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Intuitionistic Fuzzy Generalized Topological Spaces 관한 연구

On Intuitionistic Fuzzy Generalized Topological Spaces

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요 약

intuitionistic fuzzy generalized topological space와 intuitionistic gradation of generalized openness의 개념을 소개한다. 한편 IFG-mapping, weak IFG-mapping과 IFG-open mapping의 개념을 소개하며 특성을 조사한다.

Abstract

In this paper, we introduce the concepts of intuitionistic fuzzy generalized topological spaces and intuitionistic gradation of generalized openness. We also introduce the concepts of IFG-mapping, weak IFG-mapping and IFG-open mapping, and obtain some characterizations for such mappings.

Key Words : intuitionistic gradation of generalized openness, intuitionistic fuzzy generalized topological spaces, IFG-mapping, weak IFG-mapping.

1. Intorduction

In 1992 [2, 4], Chattopadyay et al. introduced the concept of fuzzy topology redefined by a gradation of openness and investigated some fundamental properties. In [6], Ramadan called the fuzzy topology "a smooth topology" and studied several topological properties in the fuzzy topological space. Atanassov [1] introduced the concept of intuitionistic fuzzy set which is a generalization of fuzzy set in Zadeh's sense [7]. In [5], Mondal and Samanta introduced and investigated the concept of intuitionistic gradations of openness which is a generalization of the concept of gradation of openness defined by Chattopadyay. In this paper, we introduce the concept of intuitionistic gradation of generalized openness which is a generalization of the concepts of intuitionistic gradations of openness and gradations of basic properties for intuitionistic fuzzy generalized topological spaces defined by the given intuitionistic gradation of generalized openness. We also introduce the concepts of IFG-mapping, weak IFG-mapping and IFG-open mapping and obtain some characterizations for such mappings.

2. Preliminaries

Let I be the unit interval $[0,1]$ of the real line. A member $A$of I^X is called a fuzzy set of X. For any $A \in I^X$, A^c denotes the complement $1_X - A$. By 0_X and 1_X we denote constant maps on X with value 0 and 1, respectively.

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Definition 2.1 ([2,4]). Let X be a non-empty set and $\tau : I^X \to I$ be a mapping satisfying the following conditions:
(1) $\tau(0_X)=\tau(1_X)=1$;
(2) $\forall A, B \in I^X$, $\tau(A \cap B) \geq \tau(A) \wedge \tau(B)$;
(3) For every subfamily $\{A_i : i \in J\} \subseteq I^X$, $\tau(\cup_{i \in J} A_i) \geq \wedge_{i \in J} \tau(A_i)$.

Then the mapping $\tau : I^X \to I$ is called a *fuzzy topology* (or gradation of openness) on X. We call the ordered pair (X,τ) a *fuzzy topological space*. The value $\tau(A)$ is called the *degree of openness* of A.

Definition 2.2 ([1]). Let X be a nonempty set. An *intuitionistic fuzzy set* A is an ordered pair $A=\{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$ (simply, $A=(\mu_A, \gamma_A)$) where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership and the degree of nonmembership, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$.

Definition 2.3 ([5]). An intuitionistic gradation of openness (briefly IGO) of fuzzy subsets of a set $X$is an ordered pair (τ,τ^*) of functions $\tau,\tau^* : I^X \to I$ such that

(IGO1) $\tau(A)+\tau^*(A)\leq 1$, for all $A\in I^X$;
(IGO2) $\tau(0_X)=\tau(1_X)=1$, $\tau^*(0_X)=\tau^*(1_X)=0$;

(IGO3) $\forall A,B\in I^X$, $\tau(A\cap B)\geq \tau(A)\wedge\tau(B)$ and $\tau^*(A\cap B)\leq \tau^*(A)\vee\tau^*(B)$;

(IGO4) For every subfamily $\{A_i : i\in J\}\subseteq I^X$, $\tau(\cup_{i\in J}A_i)\geq \wedge_{i\in J}\tau(A_i)$ and $\tau^*(\cup_{i\in J}A_i)\leq \vee_{i\in J}\tau^*(A_i)$.

