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일반 수리 모형에서의 최적 예방 보전 정책에 관한 연구 : (θ, m) 보전 정책

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An Optimal Preventive Maintenance Policy with General Repair : (θ, m) Maintenance Policy

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본 논문에서는 시스템 연령(年齡)에 의해 보전 활동의 효과를 설명하는 일반 수리(修理) 개념을 이용한 최적 보전(保全) 정책에 대한 연구를 수행하였다. 본 논문에서는 주기적인 일반 수리와 고장 시 최소 수리가 적용되는 최적 보전 정책을 고려하였다. 따라서 일반 수리에 따른 보전 정책의 비용 함수를 도출하였고 최적 보전 정책을 도출하는 알고리즘을 제시하였고 예제를 통해 알고리즘의 성능을 분석하였다. 이 연구를 통해 시스템을 운영하는데 있어서 어느 수준의 보전 정책을 적용하며 어느 정도의 기간 동안 시스템을 유지할 것인지에 대안을 제공할 수 있을 것이다.

Keywords : Optimal Maintenance Policy, Minimal Repair, Perfect Repair, General Repair, Preventive Maintenance Model, Cost Function, Exhaustive Search, Genetic Algorithm

1. Introduction[1)]

Maintenance policies are about the decisions of the maintenance actions such as repair, replacement, inspection, and the time when the maintenance activities are conducted. The main purpose of deciding an appropriate maintenance policy is to insure the high utility of a system by reducing the chance of failures during operation. In general, increasing failure rate of a system means that the probability of failure increases as the system ages and requires repeated maintenance actions. The maintenance actions are done by reducing the age of the system. In most maintenance cases, two types of repairs are considered, that is, minimal repair and perfect repair. Minimal repair restores a failed system to a working state without altering the age of the system. For example, when a unit is out of order, minimal repair is carried out on it, but minimal repair does not improve the condition of a system. On the other hand, perfect repair makes the system as good as new one. So, perfect repair can be applied to a single component system or independent multi-component system since it is done by replacing

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an old system with new one and thus the system's age goes back to zero.

However if the old system is replaced by a used but younger system, the age of repaired system becomes between zero and previous age. Then neither perfect nor minimal repair can describe this aspect. Some researchers consider the imperfect aspect of the repair to overcome the shortcoming of the assumption with perfect repair and minimal repair. Brown and Proschan [1] examined the aging properties of a maintenance action that takes minimal or perfect repair effect at random.

Nakagawa and Yasui [5] studied optimal maintenance policies with imperfect maintenance. In this research, the effect of maintenance is assumed to take the effect of perfect or minimal repair or to break down the system at random with fixed probabilities. However, the effect of maintenance is still the extremes in these studies. Kijima et al. [4] introduced the concept of general repair and provided maintenance models with general repair. The repair that makes a system better than old one but not as good as new one is called general repair. So, in terms of age after repair, general repair stands between perfect repair and minimal repair. In this research, the level of the maintenance model is assumed to be a random variable instead of being treated as a controllable factor (see Kijima [3]).

Aging maintenance model with general repair is examined more specifically in subsection 2.1. The general repair can be justified for a system for which many used parts with various ages are available and for a non-repair maintenance action such as cleaning and adjustment of parameters. For example, consider a machine in a certain manufacturing process, a system parameter (process parameter) of the machine is set to a specified level and tends to deviate from it. Due to the out-of-specification parameter, the process may produce some defected products that cause unnecessary cost. We can assume that the machine is failed in terms of its performance in a manufacturing process although it works mechanically. Then, a required action is turning the parameter back to a specified level by cleaning and adjusting the parameter and it cannot be appropriately modeled either by minimal nor perfect repair.

Dagpunar and Jack [2] considered a maintenance model with age reduction that is the same as general repair concept and found the optimal preventive maintenance (PM) time and age reduction using Kuhn-Tucker condition to solve the maximization problem. However, the objective function which comprises of the maintenance costs up to the warranty duration is not necessarily concave function. Gu [6] considered a situation that a unit is repaired preventively with general repair of random effect after it has operated for time T. As first, he obtained the mean number of times the unit has broken down from time (k-1)T to time kT using the method of leading variables. After that, he provided an optimal preventive maintenance time T, for which the minimum total repair cost is achieved with considering an objective function with a bound condition and using Lagrange multipliers method.

In this study, an optimal maintenance policy with general repair is provided. At fixed preventive maintenance intervals, general repairs are applied and failures during operation are recovered to the working state through minimal repair. Then we will study the optimal maintenance level and number of period of operation that minimize the repair cost. The remaining of this paper consists of as follows: The aging model and the maintenance model are described in section 2. In section 2, we derive the cost structure model. Sections 3 presents algorithms for obtaining the optimal policy, and numerical examples are presented in section 4. Section 5 concludes this paper.

