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An H_∞ Output Feedback Control for Uncertain Singularly Perturbed T-S Fuzzy Systems

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Abstract

This paper deals with an H_∞ output feedback controller design for uncertain singularly perturbed T-S fuzzy systems. Integral quadratic constraints are used to describe various kinds of uncertainties of the plant. It is shown that the H_∞ norm of the uncertain singularly perturbed fuzzy system is less than γ for a sufficiently small $\varepsilon > 0$ if the H_∞ norms of both the slow and fast subsystem are less than γ. Using this fact, we develop a linear matrix inequality based design method which is independent of the singular perturbation parameter ε. A numerical example is provided to demonstrate the efficacy of the proposed design method.

Key Words : Uncertain Singularly Perturbed Systems, Linear Matrix Inequality, H_∞ Fuzzy Control, Integral Quadratic Constraints, T-S Fuzzy System

1. 서 론

There are two main difficulties in controller design for real systems: nonlinearity and uncertainty. This paper deals with an H_∞ output feedback controller design for uncertain singularly perturbed nonlinear systems. The system with both slow mode and fast mode in system dynamics can be modelled as a singularly perturbed system. For example, a mechanical system with very small mass or inertia moment, dynamic system with a fast actuator and a control system with a high gain feedback system can be modelled as a singularly perturbed system.[1]-[3] Uncertain singularly perturbed system is a singularly perturbed system with uncertain factors. Originally the singular perturbation theory has been widely used to find an approximate solution in a mathematical field of celestial mechanics and hydrodynamics. In control problem, P.Kokotovic[2] first used the singular perturbation theory in the optimal control problem to obtain an approximate solution of the Riccati differential equation. After that this effective way to design the controller for linear or nonlinear singularly perturbed system has been used by many researchers.

In past three decades, the H_∞ control problem has been extensively studied by a number of researchers.[4]-[10] This problem can be stated as follows: given a dynamic system with the exogenous input and measured output, design a control law such that the L_2 gain of the operator from the exogenous input to the regulated output is minimized or no larger than some prescribed level. In nonlinear system case, it is very difficult to solve nonlinear H_∞ control problem. The reason is that it is still very difficult to obtain the solution for the resulting Hamilton-Jacobi-Bellman (HJB) equation.

In the past two decades, there has been rapidly growing interest in approximating a nonlinear system by the Takagi-Sugeno(T-S) fuzzy model.[11]-[13]. Based on this fuzzy model, a model-based fuzzy control has been developed to stabilize the T-S fuzzy model. The T-S fuzzy model provides a powerful tool for modelling complex nonlinear systems. Unlike the conventional model where a single model is used to describe the global behavior of a system, the T-S model is essentially a multi-model approach in which simple sub-models (typically linear models) are combined to describe the global behavior of the system.

In this work, we suggest an output feedback H_∞ controller design method for a singularly perturbed T-S fuzzy system with integral quadratic constraints(IQC's). Many kinds of uncertainties such as time varying parameters, time-delay and unmodelled dynamics etc. can be described via IQC's. Various kinds of uncertainties which can be described via IQC's are presented in [14].

In section 2, we define the uncertain singularly perturbed system and formulate a control problem. In section 3, an H_∞ norm bounded condition for an uncertain singularly perturbed system with IQC's is presented in the form of linear matrix inequalities(LMI's). In section 4, we present an output H_∞ controller design method for a T-S fuzzy system with IQC's. In section 5, a numerical design example is given to demonstrate the efficacy of our method.

The notation is standard. Given matrix A, A^T is

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transposed matrix and A^{-1} is the inverse matrix of A. $A^{\perp}$ is a basis vector for null space of A^T. $A>0$ means A is a positive definite matrix. R^n is an n dimensional real vector space, $R^{n\times m}$ is an $n\times m$ dimensional real matrix set. I_n is the $n\times n$ unit matrix, and 0 is the zero matrix. $diag(A,B)$ is a diagonal matrix composed by A and B.

II. System Description and Definition

Consider the system described by the following singularly perturbed T-S fuzzy model:

Plant Rule R_i $(i=1,\cdots,r)$:

IF $\rho_1(t)$ is M_{i1} and $\cdots$ and $\rho_g(t)$ is M_{ig},

THEN

$$\begin{aligned}
\begin{bmatrix}\dot{x}_1(t)\\ \varepsilon\dot{x}_2(t)\end{bmatrix} &= A_i\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+B_{i1}w(t)+B_{i2}u(t)+\sum_{k=1}^{m}F_{ik}p_k(t)\\
&=\begin{bmatrix}A_{i11}\ A_{i12}\\ A_{i21}\ A_{i22}\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+\begin{bmatrix}B_{i11}\\ B_{i12}\end{bmatrix}w(t)\\
&\quad+\begin{bmatrix}B_{i21}\\ B_{i22}\end{bmatrix}u(t)+\sum_{k=1}^{m}\begin{bmatrix}F_{ik1}\\ F_{ik2}\end{bmatrix}p_k(t),\\
z(t) &= C_{i1}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+D_{i12}u(t)\\
&=[\,C_{i11}\ C_{i12}\,]\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+D_{i12}u(t),\\
y(t) &= C_{i2}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+D_{i21}w(t)+\sum_{k=1}^{m}G_{ik}p_k(t)\\
&=[\,C_{i21}\ C_{i22}\,]\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+D_{i21}w(t)+\sum_{k=1}^{m}G_{ik}p_k(t),\\
q_k(t) &= E_{ik}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+J_{ik}w(t)+H_{ik}p(t)\\
&=[E_{ik1}\ E_{ik2}]\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+J_{ik}w(t)+H_{ik}p(t),
\end{aligned}\tag{1}$$

where r is the number of rules. $\rho_i(t)$ and M_{ij} $(1,\cdots,g)$ are the premise variables and the fuzzy set respectively. $x(t)\in R^{n_1+n_2}$ is the state variable, $w(t)\in R^{n_w}$ is the external noise, $z(t)\in R^{n_z}$ is the controlled variable, $u(t)\in R^{n_u}$ is the input variable and $y(t)\in R^{n_y}$ is the output variable. All matrices $A_i,B_i,\cdots,H_{ik}$ are the constant matrices with compatible dimensions. $p(t)=[p_1(t)^T\cdots\ p_m(t)^T]^T$, $p_k(t)\in R^{p_k}$, $(k=1,\cdots,m)$ and $q(t)=[q_1(t)^T\cdots\ q_m(t)^T]^T$, $q_k(t)\in R^{q_k}$ $(k=1,\cdots,m)$ are the uncertainty variables satisfying the following IQC′

$$\int_0^\infty p_k(t)^Tp_k(t)dt\le\int_0^\infty q_k(t)^Tq_k(t)dt,\ k=1,\cdots,m.\tag{2}$$