Then the triplet (X,τ,τ^*) is called an *intuitionistic fuzzy topological space* (briefly IFTS) on X. τ and τ^* may be interpreted as gradation of openness and gradation of nonopenness, respectively.

Definition 2.4 ([5]). Let X be a nonempty set and two functions ψ, $\psi^*: I^X\to I$ be satisfying

(IGC1) $\psi(A)+\psi^*(A)\leq 1$ for all $A\in I^X$;

(IGC2) $\psi(0_X)=\psi(1_X)=1$, $\psi^*(0_X)=\psi^*(1_X)=0$;

(IGC3) $\forall A,B\in I^X$, $\psi(A\cup B)\geq\psi(A)\wedge\psi(B)$ and $\psi^*(A\cup B)\leq\psi^*(A)\vee\psi^*(B)$;

(IGC4) for every subfamily $\{A_i : i\in J\}\subseteq I^X$, $\psi(\cap_{i\in J}A_i)\geq \wedge_{i\in J}\psi(A_i)$ and $\psi^*(\cap_{i\in J}A_i)\leq \vee_{i\in J}\psi^*(A_i)$.

Then the ordered pair (ψ,ψ^*) is called *an intuitionistic gradation of closedness* (briefly IGC) on X ψ and ψ^* may be interpreted as gradation of closedness and gradation of nonclosedness, respectively.

Definition 2.5 ([3]). An *intuitionistic gradation of supra-openness* (briefly IGSO) of fuzzy subsets of a set $X$is an ordered pair (τ,τ^*) of functions $\tau,\tau^*: I^X\to I$ such that

(IGSO1) $\tau(A)+\tau^*(A)\leq 1$, for all $A\in I^X$;

(IGSO2) $\tau(0_X)=\tau(1_X)=1$, $\tau^*(0_X)=\tau^*(1_X)=0$;

(IGSO3) For every subfamily $\{A_i : i\in J\}\subseteq I^X$, $\tau(\cup_{i\in J}A_i)\geq \wedge_{i\in J}\tau(A_i)$ and $\tau^*(\cup_{i\in J}A_i)\leq \vee_{i\in J}\tau^*(A_i)$.

Then the triplet (X,τ,τ^*) is called an *ntuitionistic fuzzy supra topological space* (briefly IFSTS) on X. τ and τ^* may be interpreted as gradation of supra- openness and gradation of supra-nonopenness, respectively.

Definition 2.6 ([1]) Let $A=(\mu_A,\gamma_A)$ and $B=(\mu_B,\gamma_B)$ be intuitionistic fuzzy sets on X. Then
(1) $A\subseteq B$ iff $\mu_A\leq\mu_B$ and $\gamma_A\geq\gamma_B$.
(2) $A=B$ iff $A\subseteq B$ and $B\subseteq A$.
(3) $A^c=(\gamma_A,\mu_A)$.
(4) $A\cap B=(\mu_A\wedge\mu_B,\ \gamma_A\vee\gamma_B)$.
(5) $A\cup B=(\mu_A\vee\mu_B,\ \gamma_A\wedge\gamma_B)$.
(6) $0_\sim=(0_X,1_X)$ and $1_\sim=(1_X,0_X)$.

Let f be a map from a set X to a set Y. Let $A=(\mu_A,\gamma_A)$ be an intuitionistic fuzzy set of X and $B=(\mu_B,\gamma_B)$ an intuitionistic fuzzy set of Y. Then:
(1) The image of A under f, denoted by $f(A)$ is an intuitionistic fuzzy set in Y defined by

$$f(A)=(f(\mu_A),\ 1_{Y}-f(1_X-\gamma_A)).$$

(2) The inverse image of B under f, denoted by $f^{-1}(B)$ is an intuitionistic fuzzy set in X defined by

$$f^{-1}(B)=(f^{-1}(\mu_B),\ f^{-1}(\gamma_B)).$$

3. Intuitionstc fuzzy generalized topological spaces

Definition 3.1 An *intuitionistic gradation of generalized openness* (briefly IGGO) of fuzzy subsets of a set X is an ordered pair (τ,τ^*) of functions τ, $\tau^*: I^X\to I$ such that

(IGGO1) $\tau(A)+\tau^*(A)\leq 1$ for all $A\in I^X$;

(IGGO2) $\tau(0_X)=1$, $\tau^*(0_X)=0$;

(IGGO3) For every subfamily $\{A_i : i\in J\}\subseteq I^X$, $\tau(\cup_{i\in J}A_i)\geq \wedge_{i\in J}\tau(A_i)$ and $\tau^*(\cup_{i\in J}A_i)\leq \vee_{i\in J}\tau^*(A_i)$.