2. The models

In this chapter, with the notion of *virtual age* maintenance model and cost structure is presented.

2.1 Aging model

We refer to the notion of virtual age by which a system works. The actual age of a system is the elapsed time since the beginning of the operation. On the other hand, the virtual age is determined by actual age and the level of maintenance. Suppose that a system has been in use for one hundred unit times then the actual age of the system is one hundred. If the system undergoes maintenance of level of 0.5 the virtual age becomes fifty while the actual age is unchanged. The aging model we consider is (see [3]):

$$\upsilon_n = \theta(\upsilon_{n-1} + x_n),\ 0 \le \theta \le 1$$

where θ denotes maintenance level. At the time just prior to the n^{th} maintenance action, the virtual age of the system is the virtual age after $(n-1)^{th}$ maintenance action plus the elapsed time of the n^{th} operation. The n^{th} maintenance reduces the virtual age according to the level of maintenance, θ. From this model, we see that the minimal repair, $\theta = 1$, and the perfect repair, $\theta = 0$, are the extreme cases.

2.2 Cost structure

In this paper, we consider a repairable system whose time window for preventive maintenance is limited due to the high operation cost or the duty of a system. We assume that the system is replaced after finite number of period of operation. Other assumptions we make are as follows:

Assumption 1 : At time 0, the system is put into operation.

Assumption 2 : For a fixed number L, the system undergoes preventive maintenance of level θ, $0 \le \theta \le 1$ at the end of time interval kL, k = 1, 2, ⋯, m-1, where m and L represent the number of repairs until a system falls into disuse and the given repair time, respectively.

Assumption 3 : The system is replaced by new system at mL. If the system fails, minimal repair is conducted and the system is put into operation.

Assumption 4 : The durations of repair and preventive maintenance are assumed to be negligible.

We are interested in minimizing the average maintenance cost per unit time on the infinite time span. So, we can define that the objective function is the expected cost per preventive maintenance (PM) cycle on infinite time horizon and the decision variables of interest are the maintenance level at each interval and the replacement cycle of the system. In this paper, we assume that the same maintenance level is applied throughout the life of the system. The system is replaced after m-1 PM's when a cycle is composed of m intervals. The cost of a cycle is composed of m-1 PM costs, one replacement cost and the cost of the failures. As the virtual age increases, the expected number of failure in an interval increases. The cost of a PM cycle is random since the number of failures in a PM cycle is random. Then, the cost of a cycle is:

$$C(\theta, mL) = (m-1)c(\theta) + c_r + c_f E\left\{\sum_{j=1}^{m}\left[N\left(\sum_{k=0}^{j-1}\theta^k\right) - N\left(\sum_{k=1}^{j-1}\theta^k\right)\right]\right\},$$

where $\sum_{k=1}^{0}\theta^k \equiv 0$. The average cost of each PM interval is shown in <Table 1>.

<Table 1> Cost of Each PM Interval

Interval	Virtual age at starting point	Cost
1	0	$c_f N(L) + c(\theta)$
2	θL	$c_f[N(\theta L + L) - N(\theta L)] + c(\theta)$
3	$(\theta^2 + \theta)L$	$c_f[N(\theta^2 L + \theta L + L) - N(\theta^2 L + \theta L)] + c(\theta)$
…	…	…
m	$L\sum_{k=1}^{m-1}\theta^k$	$c_f[N(L\sum_{k=0}^{m-1}\theta^k) - N(L\sum_{k=1}^{m-1}\theta^k)] + c_r$

Since the cycle of the system is regenerative, we can get the following equation which represents the expected cost by renewal reward theorem.

$$k(\theta, mL) \equiv \lim_{t\to\infty}\frac{C(\theta, t)}{t} = \frac{C(\theta, mL)}{mL}$$

The expected number of failures up to time t:

$$E[N(t)] = \int_0^t h(x)dx$$

If let L = 1 without loss of generality, then the objective function is,

$$\min k(\theta, m) = \frac{1}{m}\left\{(m-1)c(\theta) + c_r + c_f\sum_{j=1}^{m}\left[H\left(\sum_{k=0}^{j-1}\theta^k\right) - H\left(\sum_{k=1}^{j-1}\theta^k\right)\right]\right\},$$

subject to

$$0 \le \theta \le 1 \text{ and } m \in Z^+,$$

where Z^+ is positive integer.

