

쇼케이 적분 기준을 통한 구간치 필요측도에 관한 연구

A study on interval-valued necessity measures through the Choquet integral criterion

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Abstract

Y. Réballé[Fuzzy Sets and Systems, vol.157, pp.3025-2039, 2006] discussed the representation of necessity measure through the Choquet integral criterion. He also considered a decision maker who ranks necessity measures related with Choquet integral representation. Our motivation of this paper is that a decision maker have an "ambiguity" necessity measure to present preferences. In this paper, we discuss the representation of interval-valued necessity measures through the Choquet integral criterion.

Key Words : non-additive measures, necessity measures, Choquet integrals.

1. Introduction

Murofushi and Sugeno[10] have been studying Choquet integrals which allow to define necessity measures and risk measure. Y. Réballé[11] discussed the representation of necessity measure through the Choquet integral criterion. He also considered a decision maker who ranks necessity measures related with Choquet integral representation. Another researchers have been studying topics related with Choquet integrals, for examples, preference representation theorem(Y. Réballé [11]), integral representation(D. Schmeidler[12]), subjective probability and expected utility without additivity(D. Schmeidler[13]), interval-valued Choquet price functionals(L. Jang[8]), applications in pricing risks(L. Jang[9]), etc.

Motivation of this paper is that a decision maker have ambiguity necessity measures to present risky prospects in the sense of mathematical theory. We note that ambiguity measures can be represented by interval-valued necessity measure. We can see that this idea is similar to the concept of the Choquet integral of an interval-valued measurable function (see [1, 2, 5-9]).

In this paper, we discuss the representation of interval-valued necessity measures through the Choquet integral criterion.

2. Definitions and Preliminaries

In this section we list the set-theoretical arithmetic operations on the set of subintervals of an unit interval $I=[0,1]$ in \mathbb{R} . We denote $[I]$ by

$$[I] = \{\bar{a} = [a^-, a^+] \mid a^-, a^+ \in I \text{ and } a^- \leq a^+\}.$$

For any $a \in I$, we define $a = [a, a]$. Obviously, $a \in [I]$.

Definition 2.1 ([7-9]) If $\bar{a}, \bar{b} \in [I], k \in I$, then we define

- (1) $\bar{a} + \bar{b} = [a^- + b^-, a^+ + b^+]$,
- (2) $k\bar{a} = [ka^-, ka^+]$,
- (3) $\bar{a} \wedge \bar{b} = [a^- \wedge b^-, a^+ \wedge b^+]$,
- (4) $\bar{a} \vee \bar{b} = [a^- \vee b^-, a^+ \vee b^+]$,
- (5) $\bar{a} \leq \bar{b}$ if and only if $a^- \leq b^-$ and $a^+ \leq b^+$,
- (6) $\bar{a} < \bar{b}$ if and only if $\bar{a} \leq \bar{b}$ and $\bar{a} \neq \bar{b}$,
- (7) $\bar{a} \subset \bar{b}$ if and only if $b^- \leq a^-$ and $a^+ \leq b^+$.

Theorem 2.2 ([7-9]) Let $\bar{a}, \bar{b} \in [I]$. Then the followings hold.

- (1) idempotent law: $\bar{a} \wedge \bar{a} = \bar{a}, \bar{a} \vee \bar{a} = \bar{a}$,
- (2) commutative law: $\bar{a} \wedge \bar{b} = \bar{b} \wedge \bar{a}, \bar{a} \vee \bar{b} = \bar{b} \vee \bar{a}$,
- (3) associative law: $(\bar{a} \wedge \bar{b}) \wedge \bar{c} = \bar{a} \wedge (\bar{b} \wedge \bar{c})$,
 $(\bar{a} \vee \bar{b}) \vee \bar{c} = \bar{a} \vee (\bar{b} \vee \bar{c})$,
- (4) absorption law: $\bar{a} \wedge (\bar{a} \vee \bar{b}) = \bar{a} \vee (\bar{a} \wedge \bar{b}) = \bar{a}$,
- (5) distributive law: $\bar{a} \wedge (\bar{b} \vee \bar{c}) = (\bar{a} \wedge \bar{b}) \vee (\bar{a} \wedge \bar{c})$,
 $\bar{a} \vee (\bar{b} \wedge \bar{c}) = (\bar{a} \vee \bar{b}) \wedge (\bar{a} \vee \bar{c})$,