The normalized membership function $\mu_i(\rho(t))$ of the inferred fuzzy set $h_i(\rho(t))$ is defined as follows:

$$\mu_i(\rho(t))=\frac{h_i(\rho(t))}{\sum_{i=1}^{r}h_i(\rho(t))},\ h_i(\rho(t))=\prod_{j=1}^{g}M_{ij}(\rho_j(t)).\tag{3}$$

For all $t\geqq 0$, we assume $h_i(\rho(t))\ge 0$ $(i=1,\cdots,r)$ and $\sum_{i=1}^{r}h_i(\rho(t))>0$. Then we get

$$\mu_i(\rho(t))\ge 0,\ (i=1,\cdots,r),\quad \sum_{i=1}^{r}\mu_i(\rho(t))=1.$$

For simplicity, by defining $\mu_i=\mu_i(\rho(t))$, $\mu^T=[\mu_1\cdots\mu_r]$, (1) can be written as follows :

$$\begin{aligned}
\begin{bmatrix}\dot{x}_1(t)\\ \varepsilon\dot{x}_2(t)\end{bmatrix}&=\begin{bmatrix}A_{11}(\mu)\ A_{12}(\mu)\\ A_{21}(\mu)\ A_{22}(\mu)\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+\begin{bmatrix}B_{11}(\mu)\\ B_{12}(\mu)\end{bmatrix}w(t)\\
&\quad+\begin{bmatrix}B_{21}(\mu)\\ B_{22}(\mu)\end{bmatrix}u(t)+\sum_{k=1}^{m}\begin{bmatrix}F_{k1}(\mu)\\ F_{k2}(\mu)\end{bmatrix}p_k(t)\\
&=\sum_{i=1}^{r}\mu_i\Bigg(\begin{bmatrix}A_{i11}\ A_{i12}\\ A_{i21}\ A_{i22}\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+\begin{bmatrix}B_{i11}\\ B_{i12}\end{bmatrix}w(t)\\
&\quad+\begin{bmatrix}B_{i21}\\ B_{i22}\end{bmatrix}u(t)+\sum_{k=1}^{m}\begin{bmatrix}F_{ik1}\\ F_{ik2}\end{bmatrix}p_k(t)\Bigg),\\
z(t)&=[\,C_{11}(\mu)\ C_{12}(\mu)\,]\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+D_{12}(\mu)u(t)\\
&=\sum_{i=1}^{r}\mu_i\Bigg([\,C_{i11}\ C_{i12}\,]\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+D_{i12}u(t)\Bigg),\\
y(t)&=[\,C_{21}(\mu)\ C_{22}(\mu)\,]\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+D_{21}(\mu)w(t)\\
&\quad+\sum_{k=1}^{m}G_k(\mu)p_k(t)\\
&=\sum_{i=1}^{r}\mu_i\Bigg([\,C_{i21}\ C_{i22}\,]\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+D_{i21}w(t)\\
&\quad+\sum_{k=1}^{m}G_{ik}p_k(t)\Bigg),\\
q_k(t)&=[E_{k1}(\mu)\ E_{k2}(\mu)]\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+J_k(\mu)w(t)\\
&\quad+H_k(\mu)p_k(t)\\
&=\sum_{i=1}^{r}\mu_i\Bigg([E_{ik1}\ E_{ik2}]\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+J_{ik}w(t)+H_{ik}p_k(t)\Bigg).
\end{aligned}\tag{4}$$

In (4) $\dot{x}_2$ is about $1/\varepsilon$ bigger comparing with $\dot{x}_1$ so that we call x_2 the fast variable and x_1 the slow variable. When the singular perturbation parameter ε is set to 0, the dimension of the full system (4) changes from n_1+n_2 to n_1. When $\varepsilon=0$, assuming that $A_{22}(\mu)^{-1}$ exists, the singularly perturbed fuzzy system (4) becomes as follows :

$$\begin{aligned}
\dot{x}_1(t)&=A_{11}(\mu)x_1(t)+A_{12}(\mu)x_2(t)+B_{11}(\mu)w(t)\\
&\quad+B_{21}(\mu)u(t)+\sum_{k=1}^{m}F_{k1}(\mu)p_k(t),
\end{aligned}\tag{5}$$

$$x_2(t) = -A_{22}(\mu)^{-1}(A_{21}(\mu)x_1(t) + B_{12}(\mu)w(t) + B_{22}(\mu)u(t) + \sum_{k=1}^{m} F_{k2}(\mu)p_k(t)). \quad (6)$$

Substituting (6) into (5), we obtain

$$\begin{aligned} \dot{x}_1(t) &= A_s(\mu)x_1(t) + B_{s1}(\mu)w(t) + B_{s2}(\mu)u(t) + \sum_{k=1}^{m} F_{sk}(\mu)p_k(t), \\ z(t) &= C_{s1}(\mu)x_1(t) + D_{s12}(\mu)u(t), \\ y(t) &= C_{s2}(\mu)x_1(t) + D_{s21}(\mu)w(t) + \sum_{k=1}^{m} G_{sk}(\mu)p_k(t), \\ q_k(t) &= E_{sk}(\mu)x_1(t) + J_{sk}(\mu)w(t) + H_{sk}(\mu)p_k(t), \end{aligned} \quad (7)$$

where

$$\begin{aligned} A_s(\mu) &= A_{11}(\mu) - A_{12}(\mu)A_{22}(\mu)^{-1}A_{21}(\mu), \\ B_{si}(\mu) &= B_{i1}(\mu) - A_{12}(\mu)A_{22}(\mu)^{-1}B_{i2}(\mu) \quad i = 1,2, \\ F_{sk}(\mu) &= F_{k1}(\mu) - A_{12}(\mu)A_{22}(\mu)^{-1}F_{k2}(\mu), \\ C_{si}(\mu) &= C_{i1}(\mu) - C_{i2}(\mu)A_{22}(\mu)^{-1}A_{21}(\mu) \quad i = 1,2, \\ D_{sij}(\mu) &= D_{ij}(\mu) - C_{i2}(\mu)A_{22}(\mu)^{-1}B_{j2}(\mu) \quad i,j = 1,2, \\ G_{sk}(\mu) &= G_k(\mu) - C_{22}(\mu)A_{22}(\mu)^{-1}F_{k2}(\mu), \\ E_{sk}(\mu) &= E_{k1}(\mu) - E_{k2}(\mu)A_{22}(\mu)A_{21}(\mu), \\ J_{sk}(\mu) &= J_k(\mu) - E_{k2}(\mu)A_{22}(\mu)^{-1}B_{12}(\mu), \\ H_{sk}(\mu) &= H_k(\mu) - E_{k2}(\mu)A_{22}(\mu)^{-1}F_{k2}(\mu). \end{aligned}$$

