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The Effect of Problem Posing Oriented Calculus-II Instruction on Academic Success[1]

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There are concepts in calculus which are difficult to teach and learn. One of these concepts is integration. However, problem posing has not yet received the attention it deserves from the mathematics education community. There is no systematic study that deals with teaching of calculus concepts by problem posing oriented teaching strategy. In this respect this study investigated the effect of problem posing on students' (prospective teachers') academic success when problem posing oriented approach is used to teach the integral concept in Calculus-II (Mathematics-II) course to first grade prospective teachers who are enrolled to the Primary Science Teaching Program of Education Faculty. The study used intervention–posttest experimental design. Quantitative research techniques were employed to gather, analyze and interpret the data. The sample comprised 79 elementary prospective science teachers. The results indicate that problem posing approach effects academic success in a positive way and at significant level.

Keywords: academic success, calculus course, problem solving, problem posing, mathematics teaching
ZDM Classification: D50
MSC2000 Classification: 97D10

[1] This paper is based on the first author's Ph. D. study and an earlier version of the paper was presented at the Eighth International Technology Conference held in May 6–9, 2008; Anadolu University, Eskişehir, Turkey.

* In the memory of Hayri Akay: His life was taken by lung cancer during the publication process of this paper.

INTRODUCTION

The activities in problem solving are used as a transition stage for proposing a related problem. Gonzales (1998) added problem posing as the fifth step to the four stepped problem solving strategy of Polya. National Council of Teachers of Mathematics (NCTM, 1989; 1991; 2000) advocates students' participation to the problem posing activities that includes the generation of their own problems. In particular, NCTM (2000, p. 258) document states that students must be given opportunities to "formulate interesting problems based on a wide variety of situations, both within and outside mathematics." Most authors agree that problem posing is a term used to mean both the generation of new problems and the reformulation of given problems (Cai & Hwang, 2002; English, 2003; Silver & Cai, 1996; cited by Christou, Mousoulides, Pittalis & Pitta-Pantazi, 2005). That is why we think that it is necessary to examine the problem posing instruction in Turkey.

There are very few studies about university level students' problem posing approaches in mathematics. Some of these studies show that problem posing oriented teaching increases academic success of first and second level primary school students. However, there are conflicting studies about the effect of problem posing oriented mathematics course on academic success of older students. The researches that examine students' cognitive processes in problem posing don't have definite results. There are still some unanswered questions that need to be addressed:

> "Do students need high level creativity in order to be good problem poser or to be academically successful as a result of problem posing oriented course?"
>
> "Could students who have low mathematical ability benefit from problem posing oriented course?"

Aims of This Study

The aim of the study can be described as follows;

- To maintain the passage of pre-service teachers to problem posing stage by adding a fifth step to four stepped problem posing strategy of Polya (1957) and to influence their mental processes positively (Gonzales, 1998).
- To develop classroom activities that are appropriate to problem solving and problem posing strategies in Mathematics-II courses for the pre-service science teachers.
- To explore the effect of problem posing strategies of pre-service science teachers on their academic success.

Problem of the Study

What is the effect of problem posing oriented Calculus-II (Mathematics-II) course over traditional instruction on prospective teachers' academic success?

Assumptions

During the research process:

- It is assumed that both experimental and control group students are affected equally from outer factors that could not be controlled.
- It is assumed that both experimental and control group students have equal level of interests in learning.
- It is assumed that both experimental and control group students' academic success levels are equal considering their University Entrance Exams' (OSS) points and they would take Calculus-II course for the first time.
- It is assumed that both experimental and control group students do not interact in a way that would affect the results of the study.
- It is assumed that both experimental and control group students fill in the Academic Success Test (AST) frankly.
- It is assumed that researcher carry out the research unbiased.

THEORETICAL FRAMEWORK AND REVIEW OF LITERATURE

Academic Success

Since academic success is knowledge and abilities related to the topics that are covered in a period of time, in order to elicit these, the most suitable assessment strategies should be employed. Paper-pencil tests are widely used to assess such achievements (Yaman, 2003).

Teachers use achievement exams in order to determine students' level of knowledge and skills, detect students' strong and weak aspects, and compare performances of students who have similar level of attainment (Airasian, 2000). In order to assess these, various kinds of measurement tools that differ with respect to their aims can be developed. Achievement tests are divided into two groups as standard achievement tests and teacher made tests (Mehrens & Lehmann, 1987). Standard achievement tests are tests that are developed by field experts; that have high reliability and validity. These tests have general aims and are widely used and well accepted in the field. On the other hand teacher made tests are for particular aims. They are prepared by teachers to use in his/her classrooms and their validity and reliability are not measured (Yaman, 2003).