It is easily to see that $([I], d_H)$ is a metric where d_H is the Hausdorff metric defined by

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$d_H(A, B) = \max\{\bigvee_{a \in A} \bigwedge_{b \in B} |a - b|, \bigvee_{b \in B} \bigwedge_{a \in A} |a - b|\}$
 for all $A, B \in [\bar{I}]$. Clearly, we have the following theorem for multiplication and Hausdorff metric on $[\bar{I}]$.

Theorem 2.3 ([7-9]) (1) If we define

$$\bar{a} \cdot \bar{b} = \{x \cdot y \mid x \in \bar{a}, y \in \bar{b}\}$$

for $\bar{a}, \bar{b} \in [\bar{I}]$, then $\bar{a} \cdot \bar{b} = [a^- \cdot b^-, a^+ \cdot b^+]$.

(2) If $d_H: [\bar{I}] \times [\bar{I}] \rightarrow [0, \infty)$ is the above Hausdorff metric,

$$\text{then } d_H(\bar{a}, \bar{b}) = \max\{|a^- - b^-|, |a^+ - b^+|\}.$$

Let Ω be a non-empty set and $\mathfrak{J}(\Omega)$ a non-empty family of subsets of Ω . A function $X: \Omega \rightarrow I$ is said to be $\mathfrak{J}(\Omega)$ -measurable if for every $\alpha \in (0, 1)$, $\{w \in \Omega \mid X(w) \geq \alpha\} \in \mathfrak{J}(\Omega)$.

Let $B(\Omega, \mathfrak{J}(\Omega))$ be the set of $\mathfrak{J}(\Omega)$ -measurable functions. We remark that $B(\Omega, \mathfrak{J}(\Omega))$ is not convex (see [11]). We also list non-additive measures, possibility measures, and necessity measures.

Definition 2.4 ([3, 7-9, 10-13]) A set function μ on $\mathfrak{J}(\Omega)$ is called a non-additive measure if $\mu(\emptyset) = 0$ and $\mu(A) \leq \mu(B)$ whenever $A, B \in \mathfrak{J}(\Omega)$ and $A \subset B$.

Definition 2.5 ([11, 14]) (1) A set function μ on $\mathfrak{J}(\Omega)$ is called a possibility measure if $\mu(\emptyset) = 0$ and $\mu(X) = 1$ and $\mu(\bigcup_i A_i) \leq \max_i \mu(A_i)$ for all collections $\{A_i\} \subset \mathfrak{J}(\Omega)$.

(2) A set function ν on $\mathfrak{J}(\Omega)$ is called a necessity measure if $\nu(A) = 1 - \mu(A^c)$ for all $A \in \mathfrak{J}(\Omega)$ and $A^c = \{w \in \Omega \mid w \notin A\}$.

We note that every possibility measure and necessity measure is a non-additive measure. Let us discuss the following Choquet integral.

Definition 2.4 ([3, 7-9, 10-13]) Let μ be a non-additive measure on $\mathfrak{J}(\Omega)$ and $X \in B(\Omega, \mathfrak{J}(\Omega))$. The Choquet integral of X with respect to μ is defined by

$$(C) \int f d\mu = \int_0^1 \mu_X(\alpha) d\alpha$$

where $\mu_X(\alpha) = \mu(\{w \in \Omega \mid X(w) > \alpha\})$ and the integral on the right hand side is Lebesgue integral.

