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Identifying and Sequencing of the Elementary Concepts of Measurement of Length[1]

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In this paper some attempts have been made
(a) to identify all the elementary concepts of the major concept "measurement of length" and,
(b) to find the sequential order of these elementary concepts. Total 714 elementary concepts have been identified and sequenced

Keywords: measurement, length, elementary concepts
ZDM Classification: B10, B70, C70
MSC2000 Classification: 97C90, 97C30

0. INTRODUCTION

The present world is enlightened in the light of modern science. Mathematics lies in its root. It is the only subject where our logical faculty is nurtured and nourished from the very beginning and ultimately tuning to a scientific mind-set. Accordingly Kothari Commission has stressed on mathematics education from early stage (Kothari, 1966). In India, mathematics has been made a compulsory subject up to class 10 standard of schooling. But unfortunately, the analysis of various Boards' results conducted by National Council of Educational Research and Training and edited by Buch (1991) shows that a large number of students are very poor in mathematics. But according to the great psychologist & philosopher Piaget (1973), "Every normal child is capable of learning mathematics." Here lies the challenge of mathematics education. Definitely there are

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many reasons behind this of which "improper content organization" in the syllabi is an important factor that will be discussed in this paper. In this regard, Pushpanadham (1998) stated, "Mathematics is the subject which requires proper understanding of the concepts and their interrelations." Chilana (1984) opined, "Mathematical concepts should be graded in such a way that there is a proper sequence in their growth and development." Again the comment of "Ben Clarke Pacific Institutes for Research" (2006) on Early Mathematics Assessment is "Emerging evidence suggests difficulty in mathematics is relatively stable over time. The importance of a logical scope and sequence is paramount." So it may be concluded that the proper sequence of "basic concepts" according to their logical and psychological hierarchy should be maintained in the teaching-learning system of mathematics.

Generally the process of "syllabus framing and content organization" is made on the basis of experts' opinion but no systematic analytical method is applied in this process. As a result, many gaps are developed in the entire teaching-learning system which is very harmful to the students of mathematics because mathematics is purely a logical science. Therefore, to remove the said conceptual gaps, concept identification and its proper sequencing have an important role. The researcher has identified and sequenced the basic concepts of mathematics through "text book scanning process" and "task analysis technique". In this regard, identification of the basic concepts and their sequencing is a worthwhile enterprise which has not yet been undertaken by others.

The author has made some contribution towards identification and sequencing of some basic concepts and framing of a model syllabus also (Alam, 2003a; 2003b; 2005a; 2005b; 2006a; 2006b; 2006c; 2006d; 2007a; 2007b; 2007c; 2007d; 2008a; 2008b; 2008c; 2008d; 2008e; 2008f;2009a; 2009b).

In this paper, some attempts have been made for identifying and sequencing the elementary concepts of measurement of length.

1. DEFINITIONS

1.1. Major concept and Sub-concepts

A Major concept is an idea which is complete in it and is comprehended through a sequential process of step-by-step partial comprehension of its related concepts. These related concepts are called sub-concepts which are not complete in it but are parts of the major concept.

Examples of major concepts are "addition of whole numbers including the familiarity of the numbers", "subtraction of whole numbers", "measurement of length", "fraction", "decimal", "rational number" etc.

For the major concept such as "Addition of whole numbers including the familiarity of the numbers", its sub-concepts are "addition of one-digit numbers including zero where the sum is one-digit number", "addition of one-digit numbers where the sum is two-digit number" etc.

1.2. First level sub-concepts

The sub-concepts which are obtained after immediate derivation of a major concept are called first level sub-concepts.

1.3. Second level sub-concepts and others

When the derivation of first level sub-concepts are continued, the other sub-concepts so obtained stage by stage are called second level, third level etc.

1.4. Elementary concepts

A sub-concept which cannot be split further will be called an "elementary concept".

2. OBJECTIVE OF THE STUDY

The objective of the study is to identify different elementary concepts and their sequential order of measurement of length.