We call (7) the slow subsystem. When the slow variable $x_1(t)$ is set to zero in (4), we obtain the following system :

$$\begin{aligned} \dot{x}_2(t) &= A_{22}(\mu)x_2(t) + B_{12}(\mu)w(t) + B_{22}(\mu)u(t) + \sum_{k=1}^{m} F_{k2}(\mu)p_k(t), \\ z(t) &= C_{12}(\mu)x_2(t) + D_{12}(\mu)u(t), \\ y(t) &= C_{22}(\mu)x_2(t) + D_{21}(\mu)w(t) + \sum_{k=1}^{m} G_k(\mu)p_k(t), \\ q_k(t) &= E_{k2}(\mu)x_2(t) + J_k(\mu)w(t) + H_k(\mu)p_k(t). \end{aligned} \quad (8)$$

We call (8) the fast subsystem.

In this work, we will suggest an ε independent H_∞ output feedback controller design method such that the closed loop system is stable and the H_∞ norm from the external noise to the controlled variable is less than a prescribed disturbance attenuation level γ, thus $\int_0^\infty z(\tau)^T z(\tau)d\tau < \gamma^2 \int_0^\infty w(\tau)^T w(\tau)d\tau$.

III. Bounded H_∞ norm conditions for uncertain singularly perturbed T-S fuzzy system

In this section we introduce an H_∞ norm bounded condition for uncertain singularly perturbed T-S fuzzy system. We consider an output feedback controller as in (9).

R_i: IF $\rho_1(t)$ is M_{i1} and $\cdots$ and $\rho_g(t)$ is M_{ig},
THEN

$$\begin{aligned} E_\varepsilon \dot{x}_k(t) &= \sum_{j=1}^{r} \mu_j A_{k,ij} x_k(t) + B_{k,i} y(t), \\ u(t) &= C_{k,i} x_k(t), \end{aligned} \quad (9)$$

where $E_\varepsilon = diag(I_{n_1}, \varepsilon I_{n_2})$. Using the same notation as in (4), the controller can be represented as follows:

$$\begin{aligned} E_\varepsilon \dot{x}_k(t) &= A_k(\mu)x_k(t) + B_k(\mu)y(t) \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i \mu_j (A_{k,ij} x_k(t) + B_{k,i} y(t)), \end{aligned} \quad (10)$$

$$u(t) = C_k(\mu)x_k(t) = \sum_{i=1}^{r} \mu_i C_{k,i} x_k(t).$$

Remark 1 : When $B_{1,2} = \cdots = B_{r,2}$, and $C_{1,2} = \cdots = C_{r,2}$, the controller state equation in (9) can be simplified as follows: $E_\varepsilon \dot{x}_k(t) = A_{k,i} x_k(t) + B_{k,i} y(t)$.

Applying the controller (10) to the system (4), the closed loop system becomes

$$\begin{aligned} E_{\varepsilon\varepsilon} \dot{x}_c(t) &= A_c(\mu)x_c(t) + B_c(\mu)w(t) + F_c(\mu)p(t), \\ z(t) &= [\,C_1(\mu) \quad D_{12}(\mu)C_k(\mu)\,]x_c(t) = C_c(\mu)x_c(t), \\ q(t) &= E_c(\mu)x_c(t) + J_c(\mu)w(t) + H_c(\mu)p(t), \end{aligned} \quad (11)$$

where $E_{\varepsilon\varepsilon} = \begin{bmatrix} E_\varepsilon & 0 \\ 0 & E_\varepsilon \end{bmatrix}$, $x_c(t) = \begin{bmatrix} x(t) \\ x_k(t) \end{bmatrix}$ and

$$\begin{aligned} A_c(\mu) &= \begin{bmatrix} A(\mu) & B_2(\mu)C_k(\mu) \\ B_k(\mu)C_2(\mu) & A_k(\mu) \end{bmatrix}, \\ B_c(\mu) &= \begin{bmatrix} B_1(\mu) \\ B_k(\mu)D_{21}(\mu) \end{bmatrix},\ C_c(\mu) = [\,C_1(\mu) \ \ D_{12}(\mu)C_k(\mu)\,], \\ F_c(\mu) &= \begin{bmatrix} F(\mu) \\ B_k(\mu)G(\mu) \end{bmatrix},\ E_c(\mu) = [E(\mu)\ 0], \\ J_c(\mu) &= J(\mu),\ H_c(\mu) = H(\mu). \end{aligned}$$

Using the coordinate transformation matrix M, the closed loop system (11) can be transformed into (12).

$$\begin{aligned} E_{2\varepsilon}\begin{bmatrix} \dot{\zeta}_1(t) \\ \dot{\zeta}_2(t) \end{bmatrix} &= \overline{A}_c(\mu)\begin{bmatrix} \zeta_1(t) \\ \zeta_2(t) \end{bmatrix} + \overline{B}_c(\mu)w(t) + \overline{F}_c(\mu)p(t) \\ &= \begin{bmatrix} \overline{A}_{c11}(\mu) & \overline{A}_{c12}(\mu) \\ \overline{A}_{c21}(\mu) & \overline{A}_{c22}(\mu) \end{bmatrix}\begin{bmatrix} \zeta_1(t) \\ \zeta_2(t) \end{bmatrix} + \begin{bmatrix} \overline{B}_{c1}(\mu) \\ \overline{B}_{c2}(\mu) \end{bmatrix} w(t) \\ &\quad + \begin{bmatrix} \overline{F}_{c1}(\mu) \\ \overline{F}_{c2}(\mu) \end{bmatrix} p(t), \\ z(t) &= \overline{C}_c(\mu)\zeta(t) = [\,\overline{C}_{c1}(\mu)\ \overline{C}_{c2}(\mu)\,]\begin{bmatrix} \zeta_1(t) \\ \zeta_2(t) \end{bmatrix}, \\ q(t) &= \overline{E}_c(\mu)\zeta(t) + \overline{J}_c(\mu)w(t) + \overline{H}_c(\mu)p(t) \\ &= [\,\overline{E}_{c1}(\mu)\ \overline{E}_{c2}(\mu)\,]\begin{bmatrix} \zeta_1(t) \\ \zeta_2(t) \end{bmatrix} + \overline{J}_c(\mu)w(t) \\ &\quad + \overline{H}_c(\mu)p(t), \end{aligned} \quad (12)$$

where $\zeta(t) = Mx_c(t)$ and

$$M = \begin{bmatrix} I_{n_1} & 0 & 0 & 0 \\ 0 & 0 & I_{n_1} & 0 \\ 0 & I_{n_2} & 0 & 0 \\ 0 & 0 & 0 & I_{n_2} \end{bmatrix} \qquad E_{2\varepsilon} = \begin{bmatrix} I_{2n_1} & 0 \\ 0 & \varepsilon I_{2n_2} \end{bmatrix},$$

