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Analysis of the Propagation Characteristics of Ultrasonic Guided Waves Excited by Single Frequency and Broadband Sources

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Abstract Excitation and propagation of guided waves are very complex problems in pipes due to their dispersive nature. Pipes are commonly used in the oil, chemical or nuclear industry and hence must be inspected regularly to ensure continued safe operation. The normal mode expansion(NME) method is given for the amplitude with which any propagating waveguide mode is generated in the pipes by applied surface tractions. Numerical results are calculated based on the NME method using different sources, i.e., non-axisymmetric partial loading and quasi-axisymmetric loading sources. The sum of amplitude coefficients for 0~nineth order of the harmonic modes are calculated based on the NME method and the dispersion curves in pipes. The superimposed total field which is namely the angular profile, varies with propagating distance and circumferential angle. This angular profile of guided waves provides information for setting the transducer position to find defects in pipes.

Keywords: Ultrasonic Guided Wave, Nomal Mode Expansion, Propagation Characteristics, Broadband Source, Pipe

1. Introduction

Pipes are commonly used in the oil, chemical or nuclear industry and hence must be inspected for defects regularly to ensure continued safe operation. The use of guided waves to nondestructively inspect such pipes, although not new, is increasingly adapted as a possible screening tool in the industry fields (Alleyen and Cawley, 1997; Rose et al., 1994; Shivaraj et al., 2008; Kang et al., 2009; Shin et al., 2000; Kwun et al., 2001; Alleyne et al., 2002; Song et al., 2003). Excitation and propagation of guided waves are very complex problems in pipes due to their dispersive nature. Therefore, many researchers have investigated guided wave excitation and propagation theoretically. The first exact theoretical solution of guided wave propagation in pipes was studied by Gazis(1959). He obtained the general solution of harmonic waves propagating in an infinitely long hollow cylinder. Although exact solution given above the propagation of guided wave modes in cylinders is now completely understood, there has been no extended calculations of amplitudes of each modes generated in the cylinder. The problem of the excitation of guided wave modes in infinite

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hollow cylinders by loading applied to the boundaries of the cylinder was studied by Ditri and Rose(1992). Solving the problem of generation of guided wave modes in infinite hollow cylinders by applied surface loading, has important information on the field distributions caused by certain modes and how they influence the sensitivity of that particular mode to a particular defect. Previous works done by Ditri and Rose(1992) so far had dealt with pure modes, i.e., modes with a single frequency, in other words, they did not discuss broadband sources.

An exact solution is given for the amplitude based on the NME method with which any waveguide mode is generated due to the application of surface tractions(Ditri and Rose, 1992). Also, angular profiles that provide angular distribution of particle velocity with various circumferential angles were calculated based on the NME method at a certain distance. Angular profiles excited by single frequency sources were already discussed by Li and Rose(2001, 2002). However, angular profile excited by broadband source is not discussed. They used single frequency source rather than broadband sources, which is not realistic in many practical excitation conditions. So, we calculate angular profiles excited by broadband sources to address more realistic situation.

Our goal of this paper is to analyze the field distributions caused by certain modes at a certain distance with broadband sources. Thus, in this study, we calculate angular profiles with various sources and loading conditions as follows: 1) single frequency source excitation by a single element, 2) single frequency source excitation by multi-elements, 3) broadband source excitation by a single element, and 4) broadband sources excitation by multi-elements. These four results will be compared and discussed, so that one can predict influences of source condition which is important for inspection condition optimization.

2. Normal Mode Expansion Method

In the Gazis solution, there are a doubly infinite number of propagating modes for the hollow cylinder. We denote the fields of the normal modes by two indices, n and M, so that velocity field $\vec{\nu}_n^M$ and stress field $\vec{T}_n^M$ due to the "nth" mode of the "Mth" family can be written as follows(Gazis, 1959)

$$\vec{\nu}_n^M(r,\theta)e^{i(\omega t - k_n^M z)} = \sum_{\alpha=\gamma,\theta,z} R_{n\alpha}^M(r)\Theta_\alpha^M(M\theta)e^{i(\omega t - k_n^M z)}\vec{e}_\alpha \tag{1}$$

