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Fuzzy Strongly (r,s)-Irresolute Mappings 관한 연구

On Fuzzy Strongly (r,s)-Irresolute Mappings

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요 약

fuzzy stromgly (r,s)-irresolute 함수, fuzzy strongly (r,s)-irresolute 반열린 함수의 개념을 소개하며 특성들을 조사한다.

Abstract

In this paper, we introduce the concepts of fuzzy strongly (r,s)-irresolute mappings and fuzzy strongly (r,s)-irresolute semiopen mappings and investigate some properties of such mappings.

Key Words : fuzzy strongly (r,s)-irresolute, fuzzy strongly (r,s)-irresolute semiopen, fuzzy strongly (r,s)-irresolute semiclosed.

1. Intorduction

The concept of fuzzy set was introduced by Zadeh [12]. As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Chang [2] defined fuzzy topological spaces using fuzzy sets. Chattopadhyay, Hazra and Samanta [3] introduced the concept of smooth fuzzy topological spaces which are a generalization of fuzzy topological spaces. Coker and Demirci [5] introduced intuitionistic fuzzy topological spaces in Sostak's sense as a generalization of smooth fuzzy topological spaces [10] and intuitionistic fuzzy topological spaces [4]. The concepts of fuzzy (r,s)-open sets and fuzzy (r,s)-semiopen sets [7] are introduced. Lee and Lee [9] introduced and studied the concept of fuzzy strongly (r,s)-semiopen sets. Lee and Kim [8] introduced and studied the concept of fuzzy strongly (r,s)-semicontinuous and fuzzy strongly (r,s)-semiopen mappings. In this paper, we introduce the concepts of fuzzy strongly (r,s)-irresolute mapping and fuzzy strongly (r,s)-irresolute open mapping and investigate some characterizations for them.

2. Preliminaries

Let I be the unit interval $[0,1]$ of the real line. A member μ of I^X is called a fuzzy set of X. By $\tilde{0}$ and $\tilde{1}$, we denote constant maps on X with value 0 and 1, respectively. For any $\mu \in I^X$, μ^c denotes the complement $\tilde{1}-\mu$. All other notations are standard notations of fuzzy set theory.

Let X be a nonempty set. An intuitionistic fuzzy set A is an ordered pair

$A=\{<x,\mu_A(x),\gamma_A(x)>: x\in X\}$ (simply, $A=(\mu_A,\gamma_A)$)

where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership and the degree of non-membership, respectively, and $0 \le \mu_A(x)+\gamma_A(x) \le 1$ for $x \in X$.

An intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X is an intuitionistic fuzzy set $x_{(\alpha,\beta)}=(\mu_A,\gamma_A)$ where the functions the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ are defined as follows

$$(\mu_A(y),\gamma_A(y))=\begin{cases}(\alpha,\beta), & \text{if } y=x,\\ (0,1), & \text{if } y\neq x;\end{cases}$$

and $0 \le \alpha+\beta \le 1$.

An intuitionistic fuzzy point $x_{(\alpha,\beta)}$ is said to belong to an intuitionistic fuzzy set $A=(\mu_A,\gamma_A)$ in X, denoted by $x_{(\alpha,\beta)} \in A$, if $\mu_A(x)\ge\alpha$ and $\gamma_A(x)\le\beta$ for $x\in X$.

An intuitionistic fuzzy set A in X is the union of all intuitionistic fuzzy points which belong to A.

Definition 2.1 ([1]) Let $A=(\mu_A,\gamma_A)$ and $B=(\mu_B,\gamma_B)$ be intuitionistic fuzzy sets on X. Then

(1) $A\subseteq B$ iff $\mu_A\le\mu_B$ and $\gamma_A\ge\gamma_B$.
(2) $A=B$ iff $A\subseteq B$ and $B\subseteq A$.
(3) $A^c=(\gamma_A,\mu_A)$.
(4) $A\cap B=(\mu_A\wedge\mu_B,\ \gamma_A\vee\gamma_B)$.
(5) $A\cup B=(\mu_A\vee\mu_B,\ \gamma_A\wedge\gamma_B)$.
(6) $0_\sim=(\tilde{0},\tilde{1})$ and $1_\sim=(\tilde{1},\tilde{0})$.