For the case of characteristic function, one has $(C) \int I_A d\mu = \mu(A)$ where $A \in \mathfrak{J}(\Omega)$ and I_A is the characteristic function of A . We note that if Ω is a finite set and $X \in B(\Omega, \mathfrak{J}(\Omega))$, then there is a unique decomposition of X in the following manner

$$X = \sum_{i=1}^n \alpha_i I_{A_i}, \text{ where } \alpha_1, \dots, \alpha_n > 0 \text{ and } \sum_{i=1}^n \alpha_i \leq 1 \text{ and } A_1 \supset A_2 \supset \dots \supset A_n \neq \emptyset \text{ and } A_i \in \mathfrak{J}(\Omega) \text{ for all}$$

$i = 1, 2, \dots, n$, possibly $A_1 = \Omega$ if $\Omega \in \mathfrak{J}(\Omega)$. The computation of the Choquet integral of X with respect to μ gives

$$(C) \int X d\mu = \sum_{i=1}^n \alpha_i \mu(A_i).$$

Definition 2.5 ([3, 7-9, 10-13]) Let $X, Y \in B(\Omega, \mathfrak{J}(\Omega))$. We say that X and Y are comonotonic, in symbol $X \sim Y$ if

$$X(w) < X(w') \Rightarrow Y(w) \leq Y(w')$$

for all $w, w' \in \Omega$.

Definition 2.6 ([11]) We say that the Choquet integral has comonotonic affinity if $X, Y \in B(\Omega, \mathfrak{J}(\Omega))$ and $\alpha \in (0, 1)$ and $\alpha X + (1 - \alpha) Y \in B(\Omega, \mathfrak{J}(\Omega))$ and $X \sim Y$, then

$$(C) \int (\alpha X + (1 - \alpha) Y) d\mu = \alpha (C) \int X d\mu + (1 - \alpha) (C) \int Y d\mu.$$

Now, we introduce the following basic properties of the comonotonicity and the Choquet integral.

Theorem 2.7 ([3, 10-13]) Let $X, Y, Z \in B(\Omega, \mathfrak{J}(\Omega))$. Then we have the following.

- (1) $X \sim X$,
- (2) $X \sim Y \Rightarrow Y \sim X$,
- (3) $X \sim a$ for all $a \in I$,
- (4) $X \sim Y$ and $X \sim Z \Rightarrow X \sim Y + Z$.

Theorem 2.8 ([3, 10-13]) Let $X, Y \in B(\Omega, \mathfrak{J}(\Omega))$. Then we have the following.

- (1) If $X \leq Y$, then $(C) \int X d\mu \leq (C) \int Y d\mu$.
- (2) If $A \subset B$ and $A, B \in \mathfrak{J}(\Omega)$, then

$$(C) \int_A X d\mu \leq (C) \int_B X d\mu.$$

- (3) If $X \sim Y$ and $a, b \in I$, then

$$(C) \int (aX + bY) d\mu = a(C) \int X d\mu + b(C) \int Y d\mu.$$

- (4) If $(X \vee Y)(w) = X(w) \vee Y(w)$ and $(X \wedge Y)(w) = X(w) \wedge Y(w)$ for all $w \in \Omega$, then

$$(C) \int X \vee Y d\mu \geq (C) \int X d\mu \vee (C) \int Y d\mu$$

and

$$(C) \int X \wedge Y d\mu \leq (C) \int X d\mu \wedge (C) \int Y d\mu.$$

3. Non-additive interval-valued measures

Definition 3.1 An interval-valued set function

$\bar{\mu}: \mathfrak{J}(\Omega) \rightarrow [I]$ is a non-additive interval-valued measure if $\bar{\mu}(\emptyset) = \bar{0}$ and $\bar{\mu}(A) \leq \bar{\mu}(B)$, whenever $A, B \in \mathfrak{J}(\Omega)$ and $A \subset B$.