$$\vec{T}_n^M(r,\theta)e^{i(\omega t - k_n^M z)} = \sum_\alpha\sum_\beta R_{n\alpha\beta}^M(r)\Theta_{\alpha\beta}^M(M\theta)e^{i(\omega t - k_n^M z)}\vec{e}_\alpha\otimes\vec{e}_\beta, \alpha,\beta\in\{r,\theta,z\} \tag{2}$$

where ω and k are the angular frequency and wave number, respectively. $\vec{\nu}_n^M$ and $\vec{T}_n^M$ are independent of the cylinder axial direction, z, and the time, t. The particle displacement can be decomposed in to three directions: $\vec{e}_r$, $\vec{e}_\theta$, $\vec{e}_z$, which are the unit vectors in a cylindrical coordinate system as shown in Fig. 1.

Functions $R(r)$ and $\Theta(M\theta)$ denote the radial and angular distributions of the component of the stress produced by the "nth" mode of the "Mth" family respectively. Θ_r^M, Θ_θ^M and Θ_z^M are sinusoidal functions, $\cos(M\theta)$ and/or $\sin(M\theta)$. When $M = 0$, the modes are axisymmetric and for $M > 0$, we have a doubly infinite number of

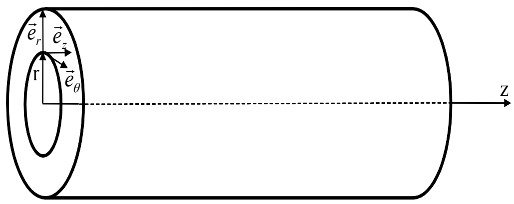

Fig. 1 Cylindrical coordinate system for the pipe

flexural modes(Ditri and Rose, 1992). The dispersion curves for the axisymmetric longitudinal and non-axisymmetric flexural modes are shown in Fig. 2, which, as an example, is for a ASTM A106 carbon steel pipe with 165.2 mm and 7.1 mm wall thickness.

2.1 Orthogonality of Normal Modes

In order to use the NME method, the orthogonality relation between the waveguide modes should be established. To do this, Ditri and Rose derived with the differential form of the complex reciprocity relation as follows(Ditri and Rose, 1992):

$$\nabla \cdot \left(V_2^* \cdot T_1 + V_1 \cdot T_2^* \right) = 0 \tag{3}$$

where V_1, T_1 and V_2, T_2 represents the particle velocity and stress fields of two different solutions to the linear elastic propagation equation. let the solutions V_1, T_1 and V_2, T_2 be different normal modes of the cylinder,

$$\begin{aligned} V_1 &= V_m^N e^{-\beta_m^N z}, T_1 = T_m^N e^{-\beta_m^N z} \\ V_2 &= V_n^M e^{-\beta_n^M z}, T_2 = T_n^M e^{-\beta_n^M z} \end{aligned} \tag{4}$$

then

$$\begin{aligned} & V_2^* \cdot T_1 + V_1 \cdot T_2^* = \\ & e^{-\left(\beta_m^N - \beta_n^{M*}\right)z} \left(V_n^{M*} \cdot T_m^N + V_m^N \cdot T_n^{M*} \right) \end{aligned} \tag{5}$$

taking divergence operator of eqn. (5) gives

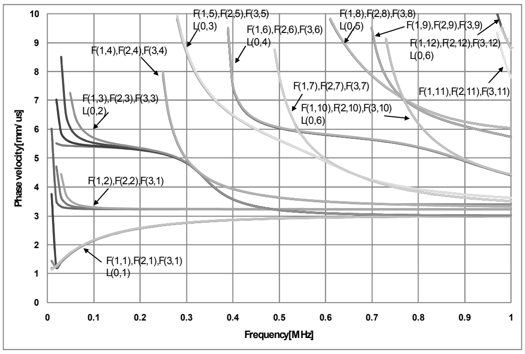

Fig. 2 Dispersion curve for ASTM A106 carbon steel pipe with thickness of 7.1 mm, outer diameter of 165.2 mm and inner diameter of 151 mm

$$\begin{aligned} & \nabla \cdot \{ \ \} = \\ & e^{-i\left(\beta_m^N - \beta_n^{M*}\right)z} \left[\nabla_{r\theta} \cdot \{ \ \} - i\left(\beta_m^N - \beta_n^{M*} \right) \{ \ \} \cdot \vec{e}_z \right] = 0 \end{aligned} \tag{6}$$

where

$$\{ \ \} = \left(V_n^{M*} \cdot T_m^N + V_m^N \cdot T_n^{M*} \right) \tag{7}$$

integrating the result, eqn. (7) over the cross section of the cylinder D gives

$$\begin{aligned} & -4i\left(\beta_m^N - \beta_n^{M*} \right) P_{nm}^{MN} = \\ & \qquad \iint_D \nabla_{r\theta} \cdot \left(V_n^{M*} \cdot T_m^N + V_m^N \cdot T_n^{M*} \right) d\sigma \end{aligned} \tag{8}$$

