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Interval-Valued Fuzzy m-semicontinuous 함수의 특성 연구

Characterizations For Interval-Valued Fuzzy m-semicontinuous Mappings On Interval-Valued Fuzzy Minimal Spaces

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요 약

[5]에서 정의된 IVF m-semiopen 집합과 IVF m-semicontinuous 함수의 기본적 성질과 특성을 조사한다.

Abstract

In [5], we introduced the concepts of IVF m-semiopen sets and IVF m-semicontinuous mappings on interval-valued fuzzy minimal spaces. In this paper, we investigate some properties of IVF m-semiopen sets and characterizations for the IVF m-semicontinuous mapping.

Key Words : interval-valued fuzzy minimal spaces, IVF m-semiopen sets, IVF m-semicontinuous

1. Intorduction and Preliminaries

Zadeh [7] introduced the concept of fuzzy set and several researchers were concerned about the generalizations of the concepts of fuzzy sets, intuitionistic fuzzy sets [1] and interval-valued fuzzy sets [3]. Alimohammady and Roohi [2] introduced fuzzy minimal structures and fuzzy minimal spaces and some results are given. In [4], Min introduced the concepts of IVF minimal structures and IVF m-continuous mappings which are generalizations of IVF topologies and IVF continuous mappings [6], respectively. In [5], Min et al. introduced the concepts of IVF m-semiopen sets and IVF m-semicontinuous mappings on interval-valued fuzzy minimal spaces. We investigated basic properties of IVF m-semiopen sets and IVF m-semicontinuous mappings. In this paper, we investigate characterizations for the IVF m-semicontinuous mapping and some properties of IVF m-semiopen sets.

Let $D[0,1]$ be the set of all closed subintervals of the interval $[0,1]$. The elements of $D[0,1]$ are generally denoted by capital letters $M, N, \cdots$ and note that $M=[M^L, M^U]$, where M^L

and M^U are the lower and the upper end points respectively. We also note that

(1) $(\forall M, N \in D[0,1])$

$(M=N \Leftrightarrow M^L=N^L,\ M^U=N^U)$.

(2) $(\forall M, N \in D[0,1])$

$(M \le N \Leftrightarrow M^L \le N^L,\ M^U \le N^U)$.

For each $M \in D[0,1]$, the complement of M, denoted by M^c, is defined by $M^c=[1-M^U, 1-M^L]$.

Let X be a nonempty set. A mapping $A: X \to D[0,1]$ is called an *interval-valued fuzzy set* (simply, IVF set) in X. For each $x \in X$, $A(x)$ is a closed interval whose lower and upper end points are denoted by $A(x)^L$ and $A(x)^U$, respectively. For any $[a,b] \in D[0,1]$, the IVF set whose value is the interval $[a,b]$ for all $x \in X$ is denoted by $\widetilde{[a,b]}$. We denote $\tilde{0}$ and $\tilde{1}$ as follows: $\tilde{0}=[0,0]$, $\tilde{1}=[1,1]$. In particular, for any $c \in [a,b]$, the IVF set whose value is $c(x)=[c,c]$ for all $x \in X$ is denoted by simply $\tilde{c}$. For a point $p \in X$ and for $[a,b] \in D[0,1]$ with $b>0$, the IVF set which takes the value $[a,b]$ at p and $\tilde{0}$ elsewhere in X is called an *interval-valued fuzzy point* (simply, IVF point) and is denoted by $[a,b]_p$. In particular, if $b=a$, then it is also denoted by a_p. We denote the set of all IVF sets in X by IVF(X). An IVF point M_x, where $M \in D[0,1]$, is said to belong to an IVF set A in X, denoted by $M_x \tilde{\in} A$, if $A(x)^L \ge M^L$ and $A(x)^U \ge M^U$. In [6], it has been shown that $A=\cup\{M_x : M_x \tilde{\in} A\}$.

For every $A, B \in IVF(X)$, we define

$A=B \Leftrightarrow (\forall x \in X)(A(x)^L=B(x)^L,\ A(x)^U=B(x)^U)$.

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$A \subseteq B \Leftrightarrow (\forall x \in X)(A(x)^L \subseteq B(x)^L,\ A(x)^U \subseteq B(x)^U)$.

The complement A^c of A is defined by

$[A^c(x)]^L = 1 - A(x)^U$ and $[A^c(x)]^U = 1 - A(x)^L$ for all $x \in X$.