It is easily to see that for each $\bar{\mu}$, there are uniquely two non-additive measures μ^- and μ^+ on $\mathfrak{J}(\Omega)$ such that $\bar{\mu} = [\mu^-, \mu^+]$.

Definition 3.2 The Choquet integral with respect to $\bar{\mu} = [\mu^-, \mu^+]$ of $X \in B(\Omega, \mathfrak{J}(\Omega))$ is defined by

$$(C) \int X d\bar{\mu} = [(C) \int X d\mu^-, (C) \int X d\mu^+].$$

Then we have the following characterization of the functional which is representable as the Choquet integral with respect to $\bar{\mu}$. We recall that a mapping $l: B(\Omega, \mathfrak{J}(\Omega)) \rightarrow I$ is said to be comonotonic affine functional if for all $X, Y \in B(\Omega, \mathfrak{J}(\Omega))$ and $\alpha \in (0, 1)$,

$$l(\alpha X + (1-\alpha)Y) = \alpha l(X) + (1-\alpha)l(Y)$$

and it is said to be monotone if for all $X, Y \in B(\Omega, \mathfrak{J}(\Omega))$ and $X \leq Y$, $l(X) \leq l(Y)$.

Definition 3.3 A mapping $T: B(\Omega, \mathfrak{J}(\Omega)) \rightarrow [I]$ is said to be comonotonic affine interval-valued functional if for all $X, Y \in B(\Omega, \mathfrak{J}(\Omega))$ and $\alpha \in (0, 1)$,

$$T(\alpha X + (1-\alpha)Y) = \alpha T(X) + (1-\alpha)T(Y)$$

and it is said to be monotone if for all $X, Y \in B(\Omega, \mathfrak{J}(\Omega))$ and $X \leq Y$, $T(X) \leq T(Y)$.

Note that a mapping $T = [l_1, l_2]: B(\Omega, \mathfrak{J}(\Omega)) \rightarrow [I]$ is a monotone comonotonic affine interval-valued functional if and only if $l_i: B(\Omega, \mathfrak{J}(\Omega)) \rightarrow I$ is a monotone comonotonic affine interval-valued functional for $i = 1, 2$.

Theorem 3.4 If $T = [l_1, l_2]: B(\Omega, \mathfrak{J}(\Omega)) \rightarrow [I]$ is a monotone and comonotonic affine interval-valued functional, then there exists a monotone non-additive interval-valued measure $\bar{\mu} = [\mu^-, \mu^+]$ uniquely defined by

$$\forall A \in \mathfrak{J}(\Omega), \bar{\mu}(A) = [\mu^-(A), \mu^+(A)]$$

such that

$$T(X) = (C) \int X d\bar{\mu}, \forall X \in B(\Omega, \mathfrak{J}(\Omega)).$$

Conversely, if $\bar{\mu} = [\mu^-, \mu^+]$ is a non-additive interval-valued measure on $\mathfrak{J}(\Omega)$, then $(C) \int (\cdot) d\bar{\mu}$ is a monotone and comonotonic affine interval-valued functional.

Proof. (\Rightarrow) From the above note, we obtain $l_i: B(\Omega, \mathfrak{J}(\Omega)) \rightarrow I$ is a monotone comonotonic affine interval-valued functional for $i = 1, 2$. By Theorem 2.1[11], there exist two non-additive measures μ_1, μ_2

such that

$$l_i(X) = (C) \int X d\mu_i, \forall X \in B(\Omega, \mathfrak{J}(\Omega)), \text{ for } i = 1, 2.$$

Thus for all $X \in B(\Omega, \mathfrak{J}(\Omega))$,

$$\begin{aligned} T(X) &= [l_1(X), l_2(X)] \\ &= [(C) \int X d\mu_1, (C) \int X d\mu_2] \\ &= (C) \int X d\bar{\mu}. \end{aligned}$$