3. Algorithms for obtaining the optimal maintenance policy and numerical examples

Generally, optimization problems consisting of objective function, constraints, and decision variables with certain restrictions are to find an optimal solution. The most popular optimization problem is a linear programming problem consisting of a linear function subject to linear constraints. Many researchers have developed various useful approaches for finding optimal solutions to different kinds of optimization problems. However, there are a lot of optimization problems that cannot be solved by systematic algorithm such as the simplex method and it is difficult to evaluate the result of the decision making. In this section, two search algorithms, that is, an exhaustive search algorithm and a genetic algorithm, will be provided to find optimal PM levels and the number of PM intervals of a system.

3.1 Exhaustive Search Algorithm

When the objective function is non-convex function, the decision variables are the levels of PM and the number of PM intervals after the system is replaced. We will exploit the two facts that : (i) the objective function is continuous in θ for all m, and (ii) the lower bound of an interval of θ can be obtained. Since every term of lower bound is also continuous in θ, by dividing the interval of θ smaller, the distance between the objective function values of the two end points and the lower bound of an interval becomes closer. Thus, we can get a function value within ε > 0 from the lowest lower bound of intervals. For illustration, for an arbitrarily chosen θ such that $0 \le \theta_{i-1} \le \theta \le \theta_i \le 1$, define $k(\theta) \equiv \min_{1\le m\le n} k(\theta, m)$ which is the minimum of the objective function given θ where n is the maximum number of cycle of a system. We assume that the number of PM is bounded from above because in real situation none of the system is used forever. So, the lower bound can be defined as follows:

$$\tilde{k}(\theta, m) = \frac{1}{m}\left\{ \begin{array}{l} (m-1)c(\theta_i) + c_r \\ + c_f \sum_{j=1}^{m}\left[H\left(\sum_{k=0}^{j-1}\theta_{i-1}^k\right) - H\left(\sum_{k=1}^{j-1}\theta_{i-1}^k\right)\right] \end{array} \right\}$$

Note that PM cost $c(\theta)$ is a decreasing function in θ and $H\left(\sum_{k=0}^{j-1}\theta^k\right) - H\left(\sum_{k=1}^{j-1}\theta^k\right)$ is an increasing function in θ. Thus, $c(\theta) \ge c(\theta_i)$ and

$$H\left(\sum_{k=0}^{j-1}\theta^k\right) - H\left(\sum_{k=1}^{j-1}\theta^k\right) \le H\left(\sum_{k=0}^{j-1}\theta_{i-1}^k\right) - H\left(\sum_{k=1}^{j-1}\theta_{i-1}^k\right).$$

Then

$$k(\theta, m) \le \tilde{k}(\theta, m), \quad \forall m.$$

Therefore,

$$k(\theta) \ge \tilde{k}(\theta),$$

where $\tilde{k}(\theta) \equiv \min_{1\le m\le n} \tilde{k}(\theta, m)$., Using the continuity of $k(\theta, m)$ in θ for all m, we can get a lower bound of an interval within a given distance ε > 0. The algorithm is as follows:

Step 1	Set $\Theta = \{\theta : 0 \le \theta \le 1\}$
Step 2	Divide the existing line segment Θ into l subintervals $\Theta_1, \cdots, \Theta_l$ such that $0 \le \theta_0 \le \cdots \le \theta_l \le 1$, where $\Theta_i = \{\theta : \theta_{i-1} \le \theta \le \theta_i\}$.
Step 3	For i = 0 to l, evaluate $k(\theta_i)$. Set $\hat{k}(\theta_i) = \min_i k(\theta_i)$.
Step 4	For i = 0 to l, evaluate $\tilde{k}_i(\theta) = \min_{1\le m\le n} \tilde{k}(\theta, m)$ for $\theta \le \Theta_i$. If $\hat{k}(\theta) - \tilde{k}_i(\theta) < \varepsilon$, then $\Theta \setminus \Theta_i$.
Step 5	If $\Theta = \phi$, then return $\hat{k}(\theta)$ and stop. Else, go to step 2.

We consider the following conditions for numerical example: $T \sim$ Weibull (1, 1.2), c_r = 100, c_f = 60, $c(\theta) = c_r(1-\theta^2)$, and ε = 0 : 5. The upper bound of m is set to be 100 in this example. The optimal solution is obtained from two iterations. In the first iteration, the line segment of θ from 0 to 1 is divided into 10 intervals. The lower bound of each interval and function value of each end point is evaluated. We can see that the stopping criterion is not satisfied. The intervals from 0 to 0.9 are discarded because the lower bounds of them are worse than the function value at 1, k(1). Now the interval from 0.9 to 1 is left. Thus, the second iteration should begin. The interval for the second iteration is divided into 10 smaller intervals. And lower bounds and end point values are evaluated. The stopping criteria is met with k(1). The obtained optimal solution is (θ^*, m^*) = (1, 6). The interpretation of the optimal solution is that no PM is conducted at the end of every operation interval and the system is replaced every 6 operation intervals. <Table 3> provides the optimal solutions with various costs and PM cost functions.

