first and last crossing travels are the same as the others. Thus, the cumulative distribution function of the distance for gantry travels considering the extra distance can be expressed as follows.

$$P\left(\frac{b_l}{k}-i\le z\right)=P\left(i\ge\frac{b_l}{k}-z\right)$$
$$=1-P\left(i\le\frac{b_l}{k}-z\right)$$
$$=1-b_l\left[1-\left(1-\frac{b_l}{k}+z\right)^n\right] \tag{B-1}$$

Here, k is a constant for the Z pattern length and n is the pre-determined number of trucks in a block. The shape of the formula follows the distribution of an interval between two consecutive positions of trucks. It is assumed that there is more than one truck within a side of a pattern. Thus, the formula considers that the YC does not move forward when it moves from one side to the other side within a pattern.

The normalized time for the trolley movement, t_x, follows a uniform distribution $U(0, b_f)$.

$$P(t_x\le z)=\begin{cases}\frac{z}{b_f} & 0\le z\le b_f\\ 1 & b_f<z\le 1\end{cases} \tag{B-2}$$

The normalized time for a YC traveling in the gantry direction, ty, can be represented as follows.

$$P(t_y\le z)=1-P\left(i\le\frac{1}{k}-z\right)$$
$$=1-\left[1-\left(1-\left(\frac{1}{k}-z\right)\right)^n\right]$$
$$=\left(1-\frac{1}{k}+z\right)^n \tag{B-3}$$

The variable z ranges from $1/k - 1$ to $1/k$. However, because it is assumed that there is at least one truck on one side of a pattern, the range is from 0 to $1/k$ and $P(t_y \le z)$ can be defined again as follows.

$$P(t_y \le z) = \begin{cases} \left(1 - \frac{1}{k} + z\right)^n & 0 \le z \le \frac{1}{k} \\ 1 & \frac{1}{k} < z \le 1 \end{cases} \tag{B-4}$$

Hence, $H(z)$ and $h(z)$ can be divided by the relation between b_f and $1/k$, and expressed as follows.

$$H(z) = \begin{cases} \frac{z}{b_f}\left(1 - \frac{1}{k} + z\right)^n & 0 \le z \le b_f \\ \left(1 - \frac{1}{k} + z\right)^n & b_f < z \le \frac{1}{k} \\ 1 & \frac{1}{k} < z \le 1 \end{cases} \quad \text{, where } b_f \le \frac{1}{k} \tag{B-5}$$

$$h(z) = \begin{cases} \frac{1}{b_f}\left(1 - \frac{1}{k} + z\right)^n + \frac{nz}{b_f}\left(1 - \frac{1}{k} + z\right)^{n-1} & 0 \le z \le b_f \\ n\left(1 - \frac{1}{k} + z\right)^{n-1} & b_f < z \le \frac{1}{k} \\ 0 & \frac{1}{k} < z \le 1 \end{cases} \quad \text{, where } b_f \le \frac{1}{k} \tag{B-6}$$

$$H(z) = \begin{cases} \frac{z}{b_f}\left(1 - \frac{1}{k} + z\right)^n & 0 \le z \le \frac{1}{k} \\ \frac{z}{b_f} & \frac{1}{k} < z \le b_f \\ 1 & b_f < z \le 1 \end{cases} \quad \text{, where } b_f > \frac{1}{k} \tag{B-7}$$

$$h(z) = \begin{cases} \frac{1}{b_f}\left(1 - \frac{1}{k} + z\right)^n + \frac{nz}{b_f}\left(1 - \frac{1}{k} + z\right)^{n-1} & 0 \le z \le \frac{1}{k} \\ \frac{1}{b_f} & \frac{1}{k} < z \le b_f \\ 0 & b_f < z \le 1 \end{cases} \quad \text{, where } b_f > \frac{1}{k} \tag{B-8}$$

Thus, the expected handling times are obtained as follows.