(\Leftarrow) Let $\bar{\mu} = [\mu^-, \mu^+]$ be a non-additive interval-valued measure. If $X, Y \in B(\Omega, \mathfrak{J}(\Omega))$ and $X \leq Y$, by Theorem 2.4 (1),

$$\begin{aligned} (C) \int X d\bar{\mu} &= [(C) \int X d\mu^-, (C) \int X d\mu^+] \\ &\leq [(C) \int Y d\mu^-, (C) \int Y d\mu^+] \\ &= (C) \int Y d\bar{\mu}. \end{aligned}$$

Thus $(C) \int (\cdot) d\bar{\mu}$ is monotone. If $X, Y \in B(\Omega, \mathfrak{J}(\Omega))$, $X \sim Y$ and $\alpha \in (0, 1)$, then

$$\begin{aligned} (C) \int [\alpha X + (1-\alpha)Y] d\bar{\mu} &= [(C) \int [\alpha X + (1-\alpha)Y] d\mu^-, \\ &\quad (C) \int [\alpha X + (1-\alpha)Y] d\mu^+] \\ &= [\alpha (C) \int X d\mu^- + (1-\alpha) \int Y d\mu^-, \\ &\quad \alpha (C) \int X d\mu^+ + (1-\alpha) (C) \int Y d\mu^+] \\ &= \alpha [(C) \int X d\bar{\mu}, (C) \int X d\mu^+] \\ &\quad + (1-\alpha) [(C) \int Y d\mu^-, (C) \int Y d\mu^+] \\ &= \alpha \int X d\bar{\mu} + (1-\alpha) \int Y d\bar{\mu}. \end{aligned}$$

Thus $(C) \int (\cdot) d\bar{\mu}$ is comonotonic affine.

4. Interval-valued necessity measures

In this section, we our concern is to rank interval-valued necessity measures. Let $\wp(\Omega)$ be the power set of Ω .

Definition 4.1 (1) An interval-valued set function $\bar{\nu}: \wp(\Omega) \rightarrow [I]$ is called an interval-valued possibility measure if

$$\bar{\nu}(\emptyset) = \bar{0}, \bar{\nu}(\Omega) = 1 \text{ and } \bar{\nu}(\bigcup_i A_i) = \max_i \bar{\nu}(A_i)$$

for all collections $\{A_i\} \subset \mathfrak{J}(\Omega)$.

(2) An interval-valued set function $\bar{\nu}$ on $\wp(\Omega)$ is called an interval-valued necessity measure if $\bar{\nu}(A) = 1 - \bar{\mu}(A^c)$ for all $A \in \wp(\Omega)$ and $A^c = \{w \in \Omega | w \notin A\}$.

Let Ω be a finite non-empty set and $\wp(\Omega)$ a non-empty family of subsets of Ω . $A^u = \{B \mid A \subset B \subset \Omega\}$ stands for the upset generated by A . Then these sets of subsets of Ω are known as filters(see [11]), we denote the set of filters by $F(\Omega)$. We recall that a family F of subsets of Ω is said to be a filter if

- (i) $\emptyset \notin F, \Omega \in F,$
- (ii) $A, B \in F \Rightarrow A \cap B \in F,$
- (iii) $A \in F, A \subset B \Rightarrow B \in F.$

From Definition 4.1(2) and Proposition 2.1([11]), we obtain the following theorem.

Theorem 4.2 An interval-valued set function $\bar{\nu} = [\nu^-, \nu^+]: \wp(\Omega) \rightarrow [I]$ is an interval-valued necessity measure if and only if ν^-, ν^+ are necessity measures on $\wp(\Omega)$.