For a family of IVF sets $\{A_i : i \in J\}$ where J is an index set, the union $G = \cup_{i \in J} A_i$ and $F = \cap_{i \in J} A_i$ are defined by

$$G(x)^L = sup_{i \in J}[A_i(x)]^L,\ G(x)^U = sup_{i \in J}[A_i(x)]^U$$

and

$$F(x)^L = inf_{i \in J}[A_i(x)]^L,\ F(x)^U = inf_{i \in J}[A_i(x)]^U,$$

respectively, for all $x \in X$.

Let $f : X \to Y$ be a mapping and let A be an IVF set in X. Then the image of A under f, denoted by $f(A)$, defined as follows

$$[f(A)(y)]^L = \begin{cases} sup_{z \in f^{-1}(y)}[A(z)]^L, & \text{if } f^{-1}(y) \neq \varnothing, \\ 0, & \text{otherwise}, \end{cases}$$

$$[f(A)(y)]^U = \begin{cases} sup_{z \in f^{-1}(y)}[A(z)]^U, & \text{if } f^{-1}(y) \neq \varnothing, \\ 0, & \text{otherwise}, \end{cases}$$

for all $y \in Y$.

Let B be an IVF set in Y. Then the inverse image of B under f, denoted by $f^{-1}(B)$, defined as follows

$$[f^{-1}(B(x))]^L = [B(f(x))]^L,\ [f^{-1}(B(x))]^U = [B(f(x))]^U$$

for all $x \in X$.

Definition 1.1 ([6]). A family τ of IVF sets in X is called an *interval-valued fuzzy topology* on X if it satisfies the following properties:

(1) $\tilde{0},\ \tilde{1} \in \tau$.
(2) $A,\ B \in \tau \Rightarrow A \cap B \in \tau$.
(3) For $i \in J$, $A_i \in \tau \Rightarrow \cup_{i \in J} A_i \in \tau$.

Every member of τ is called an *IVF open set*. An IVF set A is called an *IVF closed set* if the complement of A is an IVF open set. And the pair (X,τ) is alled an *interval-valued fuzzy topological space.*

Definition 1.2 ([4]). A family M of interval-valued fuzzy sets in X is called an *interval-valued fuzzy minimal structure* on X if

$$\tilde{0},\ \tilde{1} \in M.$$

In this case, (X,M) is called an *interval-valued fuzzy minimal space* (simply, IVF *minimal space*). Every member of M is called an IVF m-open set. An IVF set A is called an IVF m-closed set if the complement of A (simply, A^c) is an IVF m-open set.

Let (X,M) be an IVF minimal space and $A \in$ IVF(X). The IVF minmal-closure and the IVF minimal-interior of A [6], denoted by $mC(A)$ and $mI(A)$, respectively, are defined as

$$mC(A) = \cap\{B \in \text{IVF}(X) : B^c \in M \text{ and } A \subseteq B\},$$

$$mI(A) = \cup\{B \in \text{IVF}(X) : B \in M \text{ and } B \subseteq A\}.$$

Theorem 1.3 ([4]). Let (X,M) be an IVF minimal space and $A,B \in$ IVF(X). Then the following properties hold:

(1) $mI(A) \subseteq A$ and if A is an IVF m-open set, then $mI(A)=A$.
(2) $A \subseteq mC(A)$ and if A is an IVF m-closed set, then $mC(A)=A$.
(3) If $A \subseteq B$, then $mI(A) \subseteq mI(B)$ and $mC(A) \subseteq mC(B)$.
(4) $mI(A \cap B) \subseteq mI(A) \cap mI(B)$ and $mC(A) \cup mC(B) \subseteq mC(A \cup B)$.
(5) $mI(mI(A))=mI(A)$ and $mC(mC(A))= mC(A)$.
(6) $\tilde{1} - mC(A)=mI(\tilde{1} - A)$ and $\tilde{1} - mI(A)=mC(\tilde{1} - A)$.