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Let f be a map from a set X to a set Y. Let $A=(\mu_A, \gamma_A)$ be an intuitionistic fuzzy set of X and $B=(\mu_B, \gamma_B)$ an intuitionistic fuzzy set of Y.

(1) The image of A under f, denoted by $f(A)$ is an intuitionistic fuzzy set in Y defined by

$$f(A)=(f(\mu_A),\ \tilde{1}-f(\tilde{1}-\gamma_A)).$$

(2) The inverse image of B under f, denoted by $f^{-1}(B)$ is an intuitionistic fuzzy set in X defined by

$$f^{-1}(B)=(f^{-1}(\mu_B),\ f^{-1}(\gamma_B)).$$

A *smooth fuzzy topology* [12] on X is a map $T: I^X \to I$ which satisfies the following properties:

(1) $T(\tilde{0})=T(\tilde{1})=1$.

(2) $T(\mu_1 \wedge \mu_2) \geq T(\mu_1) \wedge T(\mu_2)$ for $\mu_1, \mu_2 \in I^X$.

(3) $T(\vee \mu_i) \geq \wedge T(\mu_i)$ for $\mu_i \in I^X$.

The pair (X, T) is alled a smooth fuzzy topological space.

An *intuitionistic fuzzy topology* on X is a family T of intuitionistic fuzzy sets in X which satisfies the following properties:

(1) $0_\sim, 1_\sim \in T$.

(2) If $A_1, A_2 \in T$, then $A_1 \cap A_2 \in T$.

(3) If $A_i \in T$ for all $i \in I$, then $\cup A_i \in T$.

The pair (X, T) is called an intuitionistic fuzzy topological space.

Let $IF(X)$ be a family of all intuitionistic fuzzy sets of X and let $I \otimes I$ be the set of the pair (r,s) such that $r,s \in I$ and $0 \leq r+s \leq 1$.

Definition 2.2 ([6]) Let X be a nonempty set. An intuitionistic fuzzy topology in Sostak's sense (SoIFT for short) $T=(T_1, T_2)$ on X is a map $T: I(X) \to I \otimes I$ which satisfies the following properties:

(1) $T_1(0_\sim)=T_1(1_\sim)=1$ and $T_2(0_\sim)=T_2(1_\sim)=0$.

(2) $T_1(A \cap B) \geq T_1(A) \wedge T_1(B)$ and $T_2(A \cap B) \leq T_2(A) \vee T_2(B)$.

(3) $T_1(\cup A_i) \geq \wedge T_1(A_i)$ and $T_2(\cup A_i) \leq \vee T_2(A_i)$.

The $(X, T)=(X, T_1, T_2)$ is called an intuitionistic fuzzy topological space in Sostak's sense (SoIFTS for short). Also, we call $T_1(A)$ a gradation of openness of A and $T_2(A)$ a gradation of nonopenness of A.

The fuzzy (r,s)-*closure* and the fuzzy (r,s)-*interior* of A, denoted by $\mathrm{cl}(A, r, s)$ and $\mathrm{int}(A, r, s)$, respectively, are defined as

$\mathrm{cl}(A, r, s) = \cap\{B \in IF(X): A \subseteq B$ and B is fuzzy (r,s)-closed$\}$,

$\mathrm{int}(A, r, s) = \cup\{B \in IF(X): B \subseteq A$ and B is fuzzy (r,s)-open$\}$.

Definition 2.3)[9)] Let A be an intuitionistic fuzzy set in an SoIFTS (X, T_1, T_2) and $(r,s) \in I \otimes I$. Then A is said to be fuzzy strongly (r,s)-semiopen (resp., fuzzy strongly (r,s)-semclosed) if

$$A \subseteq \mathrm{int}(\mathrm{cl}(\mathrm{int}(A, r, s), r, s), r, s)$$

(resp., $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(A, r, s), r, s), r, s)) \subseteq A$).