$$
\begin{aligned}
E(z) &= \int_0^1 z h(z)\, dz \\
&= \frac{n+1}{n+2}\frac{1}{b_f}\left(1-\frac{1}{k}+b_f\right)^{n+2} - \frac{1}{n+1}\left[(2n+1)\frac{1}{b_f}\left(1-\frac{1}{k}\right)+n\right]\left(1-\frac{1}{k}+b_f\right)^{n+1} \\
&\quad +\left(1+\frac{1}{b_f}\right)\left(1-\frac{1}{k}\right)\left(1-\frac{1}{k}+b_f\right)^{n} - \frac{1}{(n+1)(n+2)}\frac{1}{b_f}\left(1-\frac{1}{k}\right)^{n+2} \\
&\quad -\left(1-\frac{1}{k}\right)+\frac{n}{n+1} \qquad \text{, where } b_f \le \frac{1}{k} \qquad \text{(B-9)}
\end{aligned}
$$

$$
\begin{aligned}
E(z) &= -\frac{1}{(n+1)(n+2)}\frac{1}{b_f}\left(1-\frac{1}{k}\right)^{n+2} + \frac{1}{b_f}\left(1-\frac{1}{k}\right)^{2} - \frac{2n+1}{n+1}\frac{1}{b_f}\left(1-\frac{1}{k}\right) \\
&\quad + \frac{n+1}{n+2}\frac{1}{b_f} + \frac{1}{2}b_f - \frac{1}{2}\frac{1}{b_f}\frac{1}{k^2} \qquad \text{, where } b_f > \frac{1}{k} \qquad \text{(B-10)}
\end{aligned}
$$

Appendix C: Handling Time Models of a YC with the Tchebychev Metric Under the Bisectional Storage Strategy

The bisectional storage strategy is a storage space strategy which divides each bay into two partitions. Under this strategy, trucks are located at the near side of a bay for storing its container. Figure C-1 shows an example of the unidirectional travel rule under the bisectional storage strategy. The bisectional storage strategy reduces the time for the trolley movement so that it will reduce the expected cycle time.

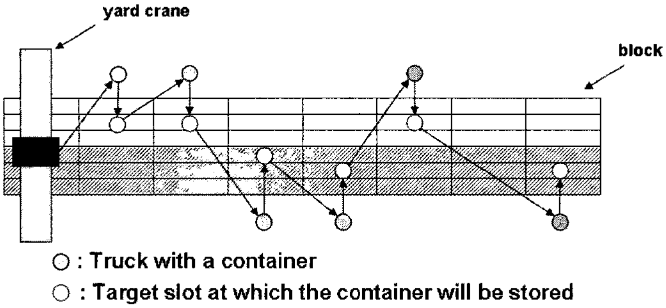

Figure C-1. An example of the sequence of receiving operations using the unidirectional travel using bisectional storage strategy

• Unidirectional travel rules

The distribution of the time for the trolley movement and the distribution of the time for YC travel in the gantry direction with a handling time of $Max\left(T_{te}^{re}, T_{ge}^{i}\right)$ can be estimated as follows.

$$P\left(t_x \le z\right) = \begin{cases} 0 & 0 \le z \le \dfrac{b_f}{2} \\ \dfrac{2}{b_f}\left(z - \dfrac{b_f}{2}\right) & \dfrac{b_f}{2} < z \le b_f \\ 1 & b_f < z \le 1 \end{cases} \qquad \text{(C-1)}$$

$$P\left(t_y \le z\right) = 1 - \left(1 - z\right)^n \qquad \text{(C-2)}$$

Hence, the expectation and the variance are formulated as follows.

$$\begin{aligned} E(z) = & -\frac{2n+2}{n+2}\frac{1}{b_f}\left(1-b\right)^{n+2} + \frac{2}{n+1}\left[\left(2n+1\right)\frac{1}{b_f} - n\right]\left(1-b_f\right)^{n+1} \\ & +2\left(1-\frac{1}{b_f}\right)\left(1-b_f\right)^n + \frac{2n+2}{n+2}\frac{1}{b_f}\left(1-\frac{b_f}{2}\right)^{n+2} - \frac{1}{n+1}\left[\left(4n+2\right)\frac{1}{b_f} - n\right]\left(1-\frac{b_f}{2}\right)^{n+1} \\ & -\left(1-\frac{2}{b_f}\right)\left(1-\frac{b_f}{2}\right)^n + \frac{3}{4}b_f \end{aligned} \qquad \text{(C-3)}$$