Based on this relation, the orthogonal relation between wave modes can be derived in terms of an area integral, P_{nm}^{MN}(Shin, 1997),

$$P_{nm}^{MN} = -\frac{1}{4} \int\int_D \left(V_n^{M*} \cdot T_m^N + V_m^N \cdot T_n^{M*} \right) \cdot \vec{e}_z d\sigma \tag{9}$$

finally, we the orthogonality relation,

$$P_{nm}^{MN} = 0 \text{ , unless } M = N \text{ and } \beta_m^N = \beta_n^{N*} \tag{10}$$

2.2 Normal Mode Expansion Amplitude

The NME method can be achieved with the orthogonality relationship expressed by eqn. (9). The goal is to find the amplitudes of each of the modes generated in the cylinder due to the application of specified surface loadings on the boundaries of the cylinder. The general particle velocity can be expressed as follows:

$$\vec{v} e^{i\omega t} = \sum_{M,n} A_n^M \vec{v}_n^M e^{i\omega t} \tag{11}$$

where A_n^M is the normal mode expansion amplitude of the nth mode of Mth circumferential order, contains $e^{i(-k_n^M z)}$, assuming waves are propagating in the $+z$ direction. Using total field expressed as eqn. (11) and the field of mode n of circumferential order M as expressed in eqns. (1) and (2). The basis form for solving various source loading problem can be expressed as follows:

$$A_{+n}^{M}\left(z\right)= -\frac{R_{nr}^{M*}\left(b\right)e^{-ik_n^M z}}{4P_{nn}^{MM}}\left\langle \Theta_r^M , p_1\left(\theta\right)\right\rangle \cdot \left\langle p_2\left(\theta\right), e^{ik_n^M z}\right\rangle \quad (12)$$

where $R_{nr}^{M^*}(b)$ is the r velocity component of given mode at outer surface, $4P_{nn}^{MM}$ is the power carried by the mode, $\langle \Theta_r^M, p(\theta)\rangle$ is angular variation of applied traction loading and $\langle p_2(\theta), e^{ik_n^M z}\rangle$ is axial variation of applied traction loading. In following section, numerical calculations of non-axisymmetric partial loading and quasi-axisymmetric loading are discussed with eqn. (12).

3. Boundary Conditions of Guided Wave Propagation

Let us consider the cylindrical pipe which has the outer diameter of 165.2 mm and wall thickness of 7.1 mm, and is made of ASTM A106 carbon steel pipe. We provided numerical calculations of angular profiles excited by four different sources, i.e., 1) single frequency source excitation by the single element, 2) single frequency source excitation by the multi-elements, 3) broadband sources excitation by the single element, and 4) broadband sources excitation by the multi-elements.

3.1 Single Frequency Source Excitation by the Single Element

A Single frequency source excitation by single element is non-axisymmetric and contacts only the pipe surface over some circumferential angle. Angular and axial variations can be reformulated as:

$$\begin{aligned}\left\langle \Theta_r^M , p_1(\theta)\right\rangle &= bP_{10}\ for\ M=0\\ &=\left(bP_{10}\right)\frac{\sin\left[\left(\alpha/2\right)M\right]}{M},\ for\ M\geq 1\end{aligned} \quad (13)$$

where P_{10} and P_{20} is uniform pressure loading conditions, α is circumferential contact angle,

$$\left\langle p_2(z), e^{ik_n^M z}\right\rangle = \frac{2P_{20}}{\beta_n^M}\sin\left(k_n^M L\right) for\ z\geq L \quad (14)$$

and L is a half of axial loading length. α is 17.6° and L is a half inch. Fig. 3 shows schematic view of single elements source conditions.