Then the triplet (X,τ,τ^*) is called an *intuitionistic fuzzy generalized topological space* (briefly IFGTS) on X. τ and τ^* may be interpreted as gradation of generalized openness and gradation of generalized nonopenness, respectively.

Obviously we get the following implications:

gradation of openness $\Rightarrow$ intuitionistic gradation of openness $\Rightarrow$ intuitionistic gradation of supra- openness $\Rightarrow$ intuitionistic gradation of generalized openness

Definition 3.2 Let X be a nonempty set and two functions ψ, $\psi^*: I^X\to I$ be satisfying

(IGGC1) $\psi(A)+\psi^*(A)\leq 1$ for all $A\in I^X$;

(IGGC2) $\psi(1_X)=1$, $\psi^*(1_X)=0$;

(IGGC3) for every subfamily $\{A_i : i\in J\}\subseteq I^X$, $\psi(\cap_{i\in J}A_i)\geq \wedge_{i\in J}\psi(A_i)$ and $\psi^*(\cap_{i\in J}A_i)\leq \vee_{i\in J}\psi^*(A_i)$.

Then the ordered pair (ψ,ψ^*) is called an *intuitionistic gradation of generalized closedness* (briefly IGGC) on X. ψ and ψ^* may be interpreted as gradation of generalized closedness and gradation of generalized nonclosedness, respectively.

Theorem 3.3 (1) If (τ,τ^*) is an IGGO on X, then the ordered pair (ψ,ψ^*), defined by $\psi(A)=\tau(A^c)$ and $\psi^*(A)=\tau^*(A^c)$ is an intuitionistic gradation of generalized closedness on X.

(2) If (ψ,ψ^*) is an intuitionistic gradation of generalized closedness on X, then the ordered pair (τ,τ^*), defined by $\tau(A)=\psi(A^c)$ and $\tau^*(A)=\psi^*(A^c)$ is an intuitionistic gradation of generalized openness on X.

Proof. (1) Since $\psi(A)+\psi^*(A)=\tau(A^c)+\tau^*(A^c)$ and $\tau(A^c)+\tau^*(A^c)\le 1$, $\psi(A)+\psi^*(A)\le 1$ for all $A\in I^X$.

From $\psi(1_X)=\tau(1_X{}^c)=\tau(0_X)=1$ and $\psi^*(1_X)=\tau^*(1_X{}^c)=\tau^*(0_X)=0$, we have the condition (IGGC2).

For every subfamily $\{A_i : i\in J\}\subseteq I^X$,

$$\begin{aligned}\psi(\cap_{i\in J}A_i) &= \tau((\cap_{i\in J}A_i)^c)\\ &= \tau(\cup_{i\in J}A_i^c)\\ &\ge \wedge_{i\in J}\tau(A_i^c)\\ &= \wedge_{i\in J}\psi(A_i)\end{aligned}$$

and

$$\begin{aligned}\psi^*(\cap_{i\in J}A_i) &= \tau^*((\cap_{i\in J}A_i)^c)\\ &= \tau^*(\cup_{i\in J}A_i^c)\\ &\le \vee_{i\in J}\tau^*(A_i^c)\\ &= \vee_{i\in J}\psi^*(A_i).\end{aligned}$$

Thus we have (IGGC3).

(2) It is similar to (1).

Henceforward, the ordered pair (ψ_i,ψ_i^*) is an intuitionistic gradation of generalized closedness defined by $\psi_i(A)=\tau_i(A^c)$ and $\psi_i^*(A)=\tau_i^*(A^c)$ on an IFGTS (X,τ_i,τ_i^*) unless explicitly stated.