$$\begin{aligned} Var(z) = & \frac{2n+2}{n+3}\frac{1}{b_f}\left(1-b_f\right)^{n+3} - \frac{2}{n+2}\left[\left(3n+2\right)\frac{1}{b_f} - n\right]\left(1-b_f\right)^{n+2} \\ & + \frac{2}{n+1}\left[\left(3n+1\right)\frac{1}{b_f} - 2n\right]\left(1-b_f\right)^{n+1} - 2\left(\frac{1}{b_f} - 1\right)\left(1-b_f\right)^n \\ & - \frac{2n+2}{n+3}\frac{1}{b_f}\left(1-\frac{b_f}{2}\right)^{n+3} + \frac{1}{n+2}\left[\left(6n+4\right)\frac{1}{b_f} - n\right]\left(1-\frac{b_f}{2}\right)^{n+2} \\ & - \frac{2}{n+1}\left[\left(3n+1\right)\frac{1}{b_f} - n\right]\left(1-\frac{b_f}{2}\right)^{n+1} + \left(\frac{2}{b_f} - 1\right)\left(1-\frac{b_f}{2}\right)^n + \frac{7}{12}b_f^2 - E^2(z) \end{aligned} \qquad \text{(C-4)}$$

• **Z pick travel rules**

The distribution of the time for the trolley movement and the distribution of the time for YC travel in the gantry direction with a handling time of $Max\left(T_{te}^{re},\ T_{ge}^{p}\right)$ can be estimated as follows. In this section, only the case of $t_h < t_v$ is considered.

$$P\left(t_x \le z\right)=\begin{cases}0 & 0\le z\le \frac{b_f}{2}\\ \frac{2}{b_f}\left(z-\frac{b_f}{2}\right) & \frac{b_f}{2}<z\le b_f\\ 1 & b_f<z\le 1\end{cases} \tag{C-5}$$

$$P\left(t_y \le z\right)=\begin{cases}\left(1-\frac{1}{k}+z\right)^n & 0\le z\le \frac{1}{k}\\ 1 & \frac{1}{k}<z\le 1\end{cases} \tag{C-6}$$

Hence, the expectations can be expressed as follows.

$$E\left(z\right)=\frac{3}{4}b_f \qquad where\ \frac{1}{k}\le\frac{b_f}{2}<b_f \tag{C-7}$$

$$\begin{aligned}E\left(z\right)=&-\frac{2n+2}{n+2}\frac{1}{b_f}\left(1-\frac{1}{k}+\frac{b_f}{2}\right)^{n+2}+\frac{1}{n+1}\left[\left(4n+2\right)\frac{1}{b_f}\left(1-\frac{1}{k}\right)+n\right]\left(1-\frac{1}{k}+\frac{b_f}{2}\right)^{n+1}\\&-\left(1-\frac{1}{k}\right)\left[\frac{2}{b_f}\left(1-\frac{1}{k}\right)+1\right]\left(1-\frac{1}{k}+\frac{b_f}{2}\right)^{n}+\frac{2}{b_f}\left(1-\frac{1}{k}\right)^2-\frac{4n+2}{n+1}\frac{1}{b_f}\left(1-\frac{1}{k}\right)\\&+\frac{2n+2}{n+2}\frac{1}{b_f}-\frac{n}{n+1}+\left(1-\frac{1}{k}\right)+b_f-\frac{1}{b_f}\frac{1}{k^2} \qquad where\ \frac{b_f}{2}<\frac{1}{k}\le b_f\end{aligned} \tag{C-8}$$

$$\begin{aligned}E\left(z\right)=&\frac{2n+2}{n+2}\frac{1}{b_f}\left(1-\frac{1}{k}+b_f\right)^{n+2}-\frac{2}{n+1}\left[\left(2n+1\right)\frac{1}{b_f}\left(1-\frac{1}{k}\right)+n\right]\left(1-\frac{1}{k}+b_f\right)^{n+1}\\&+2\left(1-\frac{1}{k}\right)\left[\frac{1}{b_f}\left(1-\frac{1}{k}\right)+1\right]\left(1-\frac{1}{k}+b_f\right)^{n}-\frac{2n+2}{n+2}\frac{1}{b_f}\left(1-\frac{1}{k}+\frac{b_f}{2}\right)^{n+2}\end{aligned}$$

$$+\frac{1}{n+1}\left[(4n+2)\frac{1}{b_f}\left(1-\frac{1}{k}\right)+n\right]\left(1-\frac{1}{k}+\frac{b_f}{2}\right)^{n+1}$$

$$-\left(1-\frac{1}{k}\right)\left[\frac{2}{b_f}\left(1-\frac{1}{k}\right)+1\right]\left(1-\frac{1}{k}+\frac{b_f}{2}\right)^{n}+\frac{n}{n+1}-\left(1-\frac{1}{k}\right)$$

$$\text{where } \frac{b_f}{2}<b_f<\frac{1}{k} \qquad \text{(C-9)}$$