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Investigating the relation between secondary school students' achievement in forming and solving equations[1]

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This study investigates relationships between 7th and 8th grade students' achievements in forming and solving equations of. Study was conducted on randomly selected 7th and 8th grade students of elementary schools in Konya City, Turkey. 145 students (99 female, 46 male) participated in the research. Data were collected by an 'Equation Test'. The test which is suitable for equation types in 7th Grade Elementary Mathematics Curriculum. It was developed by the researchers. The relationships between achievements in forming and solving equations were examined by dependent samples t-test. The t-test results show that there is a significant difference. This difference is in the favor of equation solving ($p > 0.05$). In other word, students are more successful in equation solving. In addition, students' achievements about different types of equations were investigated. The results show that the students have the highest achievement in $ax = b$ type and the lowest achievement in $(ax + b)/c = (dx + e)/f$ type of equations.

Keywords: forming equation, solving equation, problem solving, secondary school mathematics
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INTRODUCTION

There has been a curriculum reform act in Turkey to raise the quality in education and so that in mathematics education as well recently. The new form of elementary mathematical curriculum was piloted in 2005 and was started to be applied in 2006 as representing a curriculum in which students centered education is important and seems to have constructivist traces rather than traditional curriculum.

Curriculum reform act has also influenced the algebra topic. There have bee innovations made on algebra learning field and its' subtopic equation in the learning field such that the equations topic that was in 7^{th} and 8^{th} classes was widen to the 6^{th}, 7^{th} and 8^{th} classes in the new program. For this reason, students meet this topic as soon as they start to the first year of secondary stage of elementary school.

Algebra is basically the generalized form of arithmetic so that makes students' experience in arithmetic essential with respect to algebra achievement. Stacey & MacGregor (1999) stated that, the reason behind students' difficulties in solving algebra problems is not to understand the basic solving process of algebraic method, but is students' experiences in arithmetic and was appeared in the followings:

- At the meaning that is given to the unknown
- At the comments about what the equation is
- At the methods that used in solving the equations

Variables and equality might be seen as two basic and critical concepts in teachin equations. The equality concept is especially one of students' experiences in arithmetic as mentioned. In the relevant researches conducted, it is found that students perceive equality as a sign saying "do operations" as it happens in addition, subtraction, multiplication and division, moreover, there is a common mistake students perform as the operation should take place at the left side of the equality and the result should take place at the right side of the equality (Falkner *et al.*, 2000). For example when students were asked to solve $8+7+6=8+?$, most of the students wrote 15 to the place of "?" (McNeil & Alibali, 2005).

The other important concept in teaching of equations is variables which are the symbols of algebra language mathematics. So it might be said that understanding the variable concept is the key of students'achievement in algebra. Understanding the variable concepts basically represents the abilities transferring arithmetic to algebra and is needed for meaningful use of the advanced level mathematical concepts (Greene & Findell, 1999). Variable has different meanings which are;

A value that is not known yet, such as $5x+3=13$

A set of values, such as $y=2x+3$

Generalizing structure, such as $x+y=y+x$

These meanings couldn't usually be perceived completely by students. For example when x and y values were asked to the children in the example of $x+y=y+x$ answers revealed a mistake that students thinks a variable can have only a value. Beyond the need of learning of algebra, students have difficulties about commenting on the algebraic symbol (Espy, 1998).

Tehre are two basic components of equations which are solving the equations and forming the equations. To learn how to solve equations requires more than just memorizing some rules. The students, who merely learn algorithmic methods to solve equations, have difficulties different form of the equations. Perso (1996) also found that the children, who have tendency like this, have mistakes in solving equations (Capraro & Joffrion 2005).