The achievement test used in this study is prepared as standard achievement test. The questions in the test are prepared by experts as classical format. Content validity and its reliability are measured.

Problem Solving and Problem Posing

Problem solving has a fundamental role in the learning and understanding of mathematics (NCTM, 2000; Schoenfeld, 1985; Polya, 1973). NCTM (2000) defines problem solving as "engaging in a task for which the solution method is not known in advance," and suggests that "solving problems is not only a goal of learning mathematics but also a major means of doing so" (2000, p. 52). Schoenfeld (1985) points out, what might be a significant task for one student could be routine or second-nature for another. Here lies one challenge for the mathematics teacher: picking quality problems of which solution strategies are not immediately known to each student, but which are within each student's grasp (Perrin, 2007).

Central to the idea of problem posing is the regarding students as interactive, social learners involved in the project of knowledge creation (Friere, 1970; Gregson, 1994; Wallerstein, 1987). Freire (1970) contrasted problem-posing education with teacher-dominated education, which he deemed it as "banking" education (Lewis, Petrine & Hill, 1998). The core of mathematical investigations and scientific research entails problem posing and solving activities. Beside this problem posing being an important component of problem solving process lies at the hearth of mathematical activities (Kilpatrick, 1987). Problem posing is defined as reformulating given problems and generating new problems (Silver, 1994). Problem posing is not limited to generating new problems from given mathematical situations or by changing the conditions of given problems.

Problem posing also entails reformulating given problems and generalization for the solution. Problem posing is not independent from problem solving (Silver, 1994; Cai & Hwang, 2002; English, 2003; Silver & Cai, 1996; Lewis, 1998). There is a close relationship between problem posing and solving as a cognitive process (Lowrie, 2002). Problem posing is most closely associated with the "looking back" stage of Polya's four steps to problem solving. This is considered by Polya to be the most important step (Silver, Mamona-Downs, Leung, Shukkwan & Kenney, 1996). In scientific inquiry formulating a good problem can be more important than discovering a solution for the problem (Einstein & Infeld, 1938; Cai & Hwang, 2003). One of the important consensuses in mathematics education is to provide opportunities to students in mathematics lessons for developing their problem posing skills (Brown & Walter, 2005; NCTM, 2000).

Silver (1994) argued that problem posing is appealing as a teaching method for the following reasons:

(a) It has relationship to creativity and exceptional mathematics ability,
(b) It may be used to improve students' problem solving ability,
(c) It may be used to assess students' understanding of mathematics,
(d) It may be used to improve students' disposition towards mathematics, and
(e) It may be used to help students become autonomous learners.

Silver (1994, p. 22) also claimed that problem posing was "central to the discipline of mathematics and the nature of mathematics thinking." Furthermore, problem posing is an alternative method for students who are less successful in problem solving. Since students write problems in their dialect, the role of grammar, reading abilities become less important than in the case of textbook problems. Choosing problem situations from students' familiar contexts makes the problem more interesting. Students who pose the problems become more interested and motivated in finding a solution to the problem. In addition, as students become more familiar with the problem, they determine the important parts of the problem structure and catch clues for the solution (Dickerson, 1999).

Problem posing is a strategy that aims to increase students' mathematical success by helping students to deal with meaningful and real mathematics (Silver, 1994). This study investigates the effect of problem posing oriented Calculus-II course on academic success. It is important to determine whether or not problem posing strategies are suitable for all students regardless of their past success in mathematics. Although it is favored by some schools, method of increasing success by forming homogeneous groups is failed (Schmidt et al., 1998; Dickerson, 1999). Although it is recommended by NCTM, and its effectiveness is reported by some studies, problem posing has not been drawn its deserved attention from mathematics educators (English, 1998). Problem posing will help teachers to get their students to advance levels which are advocated by TIMSS (1998). To sum up, problem posing stands out as a teaching strategy that improves mathematics teaching (Dickerson, 1999).

Calculus Course

Calculus course can be regarded as an arch that is built up by arranging ideas instead of stones. Derivative concept is one of the feet of this arch and integral concept is the other foot. Newton and Leibnitz found a key stone that joins these two feet at the top. The name of this key stone is the Fundamental Theorem of Calculus. At the foundation of this arch, the concept of limit lies all along (King, 1992). Hence, we see a many researches that deal with teaching of calculus concepts in the past 30 years (Tall, 1985; Ubuz, 1996; Davidenko, 1997; Tall, 1997; Ubuz & Kırkpınar, 2000). It is reported by many researchers that we have difficulties in teaching calculus concepts (Tall, 1977; Tall &

Schwarzenberger, 1978; Ervynck, 1981; Tall & Vinner, 1981; Davis & Vinner, 1986; Tall, 1993; Monaghan, Sun & Tall, 1994; Tall, 1997; Peker, 2003).