Let A be an intuitionistic fuzzy set in an SoIFTS (X, T_1, T_2) and $(r,s) \in I \otimes I$.

The fuzzy strongly (r,s)-semiclosure and the fuzzy strongly (r,s)-semiinterior of A, denoted by $\mathrm{sscl}(A, r, s)$ and $\mathrm{ssint}(A, r, s)$, respectively, are defined as

$\mathrm{sscl}(A, r, s) = \cap\{B \in IF(X): A \subseteq B$ and B is fuzzy strongly (r,s)-semiclosed$\}$,

$\mathrm{ssint}(A, r, s) = \cup\{B \in IF(X): B \subseteq A$ and B is fuzzy strongly (r,s)-semiopen$\}$.

Definition 2.4. ([5, 8, 11]) Let a mapping $f: (X, T_1, T_2) \to (Y, U_1, U_2)$ be on SoIFTS's X, Y and $(r,s) \in I \otimes I$. Then f is said to be

(1) fuzzy (r,s)-continuous if for each fuzzy (r,s)-open set B of Y, $f^{-1}(B)$ is a fuzzy (r,s)-open set in X,

(2) fuzzy strongly (r,s)-semicontinuous if for each fuzzy (r,s)-open set B of Y, $f^{-1}(B)$ is a fuzzy strongly (r,s)-semiopen set in X,

(3) fuzzy (r,s)-open if for each fuzzy (r,s)-open set B of X, $f(B)$ is a fuzzy (r,s)-open set in Y,

(4) fuzzy strongly (r,s)-semiopen if for each fuzzy (r,s)-open set B of X, $f(B)$ is fuzzy strongly (r,s)-semiopen in Y.

3. Main Results

Definition 3.1. Let $f: (X, T_1, T_2) \to (Y, U_1, U_2)$ be a mapping on SoIFTS's X, Y and $(r,s) \in I \otimes I$. Then f is said to be fuzzy strongly (r,s)-irresolute if for each fuzzy (r,s)-open set U of Y, $f^{-1}(U)$ is fuzzy strongly (r,s)-semiopen in X.

Remark 3.2. Let $f: (X, T_1, T_2) \to (Y, U_1, U_2)$ be a mapping from SoIFTS's X, Y and $(r,s) \in I \otimes I$. Every fuzzy strongly (r,s)-irresolute mapping is fuzzy strongly (r,s)-continuous but the converse need not be true.

fuzzy (r,s)-continuous $\Rightarrow$ fuzzy strongly (r,s)-semicontinuous $\Leftarrow$ fuzzy strongly (r,s)-irresolute

Example 3.3. Let $X=\{x, y, z\}$ and let A_1 and A_2 be intuitionistic fuzzy sets of X defined as

$A_1(x)=(0.7, 0.2)$, $A_1(y)=(0.8, 0.2)$, $A_1(z)=(0.6, 0.1)$;
$A_2(x)=(0.2, 0.8)$, $A_2(y)=(0.1, 0.9)$, $A_2(z)=(0.1, 0.7)$.

Define an SoIFT $T: I(X) \to I \otimes I$ by

$$T(A)=(T_1(A),\ T_2(A)) = \begin{cases} (1,0), & \text{if } A=0_\sim, 1_\sim, \\ (\frac{1}{2},\frac{1}{3}), & \text{if } A=A_1, A_2, \\ (0,1), & otherwise; \end{cases}$$

and an SoIFT $U: I(X) \rightarrow I \otimes I$ by

$$U(A)=(U_1(A),\ U_2(A)) = \begin{cases} (1,0), & \text{if } A=0_\sim, 1_\sim, \\ (\frac{1}{2},\frac{1}{3}), & \text{if } A=A_1, \\ (0,1), & otherwise. \end{cases}$$

Let $f:(X,T) \rightarrow (X,U)$ be the identity mapping. Then f is fuzzy strongly $(\frac{1}{2},\frac{1}{3})$-semicontinuous but it is not fuzzy strongly $(\frac{1}{2},\frac{1}{3})$-irresolute.

