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지연수정전략에서의 가속 신뢰성성장 모델

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Accelerated Reliability Growth Model with Delayed Fixes

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신뢰성성장시험은 주어진 기간 동안에 고장모드를 체계적이고 영구적으로 제거하여 신뢰성을 향상시키려는 것이 목적이다. 이러한 활동은 대부분 매우 제약된 시간에 이루어져야 함으로 신제품개발프로그램에 있어서는 가속수명시험이 필수적인 신뢰성성장시험 도구로 적용된다. 본 연구에서는 가속수명시험 하에서 신뢰성성장을 계획하고 분석하는데 유용한 도구로 사용할 수 있는 가속 신뢰성성장모델을 제시하였다. 시험전략은 test-find-test 전략으로서 시험 중 고장모드가 발견되어도 이 문제에 대한 근본적 조치는 시험이 완성된 이후에 이루어지는 경우를 가정하였다. 이는 시험을 중간에 중단하고 시작하는 비용이 매우 높거나, 시스템이 고도로 복잡하여 완전히 분해하는데 어려움이 있는 경우에 적용된다.

Keywords : Accelerated Life Test, Reliability Growth, Delayed Fixes

1. Introduction

In today's environment of compressed schedules and limited testing, every opportunity to identify and correct reliability deficiencies in a new design is a prime objective. A metric for tracking system reliability before development testing based on preemptive corrective actions for potential problem modes is discussed in [7]. The tradition reliability growth models [2, 5, 10] provide assessments when the failure modes corrected are surface during the testing.

There are some different strategies in reliability growth testing. With the test-fix-test strategy, problem modes are found during testing and corrective actions for these problems are incorporated during the test. Previous works [2, 5] discussed and constructed models for the Test-Fix-Test strategy. For the Test-Find-Test strategy, problem modes are found during testing but all corrective actions for these problems are delayed and incorporated after the completion of the test.

During the design and development, various procedures have been used in practice to reduce the duration of the total test time in life test experiments. Generally, the ALT (accelerated life test) is implemented in the earliest development phases and has been used to evaluate design weaknesses and uncover specific assembly and materials problems. ALT can obtain information on lifetime characteristics quickly by providing the higher stress than usual stress for products. The products can then be tested to higher levels of stress in various ways such as constant, step and progress stress.

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Guérin [1] defined two ALT models for repairable systems : the Arrhenius-Exponential model and the Peck-Weibull model. These models allow us to improve accuracy in the estimation of reliability parameters. Crow developed the Crow-AMSAA (Army Materials Systems Analysis Activity) model for complex systems as described in MIL-HDBK-189 [8]. In this model, stable processes produce straight lines of cumulative failures versus cumulative time on log-log paper. The straight lines make the forecast of future failures easy to predict [2, 3]. Feinberg [9] describes accelerated reliability growth model linking the reliability growth and accelerated testing. This work models mathematical equations of the two areas for both iso-stress and step-stress accelerated testing.

This paper presents a reliability growth model under accelerated life test. The developed model provides assessments for the test-find-test strategy. In test-find-test strategy, failure modes are found during testing but all of the corrective actions are delayed and incorporated after the completion of the test. In these systems, corrective actions may be delayed until the end of the test since it may be too expensive to stop and then restart the test, or the equipment may be too complex for performing a complete teardown. Implementing delayed fixes usually results in a distinct jump in the reliability of the system at the end of the test phase. This scenario is also called the Crow-AMSAA Projection model [4].

2. The Model

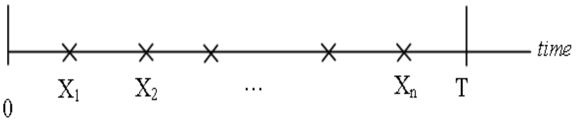
2.1 Crow-AMSAA Basic Model

In Crow-AMSAA model, the failure intensity is given by :

$$\gamma(t) = \lambda \beta t^{\beta - 1} \tag{1}$$

The parameter λ is referred to as the scale parameter and β is the shape parameter. For $\beta = 1$, there is no reliability growth. For $\beta < 1$, there is positive reliability growth. That is, the system reliability is improving due to corrective actions. For $\beta < 1$, there is negative reliability growth.