Remark 3.4 Let $\{(\tau_i,\tau_i^*) : i\in J\}$ be a family of IGGO's on X. Then the intersection $\cap_{i\in J}(\tau_i,\tau_i^*)=(\wedge_{i\in J}\tau_i,\vee_{i\in J}\tau_i^*)$ is an IGGO on X, where $(\wedge_{i\in J}\tau_i)(A)=\wedge_{i\in J}\tau_i(A)$, $(\vee_{i\in J}\tau_i^*)(A)=\vee_{i\in J}\tau_i^*(A)$.

Definition 3.5 Given a set X and IFGT's (τ_1,τ_1^*) and (τ_2,τ_2^*) on X. We say that (τ_2,τ_2^*) is finer than (τ_1,τ_1^*) (denoted by $(\tau_1,\tau_1^*)\le(\tau_2,\tau_2^*)$) is $\tau_1(A)\le\tau_2(A)$ and $\tau_1^*(A)\ge\tau_2^*(A)$ for every $A\in I^X$.

Theorem 3.6 Let (τ_r,τ_s^*) denote an IFGT on X such that $\tau_r(A)\ge r$ and $\tau_s^*(A)\le s$ for every $A\in I^X$. Then if $r\le l$ and $s\ge m$, then $(\tau_r,\tau_s^*)\le(\tau_l,\tau_m^*)$.

Proof. The proof is straightforward.

Definition 3.7 Let (X,τ,τ^*) be an IFGTS and $A\in I^X$. Then the closure of A, denoted by $\overline{A}$, is defined by $\overline{A}=\cap\{K\in I^X : \psi(K)>0$ and $\psi^*(K)\le\psi^*(A),\ A\subseteq K\}$; and the interior of A, denoted by A^o, is defined by $A^o=\cup\{K\in I^X : \tau(K)>0$ and $\tau^*(K)\le\tau^*(A),\ K\subseteq A\}$.

Theorem 3.8 Let (X,τ,τ^*) be an IFGTS and $A,B\in I^X$. Then

(1) $\psi^*(\overline{A})\le\psi^*(A)$;
(2) $\tau^*(A^o)\le\tau^*(A)$;
(3) $A\subseteq B$ and $\psi^*(B)\le\psi^*(A)\Rightarrow\overline{A}\subseteq\overline{B}$;
(4) $A\subseteq B$ and $\tau^*(A)\le\tau^*(B)\Rightarrow A^o\subseteq B^o$.

Proof. (1) From Definition 3.1 and Definition 3.7, it follows

$$\begin{aligned}\psi^*(\overline{A}) &= \psi^*(\cap\{K\in I^X : \psi(K)>0 \text{ and } \psi^*(K)\le\psi^*(A),\ A\subseteq K\})\\ &\le \vee\{\psi^*(K) : \psi(K)>0 \text{ and } \psi^*(K)\le\psi^*(A),\ A\subseteq K\})\\ &\le \psi^*(A).\end{aligned}$$

(2) It is similar to (1).
(3) We have the following:

$$\begin{aligned}\overline{A} &= \cap\{K\in I^X : \psi(K)>0 \text{ and } \psi^*(K)\le\psi^*(A),\ A\subseteq K\}\\ &\subseteq \cap\{K\in I^X : \psi(K)>0 \text{ and } \psi^*(K)\le\psi^*(B),\ B\subseteq K\}.\end{aligned}$$

Hence $\overline{A}\subseteq\overline{B}$.

(4) The proof is similar to (3).

Theorem 3.9 Let (X,τ,τ^*) be an IFGTS and $A\in I^X$. Then

(1) $(\overline{A})^c=(A^c)^o$,
(2) $\overline{A}=((A^c)^o)^c$,
(3) $(A^o)^c=\overline{A^c}$,
(4) $A^o=((\overline{A^c}))^c$.

Proof. (1) From Definition 3.7, we have

$$\begin{aligned}(\overline{A})^c &= (\cap\{K\in I^X : \psi(K)>0 \text{ and } \psi^*(K)\le\psi^*(A),\ A\subseteq K\})^c\\ &= \cup\{K^c\in I^X : \tau(K^c)=\psi(K)>0 \text{ and } \tau^*(K^c)\le\tau^*(A^c),\ K^c\subseteq A^c\}\\ &= \cup\{U\in I^X : U\subseteq A^c,\ \tau(U)>0 \text{ and } \tau^*(U)\le\tau^*(A^c)\}\\ &= (A^c)^o.\end{aligned}$$

(2), (3) and (4) are easily obtained from (1).