3. PROCEDURE

Procedure adopted has been discussed below:

3.1. Collection of text books

The prescribed text books of mathematics from Class 1 to 10 of West Bengal Board of Primary Education (WBBPE)[2], West Bengal Board of Secondary Education (WBBSE)[3], National Council of Educational Research and Training (NCERT)[4] and other available books from the market were collected in the first stage (Malhotra & Gupta, 2000; 2002a; 2002b; 2002c; NCERT, 1996; 1997; 1998; 2000; 2002a; 2002b; 2003a; 2003b; 2004a; 2004b; WBBPE, 2001a; 2001b; 2002a; 2002b; WBBSE, 1998; 2001; 2004; 2005).

[2] See http://www.iers-ptti.com/
[3] See http://www.wbbse.org/
[4] See http://www.ncert.nic.in/index.htm

3.2. Analysis of text books

These books were analyzed to identify "measurement of length" as a major concept and their elementary concepts of arithmetic mainly.

3.3. Task analysis of question papers

In the identification and sequencing of basic concepts, task analysis technique has also been applied on arithmetical and algebraic problems of mathematics question papers of 10th standard of Madhyamik Pariksha (1999, 2000) of W.B.B.S.E., Indian Certificate of Secondary Education (I.C.S.E.) Examination (2005, 2006) of the Council for the Indian School Certificate Examinations, School Certificate Examination (1999, 2000) of Visva-Bharati (a Central University).

3.4. Sequencing of concepts

The identified major concept and their elementary concepts were sequenced keeping in view the logical order of the subject and the psychological order of learners. The gaps in the concepts detected by the researcher were filled in by him at the initial stage.

3.5. Experts' opinions

The major concept and their elementary concepts with examples were given to experts for their comments. The experts were requested to add or omit or alter the sequence of concepts as they felt necessary.

Finally, the sequential form of elementary concepts of the major concept "measurement of length" incorporating the experts' opinion was developed.

4. SALIENT POINTS OF STUDY

4.1. Measurement of length

The major concept "measurement of length" has been divided under 7 first level sub-concepts which are given in Table 1. Each first level sub-concept has been divided into different levels of sub-concepts. Total 714 elementary concepts under each of the 7 first level sub-concepts are shown in Table 2.

Table 1. Measurement of length

1	2
Serial nos.	First level sub-concepts
1	Concepts of units of length
2	Conversion of units of length
3	Addition involving length
4	Subtraction involving length
5	Multiplication involving length
6	Division involving length
7	Simplification involving length

Table 2. Detailed list of different levels of sub-concepts of the major concept: measurement of length

1	3	4	5	6	7	8	9	10
First level sub-concept sl. nos.	Number of second level sub-concept	Number of third level sub-concept	Number of fourth level sub-concept	Number of fifth level sub-concept	Number of sixth level sub-concept	Number of seventh level sub-concept	Number of eighth level sub-concept	Total number of elem. concepts
1	9	1+1+1+ 1+1+1+ 1+1+2	—	—	—	—	—	10
2	2	2+2	5+5	8+8	—	—	—	16
3	2	6+4	12+8	46+32	52+32	—	—	84
4	2	6+4	12+8	46+32	52+32	—	—	84
5	2	6+4	18+12	70+48	76+48	—	—	124
6	2	6+4	18+12	72+48	—	—	—	120
7	2	2+2	4+8	16+40	79+90	182+90	186+90	276
Grand Total								714

The process of detailing out of the first level sub-concept no.3 only has been done for the economy of space. For the same reason some of the elementary concepts are elaborated and the rest are just mentioned. The notation Q for Question is used.