3.2 Single Frequency Source Excitation by the Multi-Elements

A single frequency source excitation by multi-elements is almost axisymmetric and contacts most circumferential parts of the pipe surface over circumferential angle. For an quasi-axisymmetric loading, axial variation is same as eqn. (14) and angular variation can be reformulated as:

$$\begin{aligned}\left\langle \Theta_r^M , p_1\left(\theta\right)\right\rangle &= bP_{10}\quad for\quad M=0\\ &=\frac{\left(-1\right)^n 2P_{10}a\sin\left(4n\alpha\right)}{n},\\ &\quad for\quad M\geq 8n, n\in\left\{0,1,2,\cdots\right\}\end{aligned} \quad (15)$$

Fig. 4 shows schematic view of multi-elements (8-elements) source condition. α and L are same as single element condition.

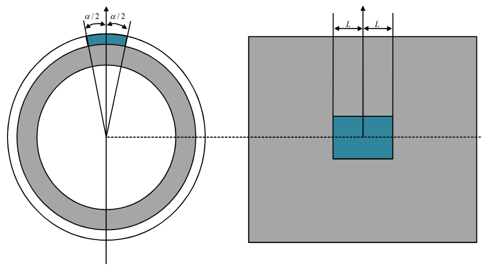

Fig. 3 Schematic view of single elements loading condition

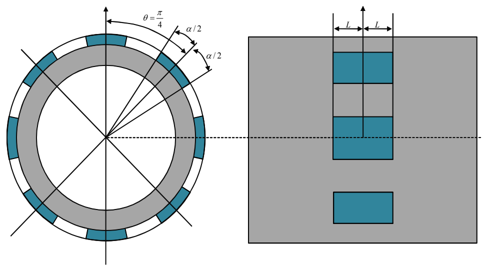

Fig. 4 Schematic view of multi-elements loading condition

3.3 Broadband Source Excitation by Single Element

For describing more realistic excitation conditions, broadband sources are introduced in this paper. Hanning window $w^M(\omega)$ as shown in Fig. 5 is adapted in order to produce wide band source.

A_{+n} is the normal mode expansion amplitude of the *n*th mode of sum of *M*th circumferential order, contains $e^{i(-k_n^M z)}$, assuming waves are propagating in the +*z* direction. Using total field expressed as eqn. (12) and the field of mode *n* of circumferential order *M* as expressed in eqns. (1) and (2). The modified form for solving broadband source with single element loading problem can be expressed as follows

$$A_{+n}(z) = \sum_M \sum_\omega A_{+n}^M(\omega, z) w^M(\omega) \tag{16}$$

3.4 Broadband Sources Excitation by Multi-Elements

With broadband sources excitation by multi-elements, angular variation is same as eqn. (15). We can define broadband sources excitation by multi-elements, recalling eqn. (16), A_{+n} is calculated by the *n*th mode of sum of *M*th circumferential order with eqn. (16). α and L are same as single element condition as shown in Fig. 4.

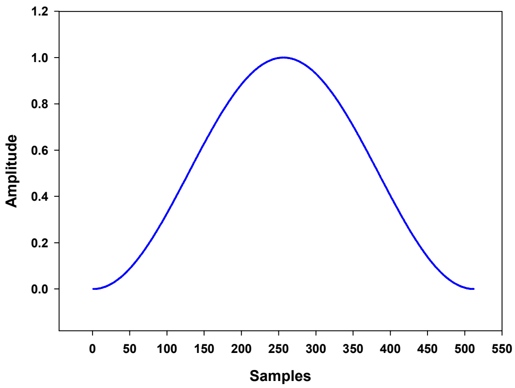

Fig. 5 Hanning window function

4. Numerical Results of Guided Wave Propagation

Using a single element, we put the transducer at the 0-degree circumferential position. Also, putting the #1 transducer at a 0-degree circumferential position when we use multi-elements. We can expect that the angular profile will be change by transducer size, center frequency, mode, circumferential order, and propagating distance.

4.1 Single Frequency Source Excitation by the Single Element

Using the normal mode expansion method, the amplitude factors with non-axisymmetric loading for 270 kHz single frequency source of each circumferential mode $L(M, 1)$ are computed (M=0,1,2,…,9), and their features are plotted in Fig. 6. At a distance near to the transducer, angular profiles are focused in transducer location. When ultrasonic guided wave propagates, the angular profiles change and spread out.