$\overline{A}_c(\mu) = MA_c(\mu)M^{-1},\ \overline{B}_c(\mu) = MB_c(\mu),$
$\overline{C}_c(\mu) = C_c(\mu)M^{-1},\ \overline{E}_c(\mu) = E_c(\mu)M^{-1},$
$\overline{F}_c(\mu) = MF_c(\mu),\ \overline{J}_c(\mu) = J_c(\mu),\ \overline{H}_c(\mu) = H_c(\mu).$

The slow subsystem of the system (12) can be expressed as follows:

$$\begin{aligned} \zeta_1(t) &= A_s(\mu)\zeta_1(t) + B_s(\mu)w(t) + F_s(\mu)p(t), \\ z(t) &= C_s(\mu)\zeta_1(t), \\ q(t) &= E_s(\mu)\zeta_1(t) + J_s(\mu)w(t) + H_s(\mu)p(t), \end{aligned} \tag{13}$$

where

$$\begin{aligned} A_s(\mu) &= \overline{A}_{c11}(\mu) - \overline{A}_{c12}(\mu)\overline{A}_{c22}^{-1}(\mu)\overline{A}_{c21}(\mu), \\ B_s(\mu) &= \overline{B}_{c1}(\mu) - \overline{A}_{c12}(\mu)\overline{A}_{c22}^{-1}(\mu)\overline{B}_{c2}(\mu), \\ C_s(\mu) &= \overline{C}_{c1}(\mu) - \overline{C}_{c2}(\mu)\overline{A}_{c22}^{-1}(\mu)\overline{A}_{c21}(\mu), \\ F_s(\mu) &= \overline{F}_{c1}(\mu) - \overline{A}_{c12}(\mu)\overline{A}_{c22}^{-1}(\mu)\overline{F}_{c2}(\mu), \\ E_s(\mu) &= \overline{E}_{c1}(\mu) - \overline{E}_{c2}(\mu)\overline{A}_{c22}^{-1}(\mu)\overline{A}_{c21}(\mu), \\ J_s(\mu) &= \overline{J}_c(\mu) - \overline{E}_{c2}(\mu)\overline{A}_{c22}^{-1}(\mu)\overline{B}_{c2}(\mu), \\ H_s(\mu) &= \overline{H}_c(\mu) - \overline{E}_{c2}(\mu)\overline{A}_{c22}^{-1}(\mu)\overline{F}_{c2}(\mu). \end{aligned}$$

The fast subsystem of system (12) can be described as follows:

$$\begin{aligned} \zeta_2(t) &= \overline{A}_{c22}(\mu)\zeta_2(t) + \overline{B}_{c2}(\mu)w(t) + \overline{F}_{c2}(\mu)p(t), \\ z(t) &= \overline{C}_{c2}(\mu)\zeta_2(t), \\ q(t) &= \overline{E}_{c2}(\mu)\zeta_2(t) + \overline{J}_c(\mu)w(t) + \overline{H}_c(\mu)p(t). \end{aligned} \tag{14}$$

Let T_{zw}^s be the operator from $w(t)$ to $z(t)$ in the slow subsystem (14). If there exist $P_s = P_s^T > 0$, and $S = S^T > 0$, $R = R^T > 0$ satisfying LMI (15) for all μ, then $\|T_{zw}^s\|_\infty < \gamma$ for all uncertainties satisfying the IQC's.

$$\begin{bmatrix} A_s(\mu)^T P_s + P_s A_s(\mu) & * & * & * & * \\ B_s(\mu)^T P_s & -\gamma I & * & * & * \\ F_s(\mu)^T P_s & 0 & -S & * & * \\ C_s(\mu) & 0 & 0 & -\gamma I & * \\ E_s(\mu) & J_s(\mu) & H_s(\mu) & 0 & -R \end{bmatrix} < 0, \tag{15}$$
$SR = I.$

Let T_{zw}^f be the operator from $w(t)$ to $z(t)$ in the fast subsystem (14). If there exist $P_f = P_f^T > 0$, and $S = S^T > 0$, $R = R^T > 0$ satisfying LMI (16) for all μ, then $\|T_{zw}^f\|_\infty < \gamma$ for all uncertainties satisfying the IQC's.

$$\begin{bmatrix} \overline{A}_{c22}(\mu)^T P_f + P_f \overline{A}_{c22}(\mu) & * & * & * & * \\ \overline{B}_{c2}(\mu)^T P_f & -\gamma I & * & * & * \\ \overline{F}_{c2}(\mu)^T P_f & 0 & -S & * & * \\ \overline{C}_{c2}(\mu) & 0 & 0 & -\gamma I & * \\ \overline{E}_{c2}(\mu) & \overline{J}_c(\mu) & \overline{H}_c(\mu) & 0 & -R \end{bmatrix} < 0, \tag{16}$$
$SR = I.$

Lemma 1[7]: When the symmetric matrix Ω and the matrix U, V is given, condition (i) and (ii) are equivalent.

(i) There exists K satisfying the following LMI.
$\Omega + UKV + (UKV)^T < 0.$
(ii) $U^{\perp T}\Omega U^{\perp} < 0,\ V^{T\perp T}\Omega V^{T\perp} < 0.$

Lemma 2: The next two conditions are equivalent.

(i) There exist symmetric positive definite matrices P_s, P_f, S and R satisfying LMI's (15) and (16) for all $\mu(t)$.
(ii) There exist a matrix $\overline{P}$ and symmetric positive definite matrices S and R satisfying LMI (17) for all $\mu(t)$.

$$\begin{bmatrix} \overline{A}_c(\mu)^T \overline{P} + \overline{P}^T \overline{A}_c(\mu) & * & * & * & * \\ \overline{B}_c(\mu)^T \overline{P} & -\gamma I & * & * & * \\ \overline{F}_c(\mu)^T \overline{P} & 0 & -S & * & * \\ \overline{C}_c(\mu) & 0 & 0 & -\gamma I & * \\ \overline{E}_c(\mu) & \overline{J}_c(\mu) & \overline{H}_c(\mu) & 0 & -R \end{bmatrix} < 0. \tag{17}$$

where $\begin{bmatrix} \overline{P}_1 & 0 \\ \overline{P}_2 & \overline{P}_3 \end{bmatrix}$, $\overline{P}_1 = \overline{P}_1^T > 0$, $\overline{P}_3 = \overline{P}_3^T > 0$.