4.2. The Second Level Sub-Concepts Of Sub-Concept No. 3

Addition involving length

3.1: Addition involving length without using decimal system.
3.2: Addition involving length with using decimal system

4.3. THE Third Level Sub-Concepts Of Sub-Concept No. 3.1

Addition involving length without using decimal system

3.1.1: Addition involving metre (m)
3.1.2: Addition involving centimeter (cm)
3.1.3: Addition involving metre (m) and centimeter (cm)
3.1.4: Addition involving kilometer (km)
3.1.5: Addition involving kilometer (km) and centimeter (cm)
3.1.6: Addition involving any units

The similar sub-concepts of sub-concept no. 3.1.2: Addition involving centimeter (cm) are given in the bracket to avoid the repetition in the different level sub-concepts of sub-concept no.3.1.1: Addition involving metre (m)

4.4. The Fourth Level Sub-Concepts Of Sub-Concept No. 3.1.1

Addition involving metre (m)

3.1.1.1 Vertical addition without carryover (3.1.2.1)
3.1.1.2 Vertical addition with carryover (3.1.2.2)

4.5. The Fifth Level Sub-Concepts of Sub-Concept No. 3.1.1.1

Vertical addition without carryover (3.1.2.1)

3.1.1.1.1 Addition (3.1.2.1.1)

Q. Add:

$$\begin{array}{r} \text{m} \\ 24 \\ \underline{32} \end{array}$$

3.1.1.1.2 Addition after transformation of addends from horizontal form to vertical form (3.1.2.1.2)

Q. Add after transformation of addends from horizontal form to vertical form 32m + 21m

A:

m
32
21

53

3.1.1.1.3 Solution of the given problem and writing proper answer (3.1.2.1.3).

Q. Solve the problem (stating the answer properly).
Raja purchased two pieces of cloth of 12m and 13m respectively for a canopy. How many metres of cloth did he purchase in total for a canopy?

A:

m
12
13

25

He purchased 25 metres cloth for a canopy.

3.1.1.1.4 Formation of the problem and working out its solution (3.1.2.1.4)

Q. Make the problem from the following mathematical expression and then solve.

m
20
+25

?

A:

Ratan has two pieces of ribbons 20m long and 25m long. What is the total length of the ribbon?

m
20
+25

45

The total length of the ribbons is 45metres.

4.6. The Fifth Level Sub-Concepts Of Sub-Concept No. 3.1.1.2:

Vertical addition with carryover (3.1.2.2)

3.1.1.2.1 Addition (3.1.2.2.1)

Q. Add

$$\begin{array}{r} \text{m} \\ 24 \\ 38 \\ \hline \end{array}$$

3.1.1.2.2 Addition after transformation of addends from horizontal form to vertical form (3.1.2.2.2)

Q. Add after transformation of addends from horizontal form to vertical form.

37m + 24m + 19m

A:

$$\begin{array}{r} \text{m} \\ 37 \\ 24 \\ +19 \\ \hline 80 \end{array}$$

3.1.1.2.3 Solution of the given problem and writing proper answer (3.1.2.2.3)

Q. Solve the problem (stating the answer properly).

Bimal bought two pieces of rope 28m long and 49m long for his cot. What length of rope was used for his cot?

A:

$$\begin{array}{r} \text{m} \\ 28 \\ +49 \\ \hline 77 \end{array}$$

77m length of rope was used for his cot.

3.1.1.2.4 Formation of the problem and working out its solution (3.1.2.2.4)

Q. Make the problem from the following mathematical expression and then solve.

$$\begin{array}{r} \text{m} \\ 27 \\ +15 \\ +19 \\ \hline ? \end{array}$$

A:

A rope has been cut into three pieces of length 27m, 15m long and 19m long respectively. What is the length of the rope?

m
27
+15
+19
61

The length of the rope is 61 metres.

4.7. The Fourth Level Sub-Concepts Of Sub-Concept No. 3.1.3

Addition involving metre (m) and centimeter (cm).
3.1.3.1 Vertical addition without carryover.
3.1.3.2 Vertical addition with carryover.

4.8. The Fifth Level Sub-Concepts Of Sub-Concept No. 3.1.3.1

Vertical addition without carryover.

3.1.3.1.1 Addition

Q. Add:

m	cm
24	21
32	17

3.1.3.1.2 Addition after transformation of addends from horizontal form to vertical form.

Q. Add after transformation of addends from horizontal form to vertical form.

31m 07cm + 7m 31cm

3.1.3.1.3 Solution of the given problem and writing proper answer.

Q. Solve the problem (stating the answer properly).

The lengths of two pieces of a bamboo are 5m 10cm and 4m 18cm respectively. What is the length of the bamboo?