<Table 2> Example of the Optimization Algorithm

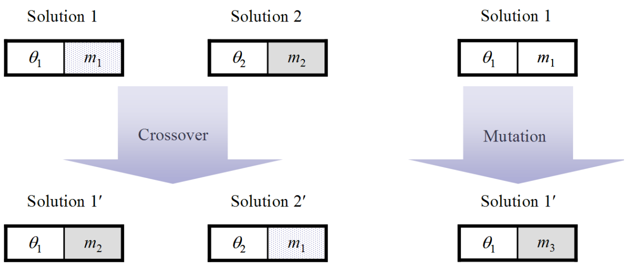
Iteration 1			Iteration 2		
Interval	$\tilde{k}(\theta)$	$k(\theta)$	Interval	$\tilde{k}(\theta)$	$k(\theta)$
0 ~ 0.1	159.01	160	0.9 ~ 0.91	114.17	114.43
0.1 ~ 0.2	159.74	160	0.91 ~ 0.92	112.91	113.17
0.2 ~ 0.3	158.06	159.53	0.92 ~ 0.93	111.63	111.90
0.3 ~ 0.4	154.37	156.75	0.93 ~ 0.94	110.34	110.61
0.4 ~ 0.5	148.92	151.97	0.94 ~ 0.95	109.03	109.30
0.5 ~ 0.6	141.91	145.37	0.95 ~ 0.96	107.71	107.98
0.6 ~ 0.7	133.61	137.11	0.96 ~ 0.97	106.37	106.64
0.7 ~ 0.8	124.15	127.20	0.97 ~ 0.98	105.02	105.29
0.8 ~ 0.9	112.71	115.67	0.98 ~ 0.99	103.64	103.91
0.9 ~ 1	99.60	102.52	0.99 ~ 1	102.26	102.5

<Table 3> Optimal Solutions with Various Cost Settings

c_r	c_f	$c(\theta)$	θ^*	m^*
100	60	$c_r(1-\theta^2)$	1	6
100	60	$c_r(1-\theta)$	1	6
100	60	$c_r(1-\theta)^2$	0.68	100
100	100	$c_r(1-\theta^2)$	1	4
100	100	$c_r(1-\theta)$	1	4
100	100	$c_r(1-\theta)^2$	0.59	100
60	100	$c_r(1-\theta^2)$	1	3
60	100	$c_r(1-\theta)$	1	3
60	100	$c_r(1-\theta)^2$	0.76	3

3.2 Genetic Algorithm

As an alternative approach for finding optimal solutions to this maintenance problem, there is the heuristic approach which is one of the most famous approaches. The term 'Heuristic' is used for algorithms with finding solutions among all possible ones, but they do not guarantee that the best will be found because the algorithm must be stopped after a specific time has been spent. But, these algorithms usually provide a solution close to the best one fast and easily. There are four famous heuristic algorithms : Genetic algorithm, Branch-and-bound algorithm, Simulated annealing, Tabu search. These algorithms are famous for the simple applicability and the excellent search performance so that these are applied to various decision making problems in engineering, social science, and management areas. Our approach to optimal maintenance level, θ, and the number of repairs, m, uses a genetic algorithm (GA). The concept underlying the GA is that of natural evolution and heredity: a population of individuals undergoes some transformation and, during this process, the individuals strive for survival. In GA, a gene is defined for a specific type of optimization problem and individuals composed with the genes are evaluated. GAs seek to breed solutions that are optimal or nearly optimal via the artificial evolution methods (or called as genetic operations) such as cloning, crossover (mating), and mutation. In this study, we use three main genetic operations : cloning, crossover (mating), and mutation for producing the required number of individuals as offspring of the next generation. Encoding scheme, and crossover and mutation operations are illustrated in <Figures 1>. We apply the genetic operations in <Figure 1> to the problem of finding optimal maintenance level and the number of repairs when a constraint exists. The coded variable representing genes in these genetic operations is a real-value encoding variable that indicates maintenance level, θ, and the number of repairs, m. The objective function of cost and constraints are given in Chapter 2.2. The design of the neighborhood as well as on the manner in which the searches conducted within the neighborhood should be carefully considered since the effec-