Theorem 3.4. Let $f:(X,T_1,T_2) \rightarrow (Y,U_1,U_2)$ be a mapping on two SoIFTS's X, Y and $(r,s) \in I \otimes I$. Then the following statements are equivalent:

(1) f is fuzzy strongly (r,s)-irresolute.

(2) $f^{-1}(B)$ is fuzzy strongly (r,s)-semiclosed for each fuzzy strongly (r,s)-semiclosed set B of Y.

(3) $f(\mathrm{sscl}(A,r,s)) \subseteq \mathrm{sscl}(f(A),r,s)$ for each intuitionistic fuzzy set A in X.

(4) $\mathrm{sscl}(f^{-1}(B),r,s) \subseteq f^{-1}(\mathrm{sscl}(B,r,s))$ for each intuitionistic fuzzy set B in Y.

(5) $f^{-1}(\mathrm{ssint}(B,r,s)) \subseteq \mathrm{ssint}(f^{-1}(B),r,s)$ for each intuitionistic fuzzy set B in Y.

Proof. (1) $\Rightarrow$ (2) It is obvious.

(2) $\Rightarrow$ (3) Let A be any intuitionistic fuzzy set in X. Since $\mathrm{sscl}(f(A),r,s)$ is a fuzzy strongly (r,s)-semiclosed set in Y, by (2), $f^{-1}(\mathrm{sscl}(f(A),r,s))$ is fuzzy strongly (r,s)-semiclosed. Thus we have

$$\mathrm{sscl}(A,r,s) \subseteq \mathrm{sscl}(f^{-1}(f(A)),r,s) \subseteq f^{-1}(\mathrm{sscl}(f(A),r,s)).$$

It implies $f(\mathrm{sscl}(A,r,s)) \subseteq \mathrm{sscl}(f(A),r,s)$.

(3) $\Rightarrow$ (4) Let B be any intuitionistic fuzzy set in Y. Then, from (3), it follows

$$f(\mathrm{sscl}(f^{-1}(B),r,s)) \subseteq \mathrm{sscl}(f(f^{-1}(B)),r,s) \subseteq \mathrm{sscl}(B,r,s).$$

Hence we have $\mathrm{sscl}(f^{-1}(B),r,s) \subseteq f^{-1}(\mathrm{sscl}(B,r,s))$.

(4) $\Rightarrow$ (5) Let B be any intuitionistic fuzzy set in Y. Then, from (4), it follows

$$\begin{aligned} f^{-1}(\mathrm{ssint}(B,r,s)) &= 1_\sim - (f^{-1}(\mathrm{sscl}(1_\sim - B,r,s))) \\ &\subseteq 1_\sim - \mathrm{sscl}(f^{-1}(1_\sim - B),r,s) \\ &= \mathrm{ssint}(f^{-1}(B),r,s). \end{aligned}$$

Hence $f^{-1}(\mathrm{ssint}(B,r,s)) \subseteq \mathrm{ssint}(f^{-1}(B),r,s)$.

(5) $\Rightarrow$ (1) Let V be a fuzzy strongly (r,s)-semiopen set of Y. From (5), $f^{-1}(V)=f^{-1}(\mathrm{ssint}(V,r,s)) \subseteq \mathrm{ssint}(f^{-1}(V),r,s)$. This implies that $f^{-1}(V)$ is a fuzzy strongly (r,s)-semiopen set.

Theorem 3.5 Let $f:(X,T_1,T_2) \rightarrow (Y,U_1,U_2)$ be a bijective mapping on two SoIFTS's X, Y and $(r,s) \in I \otimes I$. Then f is fuzzy strongly (r,s)-irresolute if and only if $\mathrm{ssint}(f(A),r,s) \subseteq f(\mathrm{ssint}(A,r,s))$ for each $A \in IF(X)$.