Suppose a development testing program begins at time 0 and is completed by time T. Let N be the total number of failures recorded and let $0 < X_1 < X_2 < \cdots < X_n < T$ denotes the N successive failure times on a cumulative time scale as depicted in <Figure 1> We assume that the Crow-AMSAA NHPP assumption applies to this set of data. Parameters of Crow-AMSAA model is derived by the maximum likelihood method :



<Figure 1> Failure times at system

$$\hat{\beta} = \frac{N}{\sum_{i=1}^{N} \frac{T}{X_i}} \tag{2}$$

$$\hat{\lambda} = \frac{\sum_{i=1}^{N} K_i}{nT^{\beta}}$$

The unbiased estimation of λ and β is:

$$\overline{\beta} = \frac{N-1}{N}\hat{\beta} \text{ and } \overline{\lambda} = \hat{\lambda} \tag{3}$$

Uner the Crow-AMSAA basic model, the achieved or demonstrated failure intensity at time T, the end of the test, is given by $\gamma(T)$. We denote the achieved failure intensity by

$$\lambda_{CA} = r(T) \tag{4}$$

This model is applicable to the case of test-fix-test strategy test plan. Procedures of parameter estimation and confidence intervals are given in [3, 6, 7, 8].

2.2 Test Plan for Test-Find-Test Strategy

In the case of the test-find-test strategy, the reliability growth test plan satisfies the following assumptions :

- The projection model places all failure into two groups, A and B. type A failure modes are all modes such that if seen during test no corrective action will be taken. This accounts for all modes for which management determines that it is not cost-effective to increase the reliability by a design change. type B failure modes are all modes such that if seen during test a corrective action will be taken. This type A and type B determination helps define the reliability growth management strategy.
- The time to take the corrective action of the failure item is negligible in comparison with test time ;

- No new errors are introduced during the corrective action process ;
- When failure occurs, if this type A failure, the test will be continued. If this is type B, at least one method has been identified to repair, replace or change component. New (or repaired) component has reliability higher than the one before;
- The components that can fail during test decrease their MTBF when temperature increases.
- Type A failure modes has constant failure intensity λ_A,
- There are M unique type B failure modes seen which means there are M distinct corrective actions incorporated into the system at the end of test. The total number of failures for the j-th observed distinct type B mode is denoted by N_j and the total number of type B failures seen during the test is : $N_B = \sum_{j=1}^{M_1} N_j$.
- The i-th type B failure mode follows the exponential distribution with failure rate λ_i, and the initial failure intensity for type B failure modes is λ_B.

2.3 Accelerated Life Test Model

One of the earliest and most successful acceleration models predicts how time-to-fail varies with temperature. This empirically based model is known as the Arrhenius equation [8] :

$$\zeta = A \cdot \exp\left(\frac{E_a}{kT}\right) \tag{5}$$

Where :

ζ is Arrhenius factor;

T denoting temperature measured in degrees Kelvin (273.16 + degrees Celsius) at the point when the failure process takes place

k is Boltzmann's constant (8.617 x 10-5 in ev/K).

A is a scaling factor that can be eliminated when calculating acceleration factors and it should be a positive constant.

E_a denotes the activation energy which is the critical parameter in the model. The value of E_aa depends on the failure mechanism and the material involved and typically ranges from 0.3 or 0.4 up to 1.5, or even higher.

The Arrhenius acceleration factor between the lifetime ζ_1 for the lower temperature T_1 and the lifetime ζ_2 for a temperature T_2 is :

$$FA = \frac{\zeta_1}{\zeta_2} = \exp\left(\frac{E_a}{k}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)\right) \tag{6}$$

2.4 Mathematics of the Reliability Growth Test Plan

Suppose a system is tested for the time duration of T. During the test, failure modes are identified, but all corrected actions are delayed until the end of the test phase. This is test-find-test-strategy. These delayed corrective actions are usually incorporated as a group and the result is generally a distinct jump in the system reliability. The projection model [2] considers this jump in reliability due to the delayed fixes [2].