3.1.3.1.4 Formation of the problem and working out its solution.

Q. Make the problem from the following mathematical expression and then solve.

m	cm
33	20
+46	19
	?

4.9. The Fifth Level Sub-Concepts Of Sub-Concept No. 3.1.3.2

Vertical addition with carryover

3.1.3.2.1 Carryover within same unit.
3.1.3.2.2 Carryover within from one unit to another unit.

4.10. The Sixth Level Sub-Concepts Of Sub-Concept No. 3.1.3.2.1

Carryover within same unit

3.1.3.2.1.1 Addition.

Q. Add:

m	cm
17	19
+24	78

3.1.3.2.1.2 Addition after transformation of addends from horizontal form to vertical form.

Q. Add after transformation of addends from horizontal form to vertical form.

38m 07cm +27m 34cm

3.1.3.2.1.3 Solution of the given problem and writing proper answer.

Q. Solve the problem (stating the answer properly).

Rahamat Ali walked 49m 16cm in the morning, 47m 37cm in the evening in a day. How many distances did he walk in that day?

3.1.3.2.1.4 Formation of the problem and working out its solution.

Q. Make the problem from the following mathematical expression and then solve

m	cm
58	25
+38	27
	?

4.11.The Sixth Level Sub-Concepts Of Sub-Concept No. 3.1.3.2.2

Carryover within from one unit to another unit.

3.1.3.2.2.1 Addition.

Q. Add:

m	cm
17	34
+24	75

3.1.3.2.2.2 Addition after transformation of addends from horizontal form to vertical form

Q. Add after transformation of addends from horizontal form to vertical form.

27m 09cm +18m 99cm

3.1.3.2.2.3 Solution of the given problem and writing proper answer.

Q. Solve the problem (stating the answer properly).

David and Javed traveled 42m 78cm and 39m 89cm respectively in the morning. How far did they travel in the morning altogether?

3.1.3.2.2.4 Formation of the problem and working out its solution.

Q. Make the problem from the following mathematical expression and then solve.

m	cm
27	26
+15	30
+19	47
	?

The similar sub-concepts of sub-concept no. 3.1.5, 3.1.6 are given in the bracket to avoid the repetition in the different level sub-concepts of sub-concept no. 3.1.4

4.12. The Fourth Level Sub-Concepts Of Sub-Concept No. 3.1.4

Addition involving Kilometre (Km)

3.1.4.1 Vertical addition without carryover (3.1.5.1, 3.1.6.1)
3.1.4.2 Vertical addition with carryover (3.1.5.2, 3.1.6.2)

4.13. The Fifth Level Sub-Concepts Of Sub-Concept No. 3.1.4.1

Vertical addition without carryover (3.1.5.1, 3.1.6.1)

3.1.4.1.1 Addition (3.1.5.1.1, 3.1.6.1.1)
3.1.4.1.2 Addition after transformation of addends from horizontal form to vertical form (3.1.5.1.2, 3.1.6.1.2)
3.1.4.1.3 Solution of the given problem and writing proper answer (3.1.5.1.3, 3.1.6.1.3)
3.1.4.1.4 Formation of the problem and working out its solution (3.1.5.1.4, 3.1.6.1.4)

4.14. The Fifth Level Sub-Concepts of Sub-Concept No. 3.1.4.2

Vertical addition with carryover (3.1.5.2, 3.1.6.2)

3.1.4.2.1 Addition (3.1.5.2.1, 3.1.6.2.1)
3.1.4.2.2 Addition after transformation of addends from horizontal form to vertical form (3.1.5.2.2, 3.1.6.2.2)
3.1.4.2.3 Solution of the given problem and writing proper answer (3.1.5.2.3, 3.1.6.2.3)
3.1.4.2.4 Formation of the problem and working out its solution (3.1.5.2.4, 3.1.6.2.4)

4.15. The Third Level Sub-Concepts of Sub-Concept No. 3.2

Addition involving length with using decimal system.