<Figure 1> Crossover Operation and Mutation Operation

tiveness of GAs depends on these two key facts when GAs are to be applied to an optimization problem. In other words, since GAs may not guarantee an optimal solution and even have the possibility of getting trapped at numerous locally optimal solutions, we selected the search procedure and the GA parameters in a way that makes moves to better solutions or steps out of a local optimum easier and execute the GA procedure several times using different population sets to increase the chances of getting closer to the globally optimal solution. The steps of the GA used are as follows:

Step 1	Determine the genetic parameters, such as initial population size, number of iterations, cloning rate, crossover rate, and mutation rate. In this study, we chose 1000 as the size of the initial population and 50 as the number of iterations for all cases.
Step 2	Generate initial populations that are solution vectors consisting of θ and m.
Step 3	Check the feasibility of the generated initial populations.
Step 4	Calculate the performance measures (or objective function values) for the generated initial populations.
Step 5	Find the best combination vector through investigation of the performance.
Step 6	Produce offspring of the next generation by considering the fitnesses of the generated initial population and using three genetic operations: cloning, crossover, and mutation. The cloning rate, crossover rate, and mutation rate that we used were 0.4, 0.4, and 0. 2, respectively.
Step 7	Stop after the given stopping criterion is satisfied.

We applied a genetic algorithm to find optimal solutions of a maintenance system, that is, the levels of PM and the number of PM intervals, and then compared the optimal solutions to those from the exhaustive search considered in this study. All the conditions are the same as the previous experimentation, that is, the maintenance level, θ, ranges from 0 to 1, the failure cost (c_f) and replacements cost (c_r) are set to be 100 and between 10 to 90 incremented by 10, respectively. As shown in <Table 4>, the optimal operation interval m^* is 2. The optimal solutions from the exhaustive search are superior to those from the genetic algorithm for all different replacement costs except when c_r = 40, 70, and 80. Thus, we can conclude that the exhaustive search might not give the optimal solutions when various replacement costs and PM cost functions are considered. Therefore, if we conduct further investigation with using the genetic algorithm, then, at least, we could obtain superior or the same optimal solutions as those from exhaustive search.

5 Conclusions

In this paper, we have developed a preventive maintenance model based on general repair model for various costs and PM cost functions. Our model can provide the optimal preventive maintenance level and the optimal system replacement time with respect to the operating cost of a system at the same time. In conclusion, if the practitioner does not have any idea of the specific preventive maintenance policy with considering a general repair, this analytical approach can be recommended to find optimal or near optimal solutions. As future research, it should be also considered to employ different distribution of the maintenance time, T, and several operating conditions more close to real situations.

<Table 4> Comparison of Optimal Solutions from Genetic Algorithm (GA) and Exhaustive Search (ES)

GA				Exhaustive Search			
c_r	θ^*	m^*	$k(\theta)$	c_r	θ^*	m^*	$k(\theta)$
90	0.8974	2	109.4	90	0.89453	2	109.39959
80	0.8768	2	104.3367	80	0.87500	2	104.33730
70	0.8625	2	99.2533	70	0.86328	2	99.25338
60	0.8407	2	94.14	60	0.83594	2	94.13931
50	0.795	2	88.9731	50	0.79688	2	88.97294
40	0.73	2	83.7064	40	0.75000	2	83.70907
30	0.6152	2	78.1968	30	0.61230	2	78.19677
20	0.0046	2	70.1053	20	0.00000	2	70.00000
10	0.0155	2	60.4398	10	0.00000	2	60.00000

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《Notation》

T : failure time of a system, a random variable

$f(t)$: probability density function (pdf) of T

$F(t)$: cumulative density function (cdf) of T

$h(t)$: failure rate $\equiv \frac{f(t)}{1-F(t)}$

$H(t) \equiv \int_0^t h(t)dt$

υ_n : virtual age after the n^{th} preventive maintenance

x_n : n^{th} duration of operation

θ : level of preventive maintenance

$N(t)$: number of failures up to time $t \geq 0$

L : length of interval of preventive maintenance

$c(\theta)$: cost of preventive maintenance

c_r : cost of replacement, $c(0)$

c_f : cost of a failure during operation

$C(t)$: total cost up to time t

D : cost of a cycle composed of m preventive maintenances

《Assumptions》

The failure time T follows a distribution $F(t)$.

The distribution $F(t)$ has a continuous increasing failure rate.

$c(\theta)$ is a decreasing function in μ.

Preventive maintenance is done periodically.

Minimal repair is applied at a failure in a negligible repair time.