In calculus courses teachers generally try to avoid problems that undervalue techniques and use informal approaches. But, whatever teaching method is used there has been dissatisfaction in calculus course for the past 18 years across various countries in the world. On the other hand whatever method is used there are some concepts in calculus course that are hard to teach and learn (Tall, 1993). Schwarzenberger (1978) discusses why calculus is not learnt easily and states that mathematical difficulties of classical analysis can not be clarified by a simple explanation. However, this idea declares the mathematical difficulties not the cognitive difficulties of calculus (Tall, 1985). Calculus course is taught to first year students of many departments in Turkish Universities. In this study, "Integrals and its applications" unit of calculus course is the topic of interest. This unit is designed in a way that students achieve skills and professional qualifications related to definite and indefinite integrals. It is divided into three groups as definite integral, indefinite integral and application of integrals according to teaching plan and problem posing approach.

In general, when we review the literature we observe scarcity of the research that deal with problem posing oriented approach in teaching of university level courses. In these studies, it is difficult to find examples of activities and materials that are suitable to problem posing oriented teaching approach. In this respect, Silver (1994) points out that: despite the interest in problem posing, we don't see systematic studies that elaborate problem posing as a part of mathematics curriculum (Lowrie & Whitland, 2000). Therefore we want to look at whether students achieve significant success as a result of problem posing oriented calculus course?

As Perrin (2007) points out that posing and then solving novel problems is more suitable to the students who are sophisticated in mathematics rather than solving problems that are modifications of preexisting problems. Furthermore, we agree with Perrin (2007) that topics in single-variable calculus such as optimization, related rates, and exponential and logistic growth provide natural contexts for problem solving activities. That is, calculus concepts are very appropriate for many other problem-solving activities (Perrin, 2007). As cited by Perrin (2007), some excellent examples of novel and interesting questions that can be explored in the calculus classroom are given by some scholars (Brannen & Ford, 2004; Campbell, 1999; Dunkels & Persson, 1980; Kenyon & Bardzell, 2001).

METHODOLOGY

This study is based on Ph. D. study of Akay (2006). Therefore, the study had two phases: pilot study and experimental phase. The pilot study was carried out in 2004–2005 academic year with 80 first year (40 experimental group students, 40 control group students) primary science teaching students. This phase lasts for 9 weeks during "Derivative and Integral" unit of Mathematics-II course. The experimental study was carried out in 2005–2006 academic year with 79 first year primary science teaching students.

Participants

As we mentioned the sample of this study consists of 79 first year primary science teaching students. These students were in two different sections and these sections are randomly selected as experimental and control group. In experimental group there were 40, in control group there were 39 prospective teachers. Prospective teachers are told that they were participants of a research study but they were not told whether they were in experimental or control group. This is one of the requirements of experimental study (Eckhardt & Ermann1977; Büyüköztürk, 2001). The participants have very close OSS points, and they are all getting similar teaching. Therefore, they have similar academic characteristics.

Research Method

This research is an experimental design research. The model of research is post-test control grouped experimental design. Such kind of design is called "quasi-experimental design." Experimental and control grouped students are considered as equal academic success at the beginning of the study since they have close OSS points, and they would take Mathematics-II course for the first time. This method can be called as non-equivalent control grouped design. That is, in post-test control grouped experimental design model, in order to compare experimental group, an equivalent control group is taken.

During research study, the independent variable that its effect on experimental group is measured is "Problem Posing Oriented Teaching, and the independent variables that its effect on control group is "Traditional Teaching Approach." The dependent variable in this research is student academic success.

Data Gathering Instruments

An academic success test is developed by the researcher to measure the effect of problem posing oriented teaching of integral unit of Mathematics Calculus-II (Mathematics-II) course.

Academic Success Test

The aim of academic success test is to determine attainment level, and problem posing and solving skills of students after problem posing oriented teaching process. While developing this test, the researcher considers the related attainment targets, checks previous final, midterm exams, and resources related to integral unit. For this aim in the study different methods were used in experimental and control groups. The developed academic success test consists of classical problems that also aim to assess problem posing and solving strategies of students. The problems that are prepared by the researcher are checked by a professor, an associate professor, and three Ph. D. students.