(proof) LMI (17) can be expressed as follows:

$$\Omega + U\overline{P}_2 V + (U\overline{P}_2 V)^T < 0, \tag{18}$$

where

$$\Omega = \begin{bmatrix} \Omega_{11} & * & * & * & * & * \\ \Omega_{21} & \Omega_{22} & * & * & * & * \\ \overline{B}_{c1}(\mu)^T \overline{P}_1(\mu) & \overline{B}_{c2}(\mu)^T \overline{P}_3 & -\gamma I & * & * & * \\ \overline{F}_{c1}(\mu)^T \overline{P}_1(\mu) & \overline{F}_{c2}(\mu)^T \overline{P}_3 & 0 & -S & * & * \\ \overline{C}_{c1}(\mu) & \overline{C}_{c2}(\mu) & 0 & 0 & -\gamma I & * \\ \overline{E}_{c1}(\mu) & \overline{E}_{c2}(\mu) & \overline{J}_c(\mu) & \overline{H}_c(\mu) & 0 & -R \end{bmatrix} < 0$$
,

$U^T = \left[\overline{A}_{c21}(\mu)\ \overline{A}_{c22}(\mu)\ \overline{B}_{c21}(\mu)\ \overline{F}_{c21}(\mu)\ 0\ 0\right],$
$V = [I\ 0\ 0\ 0\ 0\ 0],$
$\Omega_{11} = \overline{A}_{c11}(\mu)^T \overline{P}_1 + \overline{P}_1 \overline{A}_{c11}(\mu),$
$\Omega_{21} = \overline{A}_{c12}(\mu)^T \overline{P}_1 + \overline{P}_3 \overline{A}_{c21}(\mu),$
$\Omega_{22} = \overline{A}_{c22}(\mu)^T \overline{P}_3 + \overline{P}_3 \overline{A}_{c22}(\mu).$

We can write $U^{\perp}$ and $V^{T\perp}$ as follows:

$$U^{\perp}=\begin{bmatrix} I & 0 & 0 & 0 & 0 \\ u_{21} & u_{22} & u_{23} & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}, \quad V^{T\perp}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}, \qquad (19)$$

where

$$u_{21}=-\overline{A}_{c21}(\mu)^T(\overline{A}_{c22}(\mu)^{-1})^T,$$
$$u_{22}=-\overline{B}_{c2}(\mu)^T(\overline{A}_{c22}(\mu)^{-1})^T,$$
$$u_{23}=-\overline{F}_{c2}(\mu)^T(\overline{A}_{c22}(\mu)^{-1})^T$$

Using $U^{\perp}$ and $V^{T\perp}$ defined in (19), we obtain

$$U^{\perp\,T}\Omega U^{\perp}=$$
$$\begin{bmatrix} A_s(\mu)^T\overline{P}_1+\overline{P}_1A_s(\mu) & * & * & * & * \\ B_s(\mu)^T\overline{P}_1 & -\gamma I & * & * & * \\ F_s(\mu)^T\overline{P}_1 & 0 & -S & * & * \\ C_s(\mu) & 0 & 0 & -\gamma I & * \\ E_s(\mu) & J_s(\mu) & H_s(\mu) & 0 & -R \end{bmatrix}<0 \qquad (20)$$

$$V^{T\perp\,T}\Omega V^{T\perp}=$$
$$\begin{bmatrix} \overline{A}_{c22}(\mu)^T\overline{P}_3+\overline{P}_3\overline{A}_{c22}(\mu) & * & * & * & * \\ \overline{B}_{c2}(\mu)^T\overline{P}_3 & -\gamma I & * & * & * \\ \overline{F}_{c2}(\mu)^T\overline{P}_3 & 0 & -S & * & * \\ \overline{C}_{c2}(\mu) & 0 & 0 & -\gamma I & * \\ \overline{E}_{c2}(\mu) & \overline{J}_c(\mu) & \overline{H}_c(\mu) & 0 & -R \end{bmatrix}<0 \qquad (21)$$

From lemma 1, we complete the proof.

Lemma 3: If LMI (18) has a feasible solution, there exists an $\bar{\varepsilon}$ such that $\|T_{zw}\|_{\infty}<\gamma$ for all $0<\varepsilon<\bar{\varepsilon}$, where T_{zw} is the operator from $w(t)$ to $z(t)$ in the full order singularly perturbed system (12).
(Proof) In (12) we define

$$\overline{A}_c(\mu,\varepsilon)=\begin{bmatrix} \overline{A}_{c11}(\mu) & \overline{A}_{c12}(\mu) \\ \overline{A}_{c21}(\mu)/\varepsilon & \overline{A}_{c22}(\mu)/\varepsilon \end{bmatrix}, \ \overline{B}_c(\mu,\varepsilon)=\begin{bmatrix} \overline{B}_{c1}(\mu) \\ \overline{B}_{c2}(\mu)/\varepsilon \end{bmatrix},$$
$$\overline{F}_c(\mu,\varepsilon)=\begin{bmatrix} \overline{F}_{c1}(\mu) \\ \overline{F}_{c2}(\mu)/\varepsilon \end{bmatrix}.$$

The solvability of LMI (22) guarantees $\|T_{zw}\|_{\infty}<\gamma$ in (12).

$$\begin{bmatrix} \overline{A}_c(\mu,\varepsilon)^TP_{\varepsilon}+P_{\varepsilon}\overline{A}_c(\mu,\varepsilon) & * & * & * & * \\ \overline{B}_c(\mu)^TP_{\varepsilon} & -\gamma I & * & * & * \\ \overline{F}_c(\mu)^TP_{\varepsilon} & 0 & -S_{\varepsilon} & * & * \\ \overline{C}_c(\mu) & 0 & 0 & -\gamma I & * \\ \overline{E}_c(\mu) & \overline{J}_c(\mu) & \overline{H}_c(\mu) & 0 & -R_{\varepsilon} \end{bmatrix}<0. \qquad (22)$$

We seek P_{ε}, S_{ε} and R_{ε} from solutions of LMI (18). Let's $\overline{P}=\begin{bmatrix} \overline{P}_1 & 0 \\ \overline{P}_2 & \overline{P}_3 \end{bmatrix}$, S and R be any feasible solutions of LMI (18). We define $P_{\varepsilon}=\begin{bmatrix} \overline{P}_1 & \varepsilon\overline{P}_2 \\ \varepsilon\overline{P}_2 & \varepsilon\overline{P}_3 \end{bmatrix}$, $S_{\varepsilon}=S$ and $R_{\varepsilon}=R$.