Proof. Suppose that f is fuzzy strongly (r,s)-irresolute. For any intuitionistic fuzzy set A of X, since $f^{-1}(\mathrm{ssint}(f(A)r,s))$ is fuzzy strongly (r,s)-semiopen, from Theorem 3.4 and injectivity of f,

$$f^{-1}(\mathrm{ssint}(f(A),r,s)) \subseteq \mathrm{ssint}(f^{-1}(f(A)),r,s) = \mathrm{ssint}(A,r,s).$$

And from surjectivity of f, it follows

$$\mathrm{ssint}(f(A),r,s) = f(f^{-1}(\mathrm{ssint}(f(A),r,s))) \subseteq f(\mathrm{ssint}(A,r,s)).$$

For the converse, let $B \in IF(Y)$ be fuzzy strongly (r,s)-semiopen. From the hypothesis and surjectivity of f, it follows

$$\begin{aligned} f(\mathrm{ssint}(f^{-1}(B),r,s)) &\subseteq \mathrm{ssint}(f(f^{-1}(B)),r,s) \\ &= \mathrm{ssint}(B,r,s) \\ &= B. \end{aligned}$$

Since f is injective, it is $\mathrm{ssint}(f^{-1}(B),r,s) \subseteq f^{-1}(B)$. It implies $\mathrm{ssint}(f^{-1}(B),r,s)=f^{-1}(B)$. Hence f is fuzzy strongly (r,s)-irresolute.

We recall that: Let (X,T) be an SoIFTS. An intuitionistic fuzzy set A in X is said to be fuzzy (r,s)-compact [11] if for every fuzzy (r,s)-open cover $A=\{A_i \in IF(X):\ i \in J\}$ of A, there exists $J_0=\{1,2,\cdots,n\} \subseteq J$ such that $A \subseteq \cup_{i \in J_0} A_i$.

Definition 3.6 Let (X,T) be an SoIFTS. An intuitionistic fuzzy set A in X is said to be fuzzy strongly (r,s)-semicompact if for every fuzzy strongly (r,s)-semiopen cover $S=\{\ A_i \in IF(X):\ i \in J\ \}$ of A, there exists $J_0=\{1,2,\cdots,n\} \subseteq J$ such that $A \subseteq \cup_{i \in J_0} A_i$.

Theorem 3.7 Let $f:(X,T_1,T_2) \rightarrow (Y,U_1,U_2)$ be strongly (r,s)-irresolute on two SoIFTS's X, Y and $(r,s) \in I \otimes I$. If A is a fuzzy strongly (r,s)-semicompact set, then $f(A)$ is also fuzzy strongly (r,s)-semicompact.

Proof. Let $\{\ B_i \in IF(Y):\ i \in J\ \}$ be a fuzzy strongly (r,s)-semiopen cover of $f(A)$ in Y. Then $\{\ f^{-1}(B_i):\ i \in J\ \}$ is a fuzzy strongly (r,s)-semiopen cover of A in X. By definition of fuzzy strongly (r,s)

-semicompactness, there exists $J_0 = \{1, 2, \cdots, n\} \subseteq J$ such that $A \subseteq \cup_{i \in J_0}(f^{-1}(B_i))$.

This implies

$$f(A) \subseteq f(\cup_{i \in J_0}(f^{-1}(B_i))) = \cup_{i \in J_0} f(f^{-1}(B_i)) \subseteq \cup_{i \in J_0} B_i.$$

Hence $f(A) \subseteq \cup_{i \in J_0} B_i$.

Theorem 3.8 Let $f:(X, T_1, T_2) \to (Y, U_1, U_2)$ be a strongly (r,s)-semicontinuous mapping on two SoIFTS's X, Y and $(r,s) \in I \otimes I$. If A is a fuzzy strongly (r,s)-semicompact set, then $f(A)$ is fuzzy (r,s)-compact.

Proof. It is similarly proved from the above Theorem 3.7.