4.2 Single Frequency Source Excitation by Multi-Elements

Using the normal mode expansion method discussed, the amplitude factors with quasi-axisymmetric loading for 270 kHz single frequency source of each circumferential mode $L(M, 1)$ are computed (M=0,1,2,…,9), and their features are plotted in Fig. 7. Even though distance is very near to transducers location using multi-elements transducer, angular profiles are spreaded out. Compared to result of single element source, quasi-axisymmetric loading is more suitable for all section of pipe.

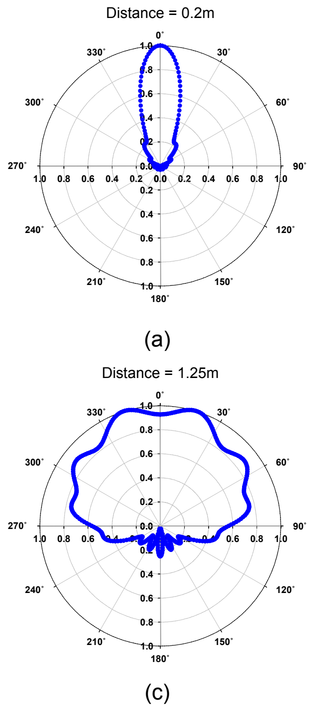

Fig. 6 Angular profiles at different distances on ASTM A106 carbon steel pipe with 165.2 mm o.d. and 7.1 mm wall thickness excited by 270 kHz single frequency and single element source (a) at 0.2 m, (b) 0.7 m (c) 1.25 m (d) 3 m

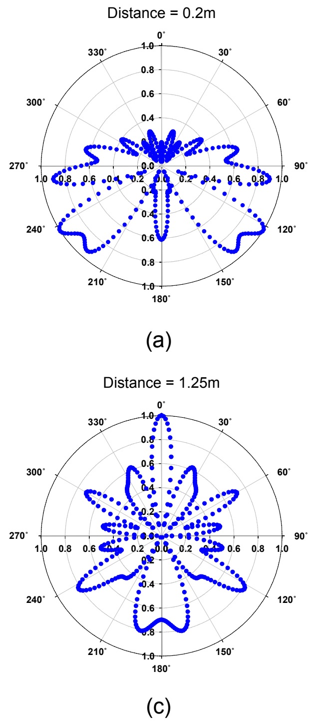

Fig. 7 Angular profiles at different distances on ASTM A106 carbon steel pipe with 165.2 mm o.d. and 7.1 mm wall thickness excited by 270 kHz single frequency and multi-element sources (a) at 0.2 m, (b) 0.7 m (c) 1.25 m (d) 3 m