2. Main Results

Definition 2.1 ([5]). Let (X,M) be an IVF minimal space and A in IVF(X). Then an IVF set A is called an *IVF m-semiopen* set in X if $A \subseteq mC(\text{mI(A)})$. An IVF set A is called an *IVF m-semiclosed* set if the complement of A is an IVF m-semiopen set. The *semi-closure* and the *semi-interio*r of A, denoted by $smC(A)$ and $smI(A)$, respectively, are defined as the following:

$$smC(A) = \cap\{F \in \text{IVF}(X) : A \subseteq F,\ F \text{ is IVF } m\text{-semiclosed in } X\}$$

$$smI(A) = \cup\{U \in \text{IVF}(X) : U \subseteq A,\ U \text{ is IVF } m\text{-semiopen in } X\}.$$

Theorem 2.2 ([5]). Let (X,M) be an IVF minimal space and $A \in$ IVF(X). Then

(1) $smI(A) \subseteq A \subseteq smC(A)$.
(2) If $A \subseteq B$, then $smI(A) \subseteq smI(B)$ and $smC(A) \subseteq smC(B)$.
(3) A is IVF m-semiopen iff $smI(A)=A$ and F is IVF m-semiclosed iff $smC(F)=F$.
(4) $smI(smI(A))=smI(A)$ and $smC(smC(A))=smC(A)$.
(5) $smC(\tilde{1} - A)=\tilde{1} - smI(A)$ and $smI(\tilde{1} - A)=\tilde{1} - smC(A)$.

Lemma 2.3 Let (X, M_X) be an IVF minimal space and $A \in \text{IVF}(X)$. Then

(1) $mI(mC(A)) \subseteq mI(mC(smC(A))) \subseteq smC(A)$.

(2) $smI(A) \subseteq mC(mI(smI(A))) \subseteq mC(mI(A))$.

(3) $mI(mC(A)) = mI(smC(A))$.

(4) $mC(mI(A)) = mC(smI(A))$.

Proof. (1) For $A \in \text{IVF}(X)$, since $A \subseteq smC(A)$ and $smC(A)$ is an IVF m-semiclosed set, we have $mI(mC(A)) \subseteq mI(mC(smC(A))) \subseteq smC(A)$.

(2) It is similar to the proof of (1).

(3) From (1) and Theorem 2.2, it follows

$$mI(mC(A)) \subseteq smC(A) \subseteq mC(A).$$

This implies $mI(mC(A)) = mI(smC(A))$.

(4) It is similar to the proof of (3).

Definition 2.4 ([5]). Let (X, M_X) and (Y, M_Y) be two IVF minimal spaces. Then $f: X \to Y$ is said to be *interval-valued fuzzy m-semicontinuous* (simply, *IVF m-semicontinuous*) if for each IVF point M_x and each IVF m-open set V containing $f(M_x)$, there exists an IVF m-semiopen set U containing M_x such that $f(U) \subseteq V$.

Theorem 2.5 Let $f: X \to Y$ be a function on IVF minimal spaces (X, M_X) and (Y, M_Y). Then f is IVF m-semicontinuous if and only if for each IVF m-open set V in Y, $f^{-1}(V) \subseteq mC(mI(f^{-1}(V)))$.

Proof. Let V be an IVF m-open set in Y and $M_x \widetilde{\in} f^{-1}(V)$. By IVF m-semicontinuity of f, there exists an IVF m-semiopen set U containing M_x such that $f(U) \subseteq V$. Thus from IVF m-semiopenness and Theorem 1.3 (3), it follows

$$M_x \widetilde{\in} U \subseteq mC(mI(U)) \subseteq mC(mI(f^{-1}(V))).$$

Hence we have $f^{-1}(V) \subseteq mC(mI(f^{-1}(V)))$.

For the converse, let M_x be an IVF point of X and V an IVF m-open set in Y containing $f(M_x)$. Then by hypothesis, we have $M_x \widetilde{\in} f^{-1}(V) \subseteq mC(mI(f^{-1}(V)))$. Put $U = f^{-1}(V)$. Then U is an IVF m-semiopen set containing M_x such that $f(U) \subseteq V$. Thus f is IVF m-semicontinuous.

Corollary 2.6. Let $f: X \to Y$ be a mapping on IVF minimal spaces (X, M_X) and (Y, M_Y). Then the following statements are equivalent:

(1) f is IVF m-semicontinuous.

(2) For each IVF m-open set V in Y, $f^{-1}(V)$ is IVF m-semiopen.

(3) For each IVF m-closed set V in Y, $f^{-1}(V)$ is IVF m-semiclosed.

Proof. It follows from Definition 2.1 and Theorem 2.5.

Corollary 2.7. Let $f: X \to Y$ be a mapping on IVF minimal spaces (X, M_X) and (Y, M_Y). Then f is IVF m-semicontinuous if and only if for each IVF m-open set V in Y, $f^{-1}(V) \subseteq mC(smI(f^{-1}(V)))$

Proof. It follows from Theorem 2.5 and Lemma 2.3.