Theorem 3.10 Let (X,τ,τ^*) be an IFGTS and $A,B\in I^X$. Then

(1) $A\subseteq\overline{A}$ and $A^o\subseteq A$,
(2) $\overline{A}\subseteq(\overline{(\overline{A})})$ and $(A^o)^o\subseteq A^o$.

Proof. (1) Obvious.
(2) Since $A \subseteq \overline{A}$ and $\psi^*(\overline{A}) \leq \psi^*(A)$, by Theorem 3.8 (3), we have $\overline{A} \subseteq \overline{(\overline{A})}$.
Similarly, we can show that $(A^o)^o \subseteq A^o$.

Theorem 3.11 Let (X,τ,τ^*) be an IFGTS and $A \in I^X$. Then
(1) $\tau(A)>0 \Rightarrow A^o=A$.
(2) $\psi(A)>0 \Rightarrow \overline{A}=A$.

Proof. (1) Let $\tau(A)>0$; then $A \in \{K \in I^X$: $\tau(K)>0$ and $\tau^*(K) \leq \tau^*(A)$, $K \subseteq A\}$, so $A \subseteq A^o$. Thus we get $A^o=A$ by Theorem 3.10.
(2) It is similar to (1).

In the above Theorem 3.11, the converse relations need not be true as the next example.

Example 3.12 Let $X=I$ and and let N denote the set of all natural numbers. For $n \in N$, we consider a fuzzy set A_n as the following:

$$A_n(x)=(\frac{n-1}{n})x \text{ for } x \in X.$$

Define an intuitionistic gradation of generalized openness τ, $\tau^*: I^X \to I$ by

$\tau(0_X)=1$, $\tau^*(0_X)=0$;
$\tau(A_n)=\frac{1}{n}$, $\tau^*(A_n)=(\frac{n-1}{2n})$;
$\tau(K)=0$, $\tau^*(K)=\frac{1}{2}$ for all other fuzzy set $K \in I^X$.

Let us take a fuzzy set A in X such that $A(x)=x$ for all $x \in I^X$. Then it follows $A^o=A$ but $\tau(A)=0$, $\tau^*(A)=\frac{1}{2}$. Thus the converse of the part (1) is not true in general. In the same way, the converse of the part (2) need not be true.

4. IFG-mappings and Weak IFG-mappings

Definition 4.1 Let (X,τ_1,τ_1^*) and (Y,τ_2,τ_2^*) be two IFGTS's. $f: X \to Y$ is called
(1) an *IFG-mapping* if $\tau_1(f^{-1}(A)) \geq \tau_2(A)$ and $\tau_1^*(f^{-1}(A)) \leq \tau_2^*(A)$ for every $A \in I^Y$;
(2) a *weak IFG-mapping* iff $\tau_2(A)>0 \Rightarrow \tau_1(f^{-1}(A))>0$ and $\tau_1^*(f^{-1}(A)) \leq \tau_2^*(A)$ for every $A \in I^Y$.

It is obvious that every IFG-map is a weak IFG-map from the above definition. But the converse is not always true as shown in the next example:

Example 4.2 Let $X=I$ and and let N denote the set of all natural numbers. For $n \in N$, we consider a fuzzy set A_n as he following:

$$A_n(x)=\frac{1}{n}x \text{ for } x \in X.$$

Define $\tau_1,\tau_1^*: I^X \to I$ by

$\tau_1(0_X)=1$, $\tau_1^*(0_X)=0$;
$\tau_1(A_n)=\frac{1}{n+2}$, $\tau_1^*(A_n)=\frac{1}{n+2}$ for each $n \in N$;
$\tau_1(A)=0$, $\tau_1^*(A)=1$ for all other fuzzy set $A \in X$.

And define $\tau_2,\tau_2^*: I^X \to I$ by

$\tau_2(0_X)=1$, $\tau_2^*(0_X)=0$;
$\tau_2(A_n)=\frac{1}{n+1}$, $\tau_2^*(A_n)=\frac{1}{n+1}$ for each $n \in N$;
$\tau_2(A)=0$, $\tau_2^*(A)=1$ for all other fuzzy set $A \in X$.