Since $\overline{P}_1$ and $\overline{P}_3$ are positive definite matrices there exists ε_1 such that P_{ε} is positive definite for all $0<\varepsilon<\varepsilon_1$. Substituting P_{ε}, S_{ε} and R_{ε} into (22) leads to

$$\begin{bmatrix} \overline{A}_c(\mu,\varepsilon)^TP_{\varepsilon}+P_{\varepsilon}\overline{A}_c(\mu,\varepsilon) & * & * & * & * \\ \overline{B}_c(\mu)^TP_{\varepsilon} & -\gamma I & * & * & * \\ \overline{F}_c(\mu)^TP_{\varepsilon} & 0 & -S_{\varepsilon} & * & * \\ \overline{C}_c(\mu) & 0 & 0 & -\gamma I & * \\ \overline{E}_c(\mu) & \overline{J}_c(\mu) & \overline{H}_c(\mu) & 0 & -R_{\varepsilon} \end{bmatrix}=$$
$$\begin{bmatrix} \overline{A}_c(\mu)^T\overline{P}+\overline{P}^T\overline{A}_c(\mu) & * & * & * & * \\ \overline{B}_c(\mu)^T\overline{P} & -\gamma I & * & * & * \\ \overline{F}_c(\mu)^T\overline{P} & 0 & -S & * & * \\ \overline{C}_c(\mu) & 0 & 0 & -\gamma I & * \\ \overline{E}_c(\mu) & \overline{J}_c(\mu) & \overline{H}_c(\mu) & 0 & -R \end{bmatrix}+$$
$$\varepsilon\begin{bmatrix} 0 & * & * & * & * & * \\ \overline{P}_2\overline{A}_{c11}(\mu) & \overline{A}_{c12}(\mu)^T\overline{P}_2^T+\overline{P}_2\overline{A}_{c12}(\mu) & * & * & * & * \\ 0 & \overline{B}_{c1}(\mu)^T\overline{P}_2^T & 0 & * & * & * \\ 0 & \overline{F}_{c1}(\mu)^T\overline{P}_2^T & 0 & 0 & * & * \\ 0 & 0 & 0 & 0 & 0 & * \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \qquad (23)$$

Since the first term of the right hand side of (23) is negative definite, there exists an ε_2 such that P_{ε}, S_{ε} and R_{ε} satisfy LMI (22) for all $0<\varepsilon<\varepsilon_2$. By choosing $\bar{\varepsilon}=\min(\varepsilon_1,\varepsilon_2)$ the proof is completed.

IV. Design of Output Feedback Controller

In this section, we suggest an output feedback controller design method based on lemma 3. By pre and post-multiplying LMI (17) by $diag(M^T,I,I)$ and $diag(M,I,I)$, we obtain

$$\begin{bmatrix} A_c(\mu)^TP+PA_c(\mu) & * & * & * & * \\ B_c(\mu)^TP & -\gamma I & * & * & * \\ F_c(\mu)^TP & 0 & -S & * & * \\ C_c(\mu) & 0 & 0 & -\gamma I & * \\ E_c(\mu) & J_c(\mu) & H_c(\mu) & 0 & -R \end{bmatrix}<0, \qquad (24)$$

where

$$P=M^T\overline{P}M=\begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix}. \qquad (25)$$

From the relationship of $\overline{P}$ and P defined in (17) and (25), the sub-matrices $P_i(i=1,\cdots,4)$ must have the structure as in (26) and (27).

$$P_i=\begin{bmatrix} P_{i1} & 0 \\ P_{i2} & P_{i3} \end{bmatrix}, \quad (i=1,\cdots,4), \qquad (26)$$

$$\begin{bmatrix} P_{11} & P_{21} \\ P_{31} & P_{41} \end{bmatrix} = \begin{bmatrix} P_{11} & P_{21} \\ P_{31} & P_{41} \end{bmatrix}^T > 0, \quad \begin{bmatrix} P_{13} & P_{23} \\ P_{33} & P_{43} \end{bmatrix} = \begin{bmatrix} P_{13} & P_{23} \\ P_{33} & P_{43} \end{bmatrix}^T > 0. \quad (27)$$

We introduce a technical lemma necessary for controller design.

Lemma 4: Suppose that P_1, Q_1 have the same matrix structure defined in (26) and (27) and $I - P_{11}Q_{11} < 0$, $I - P_{13}Q_{13} < 0$. Then there exist P and Q satisfying the following two conditions.

(i) All the sub-matrices have the same structure as in (25)-(27).

(ii) $PQ = I$.

(proof) We choose an arbitrary invertible matrices P_2 and P_3 with the following structures.

$$P_2 = \begin{bmatrix} P_{21} & 0 \\ P_{22} & P_{23} \end{bmatrix}, \; P_3 = \begin{bmatrix} P_{31} & 0 \\ P_{32} & P_{33} \end{bmatrix}, \; P_{21} = P_{31}^T, P_{23} = P_{33}^T. \quad (28)$$

We define P_4, Q_2, Q_3 and Q_4 as follows:

$$Q_2 = (I - Q_1P_1)P_3^{-1}, \; Q_3 = P_2^{-1}(I - P_1Q_1), \\ P_4 = -P_3Q_1(I - P_1Q_1)^{-1}P_2, \; Q_4 = (P_4 - P_3P_1^{-1}P_2)^{-1}. \quad (29)$$

Let P and Q be $P = \begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix}$, $Q = \begin{bmatrix} Q_1 & Q_2 \\ Q_3 & Q_4 \end{bmatrix}$. One can easily know $PQ = I$. This completes the proof.

From (11) we have

$$A_c(\mu) = \sum_{i=1}^r \sum_{j=1}^r \mu_i\mu_j A_{cij} = \sum_{i=1}^r \sum_{j=1}^r \mu_i\mu_j \begin{bmatrix} A_i & B_{i2}C_{kj} \\ B_{kj}C_{i2} & A_{kij} \end{bmatrix},$$

$$B_c(\mu) = \sum_{i=1}^r \sum_{j=1}^r \mu_i\mu_j B_{cij} = \sum_{i=1}^r \sum_{j=1}^r \mu_i\mu_j \begin{bmatrix} B_{i1} \\ B_{kj}D_{i21} \end{bmatrix},$$

$$C_c(\mu) = \sum_{i=1}^r \sum_{j=1}^r \mu_i\mu_j C_{cij} = \sum_{i=1}^r \sum_{j=1}^r \mu_i\mu_j \begin{bmatrix} C_{i1} & D_{i12}C_{kj} \end{bmatrix},$$

$$F_c(\mu) = \sum_{i=1}^r \sum_{j=1}^r \mu_i\mu_j F_{cij} = \sum_{i=1}^r \sum_{j=1}^r \mu_i\mu_j \begin{bmatrix} F_j \\ B_{kj}G_i \end{bmatrix},$$

$$E_c(\mu) = \sum_{i=1}^r \sum_{j=1}^r \mu_i\mu_j E_{cij} = \sum_{i=1}^r \sum_{j=1}^r \mu_i\mu_j \begin{bmatrix} E_i & 0 \end{bmatrix},$$

$$J_c(\mu) = \sum_{i=1}^r \sum_{j=1}^r \mu_i\mu_j J_{cij} = \sum_{i=1}^r \sum_{j=1}^r \mu_i\mu_j J_i,$$

$$H_c(\mu) = \sum_{i=1}^r \sum_{j=1}^r \mu_i\mu_j H_{cij} = \sum_{i=1}^r \sum_{j=1}^r \mu_i\mu_j H_i.$$