An effectiveness factor, d_j is defined as the fraction decrease in λ_j after a corrective action has been made for the j-th type B mode. Then, the failure rate for the i-th type B failure mode after a corrective action is $(1-d_j)\lambda_j$. In practice, for application of the projection model, the effectiveness factors are assigned based on engineering assessments, test results, etc. Studies indicate that an average effectiveness factor of about .70 is typical for a reliability growth program. Individual effectiveness factors may vary. The effectiveness factor is assigned based on engineering judgment and the predictions made based on the various factors will be affected by the quality of this assessment. Based on past experience with reliability growth analysis testing, the average effectiveness factor for all modes is likely to be in the range of 0.65 to 0.75. An individual effectiveness factor may be smaller or larger than this average, but the average over a large number of effectiveness factors during a test is likely to be in this range based on data.

With the test-find-test strategy, the system failure intensity is constant, say, λ_s, during the testing and then jumps to a lower value due to the incorporation of corrective actions. The intensity at the end of the test T, before delayed corrective actions are introduced into the system, is the achieved intensity. The reciprocal of the intensity is the achieved MTBF M_s.

The achieved failure intensity λ_s can be estimated by :

$$\lambda_s = \lambda_A + \lambda_B \tag{7}$$

where : $\hat{\lambda}_A = \dfrac{N_A}{T} \quad \hat{\lambda}_B = \dfrac{N_B}{T}$

The estimated projected failure intensity[2] is :

$$\hat{\lambda}_p = \hat{\lambda}_A + \sum_{j=1}^{M}(1-d_j)\frac{N_j}{T} + \overline{d}\hat{h}(T) \tag{8}$$

where : $\overline{d} = \dfrac{\sum_{j=1}^{M} d_j}{M}$ is average effectiveness factor, and

$$\hat{h}(T) = \hat{\lambda}\hat{\beta}T^{\hat{\beta}-1} \tag{9}$$

$\hat{\lambda}$ and $\hat{\beta}$ in (8) and (9) use only the first M failure times and unique type B failure modes.

Therefore, the achieved MTBF M_s and projected MTBF M_p can be derived as:

$$M_s = (\lambda_s)^{-1} \quad M_p = (\lambda_p)^{-1}.$$

The value S will be obtained only for the accelerated life test situation. P shows that the value will be obtained from reliability growth plan coupled with ALT.

3. Accelerated Reliability Growth Model with Test-Find-Test Strategy

To analyze the collected data during the test, the model uses Arrhenius model. The system will be tested in two levels with higher temperature T_1 and the highest temperature T_2. In each level, data will be analyzed with ALT only and Reliability Growth combines with ALT for Test-Find-Test. Therefore, λ_{s1}, λ_{s2}, λ_{p1}, λ_{p2} can be obtained. From these parameters, using Arrhenius [10] :

For both of λ_s and λ_p :

$$\lambda_i = \lambda_0 \exp\left(\frac{E_a}{k}\left(\frac{1}{T_i} - \frac{1}{T_0}\right)\right) \tag{10}$$

set :

$$x_i = -\frac{1}{k}\left(\frac{1}{T_i} - \frac{1}{T_0}\right) \tag{11}$$

then

$$\lambda_l = \lambda_o \exp(E_a x_i) \tag{12}$$

In order to define the model, the two unknown parameters E_a and λ_0 have to be evaluated. For this purpose, two tests are realized under two temperatures (T_1 and T_2). At each temperature level, x_i is estimated by equations (11). λ_s and λ_p are estimated by equations (7) and (8).

The parameters E_a and λ_0 are estimated from relationship (12) in :

$$E_a = \frac{1}{(x_2 - x_1)} log\left(\frac{\lambda_2}{\lambda_1}\right) \tag{13}$$

$$\log(\lambda_0) = \frac{x_2\log(\lambda_1) - x_1\log(\lambda_2)}{x_2 - x_1} \tag{14}$$

From equation (14); λ_{so} and λ_{po} are estimated. The MTBF M_{so} and M_{po} at normal condition are also estimated.