Then the pairs (τ_1,τ_1^*) and (τ_2,τ_2^*) are two intuitionistc gradations of generalized openness on X. Consider the identity mapping $f:(X,\tau_1,\tau_1^*) \to (Y,\tau_2,\tau_2^*)$. Then obviously f is a weak IFG-mapping. But since for each fuzzy set A_n, $\tau_2(A_n)>\tau_1(f^{-1}(A_n))$, f is not an IFG-mapping.

Theorem 4.3 Let (X,τ_1,τ_1^*) and (Y,τ_2,τ_2^*) be two IFGTS's. Then a mapping $f: X \to Y$ is a weak IFG-mapping iff for every $A \in I^Y$, $\psi_2(A)>0 \Rightarrow \psi_1(f^{-1}(A))>0$ and $\psi_1^*(f^{-1}(A)) \leq \psi_2^*(A)$.

Proof. Suppose f is a weak IFG-mapping and let $\psi(A)>0$ for $A \in I^Y$; then $\psi(A^{cc})=\tau_2(A^c)>0$. Since f is a weak IFG-mapping, it follows $\tau_1(f^{-1}(A^c))>0$ and $\tau_1^*(f^{-1}(A^c)) \leq \tau_2^*(A^c)$. Thus we get $\psi_1(f^{-1}(A))>0$ and $\psi_1^*(f^{-1}(A)) \leq \psi_2^*(A)$.

The converse is obvious.

Theorem 4.4 Let (X,τ_1,τ_1^*) and (Y,τ_2,τ_2^*) be two IFGTS's. Then $f: X \to Y$ is an IFG-mapping iff for every $A \in I^Y$, $\psi_2(A) \leq \psi_1(f^{-1}(A))$ and $\psi_1^*(f^{-1}(A)) \leq \psi_2^*(A)$.

Proof. The proof is similar to that of Theorem 4.3.

Theorem 4.5 Let (X,τ_1,τ_1^*) and (Y,τ_2,τ_2^*) be two IFGTS's. If $f: X \to Y$ is a weak IFG-mapping, then we have
(1) $f(\overline{A}) \subseteq \overline{f(A)}$ for every $A \in I^X$,
(2) $\overline{f^{-1}(A)} \subseteq f^{-1}(\overline{A})$ for every $A \in I^Y$,
(3) $f^{-1}(A^o) \subseteq (f^{-1}(A))^o$ for every $A \in I^Y$,

Proof. (1) Let $A \in I^X$; then by Definition 3.7 and Theorem 4.3, we have

$$f^{-1}(\overline{f(A)})=f^{-1}[\cap\{U\in I^Y:\psi_2(U)>0 \text{ and } \psi_2^*(U)\le \psi_2^*(f(A)),\ f(A)\subseteq U\}]$$
$$=\cap\{f^{-1}(U)\in I^X:\ \psi_1(f^{-1}(U))>0 \text{ and } \psi_1^*(f^{-1}(U))\le\psi_2^*(A),\ A\subseteq f^{-1}(U)\}.$$

From $\psi_1(f^{-1}(U))>0$ and Theorem 3.11, it follows $\overline{A}\subseteq\overline{f^{-1}(U)}=f^{-1}(U)$, and so $\cap\{f^{-1}(U)\in I^X:\psi_1(f^{-1}(U))>0$ and $\psi_1^*(f^{-1}(U))\le\psi_2^*(A),\ A\subseteq f^{-1}(U)\}\supseteq\overline{A}$.

This implies $f(\overline{A})\subseteq\overline{f(A)}$.
(2) It follows from (1).
(3) It obtains by (2) and Theorem 3.9.

Corollary 4.6 Let (X,τ_1,τ_1^*) and (Y,τ_2,τ_2^*) be two IFGTS's. If $f:X\to Y$ is an IFG-mapping, then we have
(1) $f(\overline{A})\subseteq\overline{f(A)}$ for every $A\in I^X$,
(2) $\overline{f^{-1}(A)}\subseteq f^{-1}(\overline{A})$ for every $A\in I^Y$,
(3) $f^{-1}(A^o)\subseteq (f^{-1}(A))^o$ for every $A\in I^Y$.