LMI (24) can be written as follows:

$$\sum_{i=1}^r \sum_{j=1}^r \mu_i\mu_j \Phi_{ij} = \\ \sum_{i=1}^r \sum_{j=1}^r \mu_i\mu_j \begin{bmatrix} A_{cij}^T P + PA_{cij} & * & * & * & * \\ B_{cij}^T P & -\gamma I & * & * & * \\ F_{cij}^T P & 0 & -S & * & * \\ C_{cij} & 0 & 0 & -\gamma I & * \\ E_{cij} & J_{cij} & H_{cij} & 0 & -R \end{bmatrix} < 0. \quad (30)$$

It is well known that the solvability of LMI (31) guarantees the solvability of LMI (30) for all permissible $\mu(t)$.

$$\Phi_{ii} < 0 \; (i = 1, \cdots, r) \qquad \Phi_{ij} + \Phi_{ji} < 0, \; (i = 1, \cdots, r, \; j > i). \quad (31)$$

Theorem 5: Suppose that P_1, Q_1 with matrix structure defined in lemma 4 and $S = S^T = R^{-1} > 0$, $\hat{A}_{ij}$, $\hat{B}_j$, $\hat{C}_j$ $(i, j = 1, \cdots, r)$ solve LMI's (32). There exists a sufficiently small $\bar{\varepsilon}$ such that the design objective is achieved in an uncertain singularly perturbed system (1) for all $0 < \varepsilon < \bar{\varepsilon}$.

$$\Psi_{ii} < 0, \; \Psi_{ij} + \Psi_{ji} < 0, \; i, j = 1, \cdots, r, \; j >, \quad (32)$$

where

$$\Psi_{ij} = \begin{bmatrix} \Lambda_1 & * & * & * & * & * \\ A_i^T + \hat{A}_{ij} & \Lambda_2 & * & * & * & * \\ B_{i1}^T & B_{i1}^T P_1 + D_{i21}^T \hat{B}_j^T & -\gamma I & * & * & * \\ F_j^T & F_j^T P_1 + G_i^T \hat{B}_j^T & 0 & -S & * & * \\ C_{i1}Q_1 + D_{i12}\hat{C}_j & C_{i1} & 0 & 0 & -\gamma I & * \\ E_iQ_1 & E_i & J_i & H_i & 0 & -R \end{bmatrix}.$$

$$\Lambda_1 = A_iQ_1 + Q_1^T A_i^T + \hat{C}_j^T B_{i2}^T + B_{kj}\hat{C}_j, \quad (33)$$

$$\Lambda_2 = A_i^T P_1 + P_1^T A_i + C_{i2}^T \hat{B}_j^T + \hat{B}_j C_{i2}.$$

(proof) From P_1 and Q_1, P and Q can be constructed from lemma 4. We define Π_1 and Π_2 as follows :

$$\Pi_1 = \begin{bmatrix} Q_1 & I \\ Q_3 & 0 \end{bmatrix}, \; \Pi_2 = \begin{bmatrix} I & P_1 \\ 0 & P_3 \end{bmatrix}. \quad (34)$$

Note that $P\Pi_1 = \Pi_2$. By direct matrix manipulation we obtain

$$\Psi_{ij} = \begin{bmatrix} \Pi_1^T & * & * & * & * \\ 0 & I & * & * & * \\ 0 & 0 & I & * & * \\ 0 & 0 & 0 & I & * \\ 0 & 0 & 0 & 0 & I \end{bmatrix} \Phi_{ij} \begin{bmatrix} \Pi_1 & * & * & * & * \\ 0 & I & * & * & * \\ 0 & 0 & I & * & * \\ 0 & 0 & 0 & I & * \\ 0 & 0 & 0 & 0 & I \end{bmatrix}$$

$$= \begin{bmatrix} \Lambda_1 & * & * & * & * & * \\ A_i^T + \hat{A}_{ij} & \Lambda_2 & * & * & * & * \\ B_{i1}^T & B_{i1}^T P_1 + D_{i21}^T \hat{B}_j^T & -\gamma I & * & * & * \\ F_j^T & F_j^T P_1 + G_i^T \hat{B}_j^T & 0 & -S & * & * \\ C_{i1}Q_1 + D_{i12}\hat{C}_j & C_{i1} & 0 & 0 & -\gamma I & * \\ E_iQ_1 & E_i & J_i & H_i & 0 & -R \end{bmatrix}, \quad (35)$$

where $\hat{A}_{ij} = P_1^T A_i Q_1 + \hat{B}_j C_{i2} Q_1 + P_1^T B_{i2} \hat{C}_j + P_3^T A_{kij} Q_3$, $\hat{B}_j = P_3^T B_{kj}$, $\hat{C}_j = C_{kj} Q_3$.

Now the state space realization $(A_{kij}, B_{kj}, C_{kj})$, $(i, j = 1, \cdots, r)$ of desired output feedback controller can be calculated from $\hat{A}_{ij}$, $\hat{B}_j$, $\hat{C}_j$ in (35). This completes the proof.