Definition 4.3 Interval-valued necessity measures $\bar{\nu}, \bar{\eta}$ are said to be agree if ν^-, η^- (ν^+, η^+ , resp.) are agree, that is, there is no subsets $A, B \in \wp(\Omega)$ such that

$$\begin{aligned} &\nu^-(A) > \nu^-(B) \text{ and } \eta^-(A) < \eta^-(B) \\ &(\nu^+(A) > \nu^+(B) \text{ and } \eta^+(A) < \eta^+(B), \text{ resp.}). \end{aligned}$$

From Definition 4.1 and Definition 4.3, clearly we have the following theorem.

Theorem 4.4 Let $\bar{\nu}, \bar{\eta}$ be interval-valued necessity measures and $\alpha \in (0,1)$. Then, one has

- (1) $\alpha \bar{\nu} + (1-\alpha) \bar{\eta}$ is an interval-valued necessity measure.
- (2) $\bar{\nu}, \bar{\eta}$ are agree if and only if $\alpha \bar{\nu} + (1-\alpha) \bar{\eta}$ is agree.

We recall that if $\bar{\nu} = [\nu^-, \nu^+]: \wp(\Omega) \rightarrow [I]$ is an interval-valued measure, then there is a unique decomposition of $\bar{\nu}$ over unanimity games known as Mobius transforms of ν^- and ν^+ (see [11]):

$$\bar{\nu} = \left[\sum_{j=1}^n \alpha_j u_{A_j}^-, \sum_{k=1}^m \beta_k u_{B_k}^+ \right]$$

where $\alpha_1, \dots, \alpha_n > 0, \beta_1, \dots, \beta_m > 0, \sum_{j=1}^n \alpha_j = 1, \sum_{k=1}^m \beta_k = 1,$
 $\Omega \supset A_1 \supset \dots \supset A_n \neq \emptyset, \Omega \supset B_1 \supset \dots \supset B_m \neq \emptyset,$ $u_{A_j}^-$
 ($u_{B_k}^+$, resp.) denote a unanimity game associated with ν^-
 (ν^+ , resp.), that is, elementary belief function with support A defined by,

$$\forall A \subset \Omega, u_A(B) = \begin{cases} 1 & \text{if } A \subset B \\ 0 & \text{otherwise} \end{cases}$$

or otherwise put, $\bar{\nu}$ can be expressed as follows,

$$\bar{\nu} = \left[\sum_{j=1}^n \alpha_j I_{A_j^u}, \sum_{k=1}^m \beta_k I_{B_k^u} \right]$$

where $\alpha_1, \dots, \alpha_n > 0, \beta_1, \dots, \beta_m > 0, \sum_{j=1}^n \alpha_j = 1, \sum_{k=1}^m \beta_k = 1,$
 $\emptyset \neq A_1^u \subset \dots \subset A_n^u$ and $\emptyset \neq B_1^u \subset \dots \subset B_m^u.$

As a consequence of Proposition 2.1([11]) and Proposition 2.2([11]) with ν^-, ν^+ , given non-additive measures μ^-, μ^+ defined on $\wp(\Omega)$, we can obtain the Choquet integral of an interval-valued necessity measure $\bar{\nu}$ with respect to an interval-valued nonadditive measure $\bar{\mu}$ as following:

$$(C) \int \bar{\nu} d\bar{\mu} = \left[\sum_{j=1}^n \alpha_j \mu^-(A_j^u), \sum_{k=1}^m \beta_k \mu^+(B_k^u) \right].$$

By using the above Choquet integral of an interval-valued necessity measure with respect to an interval-valued non-additive measure, we can discuss that this object is the criterion which is used to rank interval-valued necessity measures in order to obtain a weak integral representation, that is for all interval-valued necessity measures $\bar{\nu}, \bar{\eta}$:

$$\bar{\nu} \geq \bar{\eta} \Leftrightarrow (C) \int \bar{\nu} d\bar{\mu} \geq (C) \int \bar{\eta} d\bar{\mu}.$$

In the future, by using the above weak integral representation, we can study the integral representation of interval-valued preferences which are like ambiguity preferences.

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