Definition 4.7 Let (X,τ_1,τ_1^*) and (Y,τ_2,τ_2^*) be two IFGTS's. $f:X\to Y$ is called an *IFG-open mapping* (resp., *IFG-closed mappping*) iff $\tau_1(A)\le\tau_2(f(A))$ (resp., $\psi_1(A)\le\psi_2(f(A))$) and $\tau_2^*(f(A))\le\tau_1^*(A)$ (resp., $\psi_2^*(f(A))\le\psi_1^*(A)$) for every $A\in I^X$.

Theorem 4.8 Let (X,τ_1,τ_1^*) and (Y,τ_2,τ_2^*) be two IFGTS's. If a mapping $f:X\to Y$ is IFG-open, then $f(A^o)\subseteq(f(A))^o$ for every $A\in I^X$.

Proof. For every $A\in I^X$, we have

$$f(A^o)=f[\cup\{U\in I^X:\tau_1(U)>0 \text{ and } \tau_1^*(U)\le\tau_1^*(A),\ U\subseteq A\}]$$
$$=[\cup\{f(U)\in I^Y:\tau_1(U)>0 \text{ and } \tau_1^*(U)\le\tau_1^*(A),\ f(U)\subseteq f(A)\}]$$
$$\subseteq\cup\{f(U)\in I^Y:\tau_2(f(U))>0 \text{ and } \tau_2^*(f(U))\le\tau_1^*(U),\ f(U)\subseteq f(A)\}.$$

From $\tau_2(f(U))>0$ and Theorem 3.11, it follows

$$f(U)^o=f(U)\subseteq(f(A))^o.$$

Thus we get $f(A^o)\subseteq(f(A))^o$.

Theorem 4.9 Let (X,τ_1,τ_1^*) and (Y,τ_2,τ_2^*) be two IFGTS's. If $f:X\to Y$ is an injective IFG-closed mapping, then $\overline{f(A)}\subseteq f(\overline{A})$ for every $A\in I^X$.

Proof. Let $A\in I^X$; then since f is an injective IFG-closed mapping, we have

$$f(\overline{A})=f[\cap\{U\in I^X:\psi_1(U)>0 \text{ and } \psi_1^*(U)\le\psi_1^*(A),\ A\subseteq U\}]$$
$$=\cap\{f(U)\in I^X:\psi_1(U)>0 \text{ and } \psi_1^*(U)\le\psi_1^*(A),\ f(A)\subseteq f(U)\}$$
$$\supseteq\cap\{f(U)\in I^X:\psi_2(f(U))>0 \text{ and } \psi_2^*(f(U))\le\psi_1^*(U),\ f(A)\subseteq f(U)\}.$$

Thus from $\psi_2(f(U))>0$ and Theorem 3.11, we have $f(\overline{A})\supseteq\overline{f(A)}$.

References

[1] K. T. Atanassov, "Intuitionistic fuzzy sets", *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87-96, 1986.
[2] K. C. Chattopadhyay, R. N. Hazra, and S. K. Samanta, "Gradation of openness: Fuzzy topology", *Fuzzy Sets and Systems,* vol. 49, pp. 237-242, 1992.
[3] M. H. Ghanim, O. A. Tantawy, F. M. Selim, "Gradation of supra-openness", *Fuzzy Sets and Systems*, vol. 109, pp. 245-250, 2000.
[4] R. N. Hazra, S. K. Samanta, K. C. Chattopadhyay, "Fuzzy topology redefined", *Fuzzy Sets and Systems*, vol. 45, pp. 79-82, 1992.
[5] T. K. Mondal and S. K. Samanta, "On intuitionistic gradation of openness", *Fuzzy Sets and Systems*, vol. 131, pp. 323-336, 2002.
[6] A. A. Ramadan, "Smooth topological spaces", *Fuzzy Sets and Systems,* vol. 48, pp. 371-375, 1992.
[7] L. A. Zadeh, "Fuzzy sets", *Information and Control,* vol. 8, pp. 338-353, 1965.

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