Lemma 6: When $B_{12} = \cdots = B_{r2}$, $C_{12} = \cdots = C_{r2}$, Suppose that P_1, Q_1 with matrix structure defined in lemma 4 and $S=S^T=R^{-1}>0$, $\hat{A}_i$, $\hat{B}_i$, $\hat{C}_i$ $(i=1,\cdots,r)$ solve LMI's (33). Then for a sufficiently small $\varepsilon>0$, there exists an output feedback controller guaranteeing $\|T_{zw}\|_\infty<\gamma$ in the full system (4).

$$\Psi_i<0,\ i=1,\cdots,r \tag{33}$$

where

$$\Psi_i=\begin{bmatrix}\Xi_1 & * & * & * & * & * \\ A_i^T+\hat{A}_i & \Xi_2 & * & * & * & * \\ B_{i1}^T & B_{i1}^TP_1+D_{i21}^T\hat{B}_i^T-\gamma I & * & * & * \\ F_i^T & F_i^TP_1+G_i^T\hat{B}_i^T & 0 & -S & * & * \\ C_{i1}Q_1+D_{i12}\hat{C}_i & C_{i1} & 0 & 0 & -\gamma I & * \\ E_iQ_1 & E_i & J_i & H_i & 0 & -R\end{bmatrix},$$

$$\Xi_1=A_iQ_1+Q_1^TA_i^T+\hat{C}_i^TB_{i2}^T+B_{ki}\hat{C}_i, \tag{34}$$

$$\Xi_2=A_i^TP_1+P_1^TA_i+C_{i2}^T\hat{B}_i^T+\hat{B}_iC_{i2},$$
$$\hat{A}_i=P_1^TA_iQ_1+\hat{B}_iC_{i2}Q_1+P_1^TB_{i2}\hat{C}_i+P_3^TA_{ki}Q_3$$
$$\hat{B}_i=P_3^TB_{ki},\ \hat{C}_i=C_{ki}Q_3.$$

The state space realization (A_{ki}, B_{ki}, C_{ki}), $(i=1,\cdots,r)$ of the output feedback controller can be calculated from $\hat{A}_i$, $\hat{B}_i$, $\hat{C}_i$.

Remark 2: The main advantage of singular perturbation method in control problem is to avoid numerical ill-conditioning problems resulting from a small parasitic term ε. In this work, we present an ε-independent controller design method to avoid numerical ill-conditioning problems.

V. A numerical Example

In this section, we present a numerical example to illustrate the efficacy of the proposed design method. We consider an inverted pendulum controlled by a dc motor via a gear train which can be described by the following singularly perturbed T-S fuzzy system [15]-[16]:

R_1: If $x_1(t)$ is M_1,

THEN

$$\begin{aligned}E_\varepsilon\dot{x}(t)&=A_1x(t)+B_{1,1}w(t)+B_{1,2}u(t)+F_1p(t),\\ z(t)&=C_{1,1}x(t)+D_{1,12}u(t),\\ y(t)&=C_{1,2}x(t)+D_{1,21}w(t)+G_1p(t),\\ q(t)&=E_1x(t).\end{aligned} \tag{35}$$

R_2: If $x_1(t)$ is M_2,

THEN

$$\begin{aligned}E_\varepsilon\dot{x}(t)&=A_2x(t)+B_{2,1}w(t)+B_{2,2}u(t)+F_2p(t),\\ z(t)&=C_{2,1}x(t)+D_{2,12}u(t),\\ y(t)&=C_{2,2}x(t)+D_{2,21}w(t)+G_2p(t),\\ q(t)&=E_2x(t),\end{aligned} \tag{36}$$

where the membership function $M_1=1-|x_1(t)|$, $M_2=1-M_1$ and

$$E_\varepsilon=\begin{bmatrix}1&0&0\\0&1&0\\0&0&\varepsilon\end{bmatrix},\quad A_1=\begin{bmatrix}0&1&0\\9.8&0&1\\0&-1&-1\end{bmatrix},\quad A_2=\begin{bmatrix}0&1&0\\0&0&1\\0&-1&-1\end{bmatrix},$$

$$B_{1,1}=B_{2,1}=\begin{bmatrix}0.1\\0\\0\end{bmatrix},B_{1,2}=B_{2,2}=\begin{bmatrix}0\\0\\1\end{bmatrix},\quad C_{1,1}=C_{2,1}=\begin{bmatrix}0.1\\0\\0\end{bmatrix}^T,$$

$$C_{1,2}=C_{2,2}=\begin{bmatrix}1\\0\\0\end{bmatrix}^T,\ F_1=F_2=\begin{bmatrix}0\\1\\0\end{bmatrix},\quad G_1=G_2=0,$$

$$D_{1,12}=D_{2,12}=D_{1,21}=D_{2,21}=0.1,\ E_1=E_2=[0\ 0\ 0.1].$$

When $\gamma=0.5$ and $R=1$, solving LMI's (33) gives

$$P_1=\begin{bmatrix}2.5172e+1 & -6.4124e+0 & 0\\ -6.4124e+0 & 2.4170e+0 & 0\\ 5.9783e+0 & 3.9826e+2 & 3.9804e+2\end{bmatrix},$$

$$Q_1=\begin{bmatrix}2.0705e+0 & -5.7032e+0 & 0\\ -5.7032e+0 & 1.9998e+1 & 0\\ -1.3214e+0 & -1.0295e+2 & 1.1155e+2\end{bmatrix},$$

so that we obtain a controller as

$$A_{k,1}=\begin{bmatrix}-3.1807e+3 & -1.0289e+4 & -3.7080e-2\\ -1.9710e+5 & -6.3860e+5 & -1.3896e+0\\ -1.9704e+5 & -6.3840e+5 & -1.3949e+0\end{bmatrix},$$

$$A_{k,2}=\begin{bmatrix}-2.8389e+3 & -9.1858e+3 & -3.7066e-2\\ -1.9704e+5 & -6.3842e+5 & -1.3886e+0\\ -1.9681e+5 & -6.3765e+5 & -1.3939e+0\end{bmatrix},$$

$$B_{k,1}=\begin{bmatrix}-6.7547e+1\\ 4.9099e+0\\ 3.5250e-1\end{bmatrix},\ B_{k,2}=\begin{bmatrix}-7.2651e+1\\ 5.8382e+0\\ 3.2536e-1\end{bmatrix},$$

$$C_{k,1}=[4.9514e+2\ \ 1.6042e+3\ \ 9.9208e-4],$$

$$C_{k,2}=[4.9455e+2\ \ 1.6023e+3\ \ 9.8962e-4].$$

In order to see if the above controller meets the design specification, we check the solvability of LMI's (22) for the full order closed loop system. We can obtain a positive definite solution P_ε when $\gamma=0.5$, $\varepsilon\leqq 4.4\times10^{-5}$.

VI. Conclusion

In this work we have suggested a robust H_∞ output feedback controller design method for uncertain singularly perturbed T-S fuzzy systems. When the H_∞ norm of both slow and fast sub-systems of uncertain singularly perturbed system are less than γ, we proved that the H_∞ norm of the full system is also less than γ for a sufficiently small parasitic term ε. A sufficient condition guaranteing the existence of an output feedback controller satisfying the design specification has been presented in terms of ε independent linear matrix inequalities which may alleviate a numerical ill- conditioning problems resulting from the extremely small ε.

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