4. Numerical Example

Let us consider the following test condition : the activation energy is $E_a = 0.36$ eV the temperature at normal condition, high level and highest level are T_0 = 300℃, T_1 = 70℃ and T_2 = 100℃, consecutively. Each stress test will be stopped at 300 hours.

From equation (11), x_1 and x_2 can be derived as $x_1 = 4.4664$ and $x_2 = 7.1876$. From the data in <Tables 1>, the system is tested for T = 300 hours at high level with temperature T = 70°C. There is total of $N_1 = 21$ failures and all corrective actions will be incorporated at the end of 300 hours test. Each failure is as a type A (no corrective action) or type B failure mode (corrective action). There are $N_A = 3$ type A failures and $N_{B1} = 18$ type B failures during the test.

In <Table 2> there are $M_1 = 10$ unique type B failure modes seen which means there are 10 distinct corrective actions in-

<Table 1> Test-Find-Test Data with temperature T = 700℃

j	Xj	Mode	j	Xj	Mode	j	Xj	Mode
1	2.6	B1	8	74.5	B2	15	178.5	B9
2	8.6	B2	9	84.2	B6	16	212.2	B1
3	12	A	10	98.2	B7	17	229.3	B4
4	15.6	B3	11	112.3	B6	18	243.6	B6
5	32.5	B4	12	124.6	A	19	165.2	B7
6	45.5	A	13	135.6	B8	20	275.4	B4
7	50.6	B5	14	160	B3	21	290.3	B10

<Table 2> Test-Find-Test Type B Failure Mode Data and Effectiveness Factors with temperature T = 80℃

B mode j	Number N_{1j}	First Occurrence	EF d_{1j}
1	2	2.6	0.73
2	2	8.6	0.78
3	2	15.6	0.65
4	3	32.5	0.71
5	1	50.6	0.63
6	3	84.2	0.72
7	2	98.2	0.68
8	1	135.6	0.79
9	1	178.5	0.74
10	1	290.3	0.65

corporated into the system at the end of test. The total number of failures for the j-th observed distinct type B mode is denoted by N_{1j} and the total number of type B failures seen during the test is : $N_B = \sum_{j=1}^{M_1} N_{1j}$.

As shown in section 2.4. These following parameters are derived :

$$\hat{\lambda}_{A1} = \frac{3}{30} = 0.01 \qquad \hat{\lambda}_{B1} = \frac{18}{300} = 0.06$$

Therefore, $\lambda_{s1} = \lambda_{A1} + \lambda_{B1} = 0.07$

The projection model $\hat{\lambda}_1$ and $\hat{\beta}_1$ for (10) use only the M_1 first occurrence failure times of the seen and unique type B failure modes. These first occurrences are given in Table 2. Applying equations (3) to the first occurrence data in Table 2 we have:

$$\hat{\lambda}_1 = 0.6702 \qquad \hat{\beta}_1 = 0.4739$$

Based on the data in Table 2 :

$$\overline{d_1} = 0.708 \qquad \hat{h}(300) = 0.0158$$

$$\overline{d_1}\hat{h}(300) = 0.0112$$

$$\sum_{j=1}^{M}(1-d_{1j})\frac{N_{1j}}{T} = 0.0174$$

Therefore, the failure intensity of the projection is :

The estimated projected failure intensity [2] is :

$$\hat{\lambda}_{p1} = \lambda_{A1} + \sum_{j=1}^{M}(1-d_{1j})\frac{N_{1j}}{T} + \overline{d_1}\hat{h}_1(T)$$

$$\hat{\lambda}_{p1} = 0.0386$$

<Table 3> Test-Find-Test Data with temperature T=1000℃

j	Xj	Mode	j	Xj	Mode
1	5.6	A	25	140	B5
2	10.3	B1	26	144.7	A
3	16.3	A	27	149.8	B8
4	19	B2	28	155.8	B9
5	20.6	B3	29	162.3	B3
6	28.9	B2	30	169	B8
7	35.3	B4	31	178	B10
8	40.9	B2	32	180.5	A
9	49.6	B5	33	190	B11
10	50.7	A	34	199.9	B1
11	59.1	B6	35	201.2	B6
12	66.8	B7	36	212	A
13	75.4	B8	37	219	B12
14	80	B6	38	224.5	B7
15	86.3	B4	39	226.9	A
16	90.6	A	40	230.1	B13
17	98.8	B7	41	238.4	B3
18	103.4	B9	42	246	B14
19	106.8	B2	43	250.4	A
20	111	B10	44	251	A
21	116.4	B1	45	260	B9
22	123	B11	46	275.4	B14
23	129.5	B12	47	286.7	B15
24	133.5	A	48	295.8	B6