Definition 3.9 Let $f:(X, T_1, T_2) \to (Y, U_1, U_2)$ be a mapping on two SoIFTS's X, Y and $(r,s) \in I \otimes I$. Then f is called a fuzzy strongly (r,s)-irresolute semi-open (resp., fuzzy strongly (r,s)-irresolute semiclosed) mapping if for every fuzzy strongly (r,s)-semiopen (resp., fuzzy strongly (r,s)-semiclosed) set A in X, $f(A)$ is fuzzy strongly (r,s)-semiopen (resp., fuzzy strongly (r,s)-semiclosed) in Y.

Remark 3.10 Let $f:(X, T_1, T_2) \to (Y, U_1, U_2)$ be a mapping on two SoIFTS's X, Y and $(r,s) \in I \otimes I$. Every fuzzy strongly (r,s)-irresolute semiopen (resp., fuzzy strongly (r,s)-irresolute semiclosed) mapping is fuzzy strongly (r,s)-semiopen (resp., fuzzy strongly (r,s)-semiclosed) but the converse need not be true.

fuzzy (r,s)-open $\Rightarrow$ fuzzy strongly (r,s)-semiopen
$\Leftarrow$ fuzzy strongly (r,s)-irresolute semiopen

Example 3.11 In Example 3.3, consider the identity $f:(X, T) \to (X, U)$ is a fuzzy strongly (r,s)-semiopen mapping but not fuzzy strongly (r,s)-irresolute semiopen.

Theorem 3.12 Let $f:(X, T_1, T_2) \to (Y, U_1, U_2)$ be a mapping on two SoIFTS's X, Y and $(r,s) \in I \otimes I$. The the following are equivalent:

(1) f is fuzzy strongly (r,s)-irresolute semiopen.
(2) $f(\text{ssint}(A,r,s)) \subseteq \text{ssint}(f(A),r,s)$ for $A \in IF(X)$.
(3) $\text{ssint}(f^{-1}(B),r,s) \subseteq f^{-1}(\text{ssint}(B,r,s))$ for each $B \in IF(Y)$.
(4) For $B \in IF(Y)$ and each fuzzy strongly (r,s)-semiclosed set A of X with $f^{-1}(B) \subseteq A$, there exists a fuzzy strongly (r,s)-semiclosed set C of Y such that $B \subseteq C$ and $f^{-1}(C) \subseteq A$.

Proof. (1) $\Rightarrow$ (2) For $A \in IF(X)$,

$f(\text{ssint}(A,r,s))$
$= f(\cup\{ B \in IF(X): B \subseteq A,\ B$ is fuzzy strongly (r,s)-semiopen $\})$
$= \cup\{ f(B) \in IF(Y): f(B) \subseteq f(A),\ f(B)$ is fuzzy strongly (r,s)-semiopen $\}$
$\subseteq \cup\{ U \in IF(Y): U \subseteq f(A),\ U$ is fuzzy strongly (r,s)-semiopen $\}$
$= \text{ssint}(f(A),r,s)$.

Hence $f(\text{ssint}(A,r,s)) \subseteq \text{ssint}(f(A),r,s)$.

(2) $\Rightarrow$ (3) For $B \in IF(Y)$, from (2), it follows that

$$f(\text{ssint}(f^{-1}(B),r,s)) \subseteq \text{ssint}(f(f^{-1}(B)),r,s) \subseteq \text{ssint}(B,r,s).$$

Hence $\text{ssint}(f^{-1}(B),r,s) \subseteq f^{-1}(\text{ssint}(B,r,s))$.

(3) $\Rightarrow$ (4) Let A be a fuzzy strongly (r,s)-semiclosed set of X with $f^{-1}(B) \subseteq A$ for $B \in IF(Y)$. Then $1_\sim - A \subseteq 1_\sim - f^{-1}(B) = f^{-1}(1_\sim - B)$ and so $\text{ssint}(1_\sim - A,r,s) = 1_\sim - A \subseteq \text{ssint}(f^{-1}(1_\sim - B),r,s)$.