Theorem 2.8 Let $f: X \to Y$ be a function on IVF minimal spaces (X, M_X) and (Y, M_Y). Then f is IVF m-semicontinuous if and only if $f^{-1}(mI(B)) \subseteq mC(mI(f^{-1}(B)))$ for $B \in \text{IVF}(Y)$.

Proof. For each $M_x \widetilde{\in} f^{-1}(mI(B))$, since $f(M_x) \widetilde{\in} mI(B)$, there exists an IVF m-open set V such that $f(M_x) \widetilde{\in} V \subseteq B$. From IVF m-semicontinuity of f, there exists an IVF m-semiopen set U containing M_x such that $f(U) \subseteq V$. This implies $M_x \widetilde{\in} U \subseteq f^{-1}(V) \subseteq f^{-1}(B)$. Since U is IVF m-semiopen, it follows

$$M_x \widetilde{\in} U \subseteq mC(mI(U)) \subseteq mC(mI(f^{-1}(B))).$$

Hence $f^{-1}(mI(B)) \subseteq mC(mI(f^{-1}(B)))$.

For the converse, let M_x be an IVF point of X and an IVF m-open set V containing $f(M_x)$. Then by hypothesis, we have

$$f^{-1}(V) = f^{-1}(mI(V)) \subseteq mC(mI(f^{-1}(V))).$$

Thus $f^{-1}(V)$ is an IVF m-semiopen set containing M_x. Put $U = f^{-1}(V)$. Then $f(U) \subseteq V$ and so f is IVF m-semicontinuous.

Corollary 2.9. Let $f: X \to Y$ be a mapping on IVF minimal spaces (X, M_X) and (Y, M_Y). Then f is IVF m-semicontinuous if and only if $f^{-1}(mI(B)) \subseteq mC(smI(f^{-1}(B)))$ for $B \in \text{IVF}(Y)$.

Proof. It follows from Theorem 2.8 and Lemma 2.3.

Theorem 2.10 Let $f: X \to Y$ be a function on IVF minimal spaces (X, M_X) and (Y, M_Y). Then the following statements are equivalent:

(1) f is IVF m-semicontinuous.

(2) $mI(mC(f^{-1}(B))) \subseteq f^{-1}(mC(B))$ for $B \in \text{IVF}(Y)$.

(3) $f(mI(mC(A))) \subseteq mC(f(A))$ for $A \in \text{IVF}(X)$.

Proof. (1) $\Rightarrow$ (2) Let $A \in \text{IVF}(X)$. Then from Theorem 1.3 and Theorem 2.8, it follows

$$\begin{aligned} f^{-1}(mC(B)) &= f^{-1}(\tilde{1} - mI(\tilde{1} - B)) \\ &= \tilde{1} - (f^{-1}(mI(\tilde{1} - B))) \\ &\subseteq \tilde{1} - mC(mI(f^{-1}(\tilde{1} - B))) \\ &= mI(mC(f^{-1}(B))). \end{aligned}$$

(2) $\Rightarrow$ (3) For $A \in \text{IVF}(X)$, by (2), we have

$mI(mC(A)) \subseteq mI(mC(f^{-1}(f(A)))) \subseteq f^{-1}(mC(f(A)))$.

This implies $f(mI(mC(A))) \subseteq mC(f(A))$.

(3) $\Rightarrow$ (1) Let F an IVF m-closed set in Y. Then

by (3), $f(mI(mC(f^{-1}(F)))) \subseteq mC(f(f^{-1}(F))) \subseteq mC(F)=F$. This implies $f^{-1}(F)$ is an IVF m-semiclosed set. Hence by Corollary 2.6, f is IVF m-semicontinuous.

Corollary 2.11. Let $f: X \to Y$ be a function on IVF minimal spaces (X, M_X) and (Y, M_Y). Then the following statements are equivalent:

(1) f is IVF m-semicontinuous.

(2) $mI(smC(f^{-1}(B))) \subseteq f^{-1}(mC(B))$ for $B \in \mathrm{IVF}(Y)$.

(3) $f(mI(smC(A))) \subseteq mC(f(A))$ for $A \in \mathrm{IVF}(X)$.

Proof. It follows from Theorem 2.10 and Lemma 2.3.

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