Now, the system will be tested at the highest level with temperature at 1000℃. The data are shown in <Table 3> and <Table 4>.

From the table 3, there are 11 failures type A and 37 failures type B.

Similarly, calculating the following parameters:

$$\hat{\lambda}_{A2} = \frac{11}{300} = 0.0367 \qquad \hat{\lambda}_{B2} = \frac{37}{300} = 0.1233$$

and $\lambda_{s2} = \lambda_{A2} + \lambda_{B2} = 0.16$

The projection model $\hat{\lambda}_2$ and $\hat{\beta}_2$ for (10) use only the $M_2 = 15$ first occurrence failure times of the seen and unique type B failure modes. These first occurrences are given in <Table 4> Applying equations (3) to the first occurrence data in <Table 2> we have :

$$\hat{\lambda}_2 = 0.3693 \qquad \hat{\beta}_2 = 0.6494$$

<Table 4> Test-Find-Test Type B Failure Mode Data and Effectiveness Factors with temperature T=100℃

B mode j	Number Nj	First Occurrence	EF dj
1	3	10.3	0.73
2	4	19	0.78
3	3	20.6	0.65
4	2	35.3	0.71
5	2	49.6	0.63
6	4	59.1	0.72
7	3	66.8	0.68
8	3	75.4	0.79
9	3	103.4	0.74
10	2	111	0.65
11	2	123	0.64
12	2	129.5	0.48
13	1	230.1	0.69
14	2	246	0.74
15	1	286.7	0.63

And the failure intensity of the projection is :

$$\hat{\lambda}_{p2} = 0.0962$$

Using equation (14), λ_{p0} and λ_{s0} can be obtained from the value of λ_{p1}, λ_{p2} and λ_{s1}, λ_{s2}

$$\lambda_{p0} = \exp\frac{x_2 \log(\lambda_{p1}) - x_1 \log(\lambda_{p2})}{x_2 - x_1} = 0.0086$$

By analyzing test data used the model, The MTBF of the system at 300 hours at normal condition is :

$$MTBF_P(300) = \frac{1}{\lambda_{p0}} = 116.1938 \text{ (hours/failure).}$$

Similarly, if the test data is analyzed by ALT only, λ_{s0} can be obtained :

$$\lambda_{s0} = \exp\frac{x_2 \log(\lambda_{s1}) - x_1 \log(\lambda_{s2})}{x_2 - x_1} = 0.0180$$

And the MTBF at 300 hours is:

$$MTBF_S(300) = \frac{1}{\lambda_{s0}} = 55.4865 \text{ (hours/failure).}$$

The value E_a of activation energy can be derived from equation :

$$E_{pa} = \frac{1}{(x_2 - x_1)} log\left(\frac{\lambda_{p2}}{\lambda_{p1}}\right) = 0.336$$

$$E_{sa} = \frac{1}{(x_2 - x_1)} log\left(\frac{\lambda_{s2}}{\lambda_{s1}}\right) = 0.308$$

5. Conclusion

This paper shows an accelerated reliability growth model with test-find-test strategy. By using this model, the data collected from ALT test can be analyzed with reliability growth without transfer to normal condition. As shown in the example, the MTBF obtained by ALT is 55.4865 hours/failure, but if the data is analyzed by reliability growth combines ALT, the MTBF of system is 116.1938 failure/hours.

Using this model, the reliability of systems or components can meets the expected values faster than use ALT model. This model has combined both good features of Arrhenius and Crow-AMSAA model. This method is able to improve the accuracy in the estimation of MTBF for the system test which all corrective actions are implemented after test or test phase.

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