The test was piloted during 2004–2005 academic year spring term with 64 first year university students who were taking Mathematics-II course. The reliability of test is measured by 2×2 cross tabulation of the scores given by a professor and two research assistants. Pearson product-moment correlation coefficients were found as (0.98), (0.97) and (0.95) (Akay, 2006). To sum up academic success was measured by means of a test that had five problems and was developed by considering attainment targets of "Integral and its Applications Unit" of Calculus-II course. Content validity of the test was satisfied by taking expert opinions, and trying to prepare the problems in a way that they would cover the unit attainments.

Analysis of Data

In order to measure the effect of problem posing oriented course on academic success, t-test for independent groups.

Experimental Research Design

Experimental study phase is carried out by the researcher and teaching assistants during 2005–2006 academic year spring term in Integral unit of Mathematics-II course. This phase lasted 8 weeks (32 hours, four hours in each week).

It was assumed that both of the groups were affected equally by exterior factors and both of them have same degree of interest in learning. Both of the groups are assessed by the same academic success test at the end of the teaching process.

Teaching Strategy That is Followed in Experimental Group

The prospective teachers in the experimental group attended to 28 problem posing activities that were prepared previously in the pilot study in "Integral and its Applications Unit" of Mathematics-II course.

In this study, the framework of these problem posing activities was developed by the researcher and the participant teacher by considering the "*what if not*" strategy (Brown & Walter, 1993; 2005), "*semi-structured problem posing situations", "structured problem posing situation" and "free problem posing situations"* (Stoyanova & Ellerton, 1996; Stoyanova,1998).

The students in experimental group firstly study strategies based on Polya's four steps of problem solving. After this, the idea of adding fifth step to Polya's four steps is tried to be developed (Gonzales, 1998). Then, 28 different problem posing activities are applied. The given problems are chosen from textbooks (Edwards & Penney, 2001) and a webpage named "*visual calculus*"[2] in 2004. This webpage is especially beneficial for spatial reasoning that is required to solve problems related to area and volume.

Some examples of these problem posing and visual calculus activities are presented in Table 2 and Table 3.

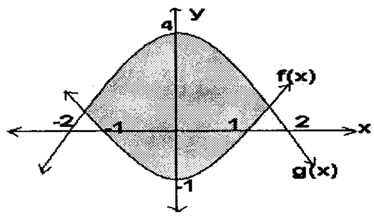
Table 2. Some problem posing activities used in experimental group

Activity 1: Get the function which anti-derivative "$x \cdot e^{\sin x} + \sin(e^x) + 5$".

Activity 2: Write a problem involving definite integral that has the result "*sin (b)-sin (a)*" where $a, b \in R$. Check your answer.

Activity 3: "When you graduate from the faculty you are going to get engaged. But you are going to design your own ring. How do you design your ring and how do you calculate its cost?

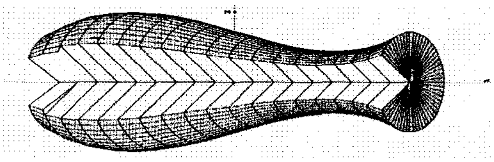
Activity 4: As can be seen in the figure shaded region is bounded by $f(x)$ and $g(x)$ parabolas. Write problems related to this figure.



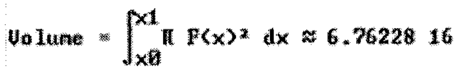
[2] http://archives.math.utk.edu/visual.calculus/
"Visual Calculus-Integration". Retrieved from:
http://archives.math.utk.edu/visual.calculus/4/index.html

Table 3. Some "Visual Calculus" activities used in experimental group

Activity 5: Draw the solid of revolution which is obtained by rotating the area of the region bounded by the graph of the function **f(x) = x³ - x + 1** the lines **x = -1, x = 1** and the **x**-axis around the **x**-axis

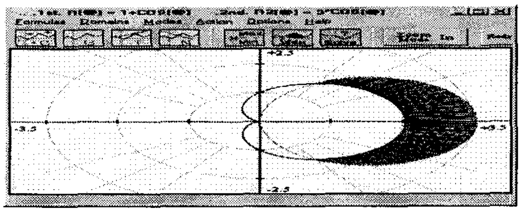


These graphs were generated by Microcalc. For this visualization, we have detailed instructions for the following software packages: Microcalc MPP3D . After the solid is graphed, then pres spacebar to see the following:

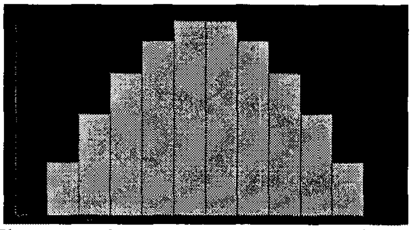


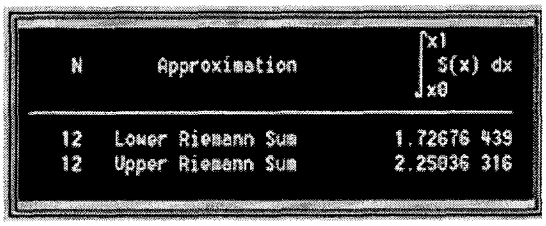
Activity 6: Find the area of the region which lies inside the graph of $g(\theta) = 3\cos(\theta)$ and outside of the graph of $f(\theta) = 1 + \cos(\theta)$

For this visualization, we have detailed instructions for the following software packages: JKGraph



Activity 7: Approximate the integral $\int_0^{\pi} \sin(x)dx$ by taking the average of the upper and lower Riemann sums with n=12.





These graphs were generated by Microcalc.

Teaching Strategy That Is Followed In Control Group

In this group integral unit is taught by employing traditional teaching strategies (teacher centered; teaching by telling, question-answer, exercises). The same textbook is used in this group also (Edwards & Penney, 2001). The mathematics course was run by the same instructor the following the same course content during the study.

FINDINGS AND INTERPRETATION

In this section comparison of data gathered from experimental and control group will be presented.

Table 4. Descriptive statistical results related to academic success of experimental and control group

Group	*N*	$\overline{X}$	S. S.	S. Error
Experiment	41	71.51	21.11	3.30
Control	38	37.21	16.12	2.61

Table 5. T-test results of post-test academic success test with respect to groups

Levene's Test		T-Test for Equality of Means				
F	*p*	*t*	*df*	*p*	Mean Dif.	S. Error Dif.
1.36	0.247	8. 07	77	0.000*	34. 30	4.25

According to results of t-test for independent groups, prospective teachers post-test scores differ significantly with respect to group variable (experiment-control) [t (77) = 8.070; $P <$,01]. The average post-test score of experimental group prospective teachers is ($\overline{X} = 71.51$) whereas this ($\overline{X} = 37.21$) for control group students. This difference is statistically significant.

RESULTS AND RECOMMENDATIONS

The aim of this study was to explore the effect of problem posing oriented teaching strategy on prospective teachers' academic success after Calculus-II (Mathematics-II) Course was taught to first grade Primary Science Teaching Program prospective teachers. The results indicate that problem posing approach effects academic success in a positive way and at significant level (see Table 4 and Table 5). That is problem posing approach employed in experimental group is more effective in increasing academic success than teacher centered traditional teaching approach. One of the underlying reasons of this effect can be that problem posing teaching strategy provides opportunities for active involvement of students. In this respect, we suggest that teachers might use problem posing and solving oriented lessons in teaching mathematics rather than just touching the topics. In activities that are directed by students, more real life situations should be considered. In order to do this, some topics in the curriculum must be combined with problem posing approach.

The academic success scores of prospective teachers in this study reflects the attainment targets of integral and its applications unit together with students' problem solving and posing skills. In this respect, problem posing oriented teaching strategy is an effective teaching method for improving prospective teachers' problem solving and posing skills. That is, there is a close relationship between academic success (related to problem posing and solving skills) and problem posing oriented course. However, it is still unclear whether or not problem posing oriented course can be beneficial to low ability students.

Our research findings suggest that primary science teaching students who can be considered as relatively less successful in mathematics increase their problem solving and posing performance as a result of problem posing oriented course. Hence, our results support the claims of Silver (1997) who asserts that problem posing oriented approach would be beneficial not only to high achievers but also low achievers. In other words, we support several researchers who suggest that problem posing is a strategy that includes all students in creative activities; not only genius or exceptional ones would engage in and accomplish creative tasks (Hadamard, 1945; Polya, 1954; Silver, 1994; 1997)

To sum up, we conclude that problem posing oriented calculus course significantly affects students' problem posing and solving skills related to integral concept. We explain this as follows: while prospective teachers are solving the given problems they have opportunity to discuss every step of problem solving when problem posing strategies are emphasized. This adds support to Gonzales (1994), Leung (1993) and Abu-Elwan's (1999) findings. We recommend that this study could be replicated with different bunch of students for teaching different calculus concepts.

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