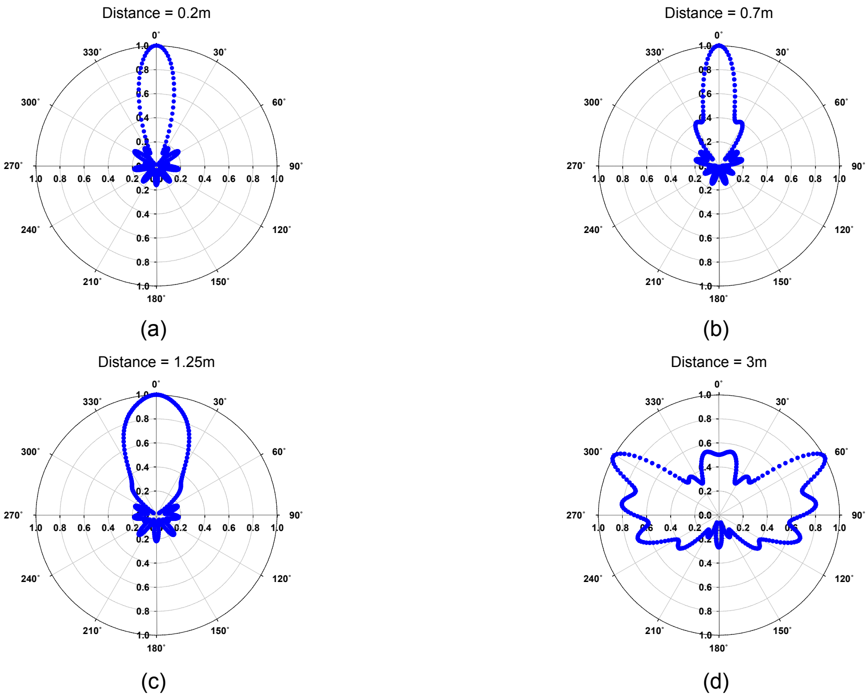

Fig. 8 Angular profiles at different distances on ASTM A106 carbon steel pipe with 165.2 mm o.d. and 7.1 mm wall thickness excited by 500 kHz broadband and single element source (a) at 0.2 m, (b) 0.7 m (c) 1.25 m (d) 3 m

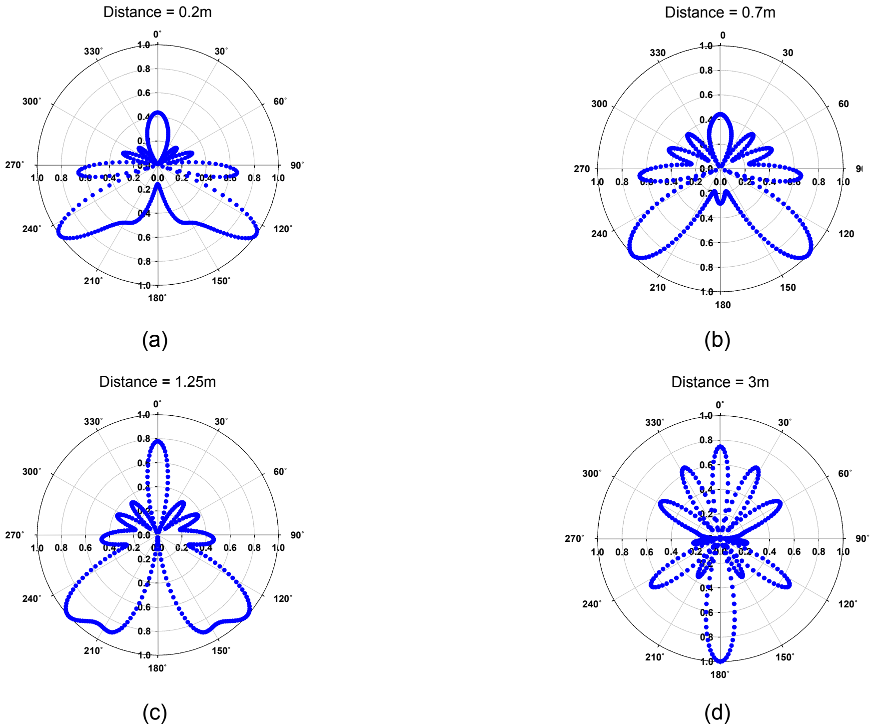

Fig. 9 Angular profiles at different distances on ASTM A106 carbon steel pipe with 165.2 mm o.d. and 7.1 mm wall thickness excited by 500 kHz broadband and multi-element sources (a) at 0.2 m, (b) 0.7 m (c) 1.25 m (d) 3 m

4.3 Broadband Sources Excitation by Single Element

The amplitude factors with partial loading (single element) for 500 kHz broadband sources (350 kHz~850 kHz) of each circumferential mode are computed, and their features are plotted in Fig. 8. Because broadband sources have finite bandwidth, there came up another small pick amplitude at distance 0.2 m at besides transducer location.

4.3 Broadband Sources Excitation by the Multi-Elements

The amplitude factors with quasi-axisymmetric loading(multi-elements) for 500 kHz broadband sources(350 kHz~850 kHz) of each circumferential mode are computed, and their features are plotted in Fig. 9. Compared to single frequency excitation, energies of broadband sources are harvested except 0-degree circumferential position.

5. Conclusion

The angular profile based on normal mode expansion method was used for studying four kinds of sources, i.e., 1) single frequency source excitation by the single element, 2) single frequency source excitation by multi-elements, 3) broadband sources excitation by the single element, and 4) broadband sources excitation by multi-elements. The contribution of angular profile with broadband sources is more realistic than the single frequency excitation. It is possible to verify the numerical results with experiments. Further research is needed for angular profile tuning(Li and Rose, 2002) with broadband sources that make possible to focus beam at desired distances.

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