3.2.1: Addition involving metre (m)
3.2.2: Addition involving centimeter (cm)
3.2.3: Addition involving kilometer (km)
3.2.4: Addition involving any units

The similar sub-concepts of sub-concept no. 3.2.2, 3.2.3, 3.2.4 are given in the bracket to avoid the repetition in the different level sub-concepts of sub-concept no. 3.2.1

4.16. The Fourth Level Sub-Concepts of Sub-Concept No. 3.2.1

Addition involving metre (m)

3.2.1.1 Vertical addition without carryover (3.2.2.1, 3.2.3.1, 3.2.4.1)
3.2.1.2 Vertical addition with carryover (3.2.2.2, 3.2.3.2, 3.2.4.2)

4.17.The Fifth Level Sub-Concepts of Sub-Concept No. 3.2.1.1

Vertical addition without carryover (3.2.2.1, 3.2.3.1, 3.2.4.1)

3.2.1.1.1 Addition (3.2.2.1.1, 3.2.3.1.1, 3.2.4.1.1)

Q. Add

23.25 m
102.13 m
1.10 m

3.2.1.1.2 Addition after transformation of addends from horizontal form to vertical form. (3.2.2.1.2, 3.2.3.1.2, 3.2.4.1.2)

Q. Add after transformation of addends from horizontal form to vertical form.

17.10 m + 211m +200.12 m

3.2.1.1.3 Solution of the given problem and writing proper answer (3.2.2.1.3, 3.2.3.1.3, 3.2.4.1.3)

Q. Solve the problem (stating the answer properly).

Mousumi bought three pieces of cloth of 21.05m long, 10.20m long and 20.12m long respectively for a canopy. How much cloth did she buy?

3.2.1.1.4 Formation of the problem and working out its solution (3.2.2.1.4, 3.2.3.1.4, 3.2.4.1.4)

27.16 m
+30.32 m
?

4.18. The Fifth Level Sub-Concepts of Sub-Concept No. 3.2.1.2:

Vertical addition with carryover (3.2.2.2, 3.2.3.2, 3.2.4.2)

3.2.1.2.1 Addition (3.2.2.2.1, 3.2.3.2.1, 3.2.4.2.1)

Q. Add:

217.25 m
+92.36 m
4.75 m

3.2.1.2.2 Addition after transformation of addends from horizontal form to vertical form (3.2.2.2.2, 3.2.3.2.2, 3.2.4.2.2)

Q. Add after transformation of addends from horizontal form to vertical form.

2.75m + 9.68m + 14.75m + 10.25m

3.2.1.2.3 Solution of the given problem and writing proper answer (3.2.2.2.3, 3.2.3.2.3,

3.2.4.2.3)

Q. Solve the problem (stating the answer properly).

Mahesh cut a bamboo into two pieces. The length of one piece is 4.7m and the length of the other is 7.85m. Find the total length of the bamboo that Mahesh cut.

3.2.1.2.4 Formation of the problem and working out its solution (3.2.2.2.4, 3.2.3.2.4, 3.2.4.2.4)

Q. Make the problem from the following mathematical expression and then solve.

$$\begin{array}{r} 3.75\ \text{m} \\ +0.85\ \text{m} \\ \hline ? \end{array}$$

5. CONCLUSION

1. This methodology will help to mark different types of conceptual gaps of mathematics in the syllabus, text books and entire teaching learning process.
2. The major concept "measurement of length" consists of 714 elementary concepts. It has been detected that a good number of conceptual gaps are present in the different Boards' syllabus and in their prescribed text books.
3. This study will also help to prepare a gapless proper syllabus, to develop a good text book and to improve the quality of teaching learning process of mathematics including "measurement of length."
4. It will also help to diagnose the particular areas of weakness of students and also in planning for necessary remedial measures.
5. Backward learners can be found easily and remedial method for them can be applied.
6. Special interest for mathematics can be enhanced which will be helpful for entire science education.

6. FURTHER STUDY

The diagnostic study may be taken on large number of samples of classes I, II, III, and IV on "measurement of length."

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