And by (3), we have

$$1_\sim - A \subseteq \text{ssint}(f^{-1}(1_\sim - B),r,s) \subseteq f^{-1}(\text{ssint}(1_\sim - B),r,s).$$

Thus $A \supseteq 1_\sim - (f^{-1}(\text{ssint}(1_\sim - B,r,s)))$
$= f^{-1}(1_\sim - \text{ssint}(1_\sim - B,r,s))$
$= f^{-1}(\text{sscl}(B,r,s))$.

Set $C = \text{sscl}(B,r,s)$. Then C is a fuzzy strongly (r,s)-semiclosed set of Y such that $B \subseteq C$ and $f^{-1}(C) \subseteq A$. Hence the statement (4) is satisfied.

(4) $\Rightarrow$ (1) Let A be fuzzy strongly (r,s)-semiopen in X. Then $f^{-1}(1_\sim - f(A)) = 1_\sim - f^{-1}(f(A)) \subseteq 1_\sim - A$.

Since $1_\sim - A$ is fuzzy strongly (r,s)-semiclosed, by (4), there exists a fuzzy strongly (r,s)-semiclosed set C such that $1_\sim - f(A) \subseteq C$ and $f^{-1}(C) \subseteq 1_\sim - A$. It implies that $1_\sim - C \subseteq f(A)$ and

$$f(A) \subseteq f(1_\sim - f^{-1}(C)) = f(f^{-1}(1_\sim - C)) \subseteq 1_\sim - C.$$

Hence $f(A)$ is a fuzzy strongly (r,s)-semiclosed set in Y.

Theorem 3.13 Let $f:(X, T_1, T_2) \to (Y, U_1, U_2)$ be a mapping on two SoIFTS's X, Y and $(r,s) \in I \otimes I$. Then the following are equivalent:

(1) f is fuzzy strongly (r,s)-irresolute semiclosed.
(2) $\text{sscl}(f(A),r,s) \subseteq f(\text{sscl}(A,r,s))$ for $A \in IF(X)$.
(3) For $B \in IF(Y)$ and each fuzzy strongly (r,s)-semiopen set A of X with $f^{-1}(B) \subseteq A$, there exists an fuzzy strongly (r,s)-semiopen set C of Y such that $B \subseteq C$ and $f^{-1}(C) \subseteq A$.

Proof. It is similarly proved from Theorem 3.12.

Theorem 3.14 Let $f:(X, T_1, T_2) \to (Y, U_1, U_2)$ be a bi-jective mapping on two SoIFTS's X, Y and $(r,s) \in$

$I \otimes I$. Then

(1) f is fuzzy strongly (r,s)-irresolute semiopen if and only if $\text{ssint}(f^{-1}(A),r,s) \subseteq f^{-1}(\text{ssint}(A,r,s))$ for each $A \in IF(Y)$.

(2) f is fuzzy strongly (r,s)-irresolute semiclosed if and only if $f^{-1}(\text{sscl}(A,r,s)) \subseteq \text{sscl}(f^{-1}(A),r,s)$ for each $A \in IF(Y)$.

Proof. (1) Suppose that f is fuzzy strongly (r,s)-irresolute semiopen. For any intuitionistic fuzzy set A of Y, from Theorem 3.12 and surjectivity of f,

$$f(\text{ssint}(f^{-1}(A),r,s)) \subseteq \text{ssint}(f(f^{-1}(A)),r,s) = \text{ssint}(A,r,s).$$

It implies $\text{ssint}(f^{-1}(A),r,s) \subseteq f^{-1}(\text{ssint}(A,r,s))$.

Conversely, let $B \in IF(X)$ be fuzzy strongly (r,s)-semiopen. Then from hypothesis and injectivity of f, it follows

$$\text{ssint}(f^{-1}(f(B)),r,s) = \text{ssint}(B,r,s) \subseteq f^{-1}(\text{ssint}(f(B),r,s)).$$

And from surjectivity of f, we have the following

$$f(\text{ssint}(B,r,s)) \subseteq f(f^{-1}(\text{ssint}(f(B),r,s))) = \text{ssint}(f(B),r,s).$$

Hence from Theorem 3.12, f is fuzzy strongly (r,s)-irresolute semiopen.

(2) It is similar to (1)

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