Another important difficulty that students perform is forming equations out of the (word) problems (Capraro & Joffrion, 2006) which might be said as a part of comprehending process (Chaiklin *et al.*, 1989). The most difficult part in solving algebraic problems for students is to understand the problem and then to form the equation. Instead of experiencing the deep cognitive process, students try to do a series of calculations to reach the answer (Stacey & MacGregor, 1999). Students do need to develop representative techniques to understand the linear equations sufficiently. In other words, students' understanding of equations is related to the students' ability to convert verbal representations to the form of equations (Silver, 2000).

In the light of literature review, in this study, it is aimed to determine the ability of students in solving and forming equations and to determine the relations between these. As the result of such determination, it may be possible to observe whether students are more successful in solving the equations or forming the equations. Then the teaching of equations might be revised with new suggestions particularly through preparing the new elementary mathematics lesson curriculum in terms of solving problems which is one of the most wanted fields to be developed in students of 4 basic ability.

2. METHOD

2.1. Research Design and Participants

This study has anti-positivist paradigm with interpretive approaches (Cohen *et al.*, 2000, p. 22). Case study (Patton, 2002) is used as a research strategy to make an in-depth examination of forming and solving equations and relation between them study.

Purposeful sampling technique (Patton, ibid.) of non-probability sampling methods, which accept individuals or events as they are, was used for the selection of the sample (Cohen *et al.*, 2000).The study is conducted with all 7th and 8th grade students of three elementary schools, which were selected randomly, in Konya, Turkey. There were 99 girls and 46 boys, 145 students in total, in these schools.

2.2 Data collecting tools

In the study, the data was collected by conducting the research instrument "equations test" which was developed by the researchers with respect to the equation types at elementary education curriculum of 7th grade. Content validity of the data collection instruments was obtained by a detailed consideration of the scope of research by four tutors in Department of Mathematics Education. In order to ensure reliability qualitative data were categorized and coded (Miles & Huberman, 1984, p. 23). Compatibility rates among these categories were then calculated. The coding revealed a compatibility rate of %92. "Equations test" has two sections "forming equation test" and "solving equation test". In the test there were totally 36 open-ended questions 18of which were about forming the equation test and 18 of which were about solving the equation best.

In the "forming the equation test, the open-ended questions were formed in six types as:

1st type: $x + a = b$;
2nd type: $ax = b$;
3rd type: $ax + b = c$;
4th type: $\frac{ax + b}{c} = d$;
5th type: $ax + b = cx + d$ and
6th type: $\frac{ax + b}{c} = \frac{ax + e}{f}$

taken from elementary school 7th grade mathematics program. There were three questions from each type so that there were 18 questions in total. The important feature of the test was that students were asked to form the equations (but not solving) from the given verbal problems by fitting to the equation types by using.

The equations which were expected to be constructed by the students in "the solving the equation test", were prepared in a random order and given to students to be solved clearly in "the solving the equation test" by the researchers.

While grading the test items the same grading rules as open-ended questions used. Moreover open-ended questions were classified with respect to equation types and were considered in terms of difficulties to define the highest point that might be obtained.

Therefore the grades which were given to types were as 2 points for items of 1st type; 2 points for items of 2nd type; 4 points for items of 3rd type; 4 points for items of 4th type; 5 points for items of 5th type and 8 points for items of 6th type. Similarly, the items in "the forming equation test" were graded in the same way with respect to the equation types.

The reliability of "the equation test"," the forming equation tests" and "the solving equation tests" was separately calculated by the Cronbach's alpha coefficient by using the data obtained from the 145 students. As a result of calculations, Cronbach's alpha coefficients for "the equation test", "the forming of equation test" and "the solving of equation test" were found as 0,96, 0,94 and 0,96 respectively.

2.4 The analyses of the data

When analyzing the data with regard to independent variable, arithmetic average, frequency, percentage and points of absolute achievement were used to describe data. When calculating the points of absolute achievement, the following formula was used;

Absolute Success Point =

$$\frac{\text{The Basic Point That Obtained From The Test(Item)}}{\text{The Maximum point That Can Be Obtained From the Test(Item)}} \times 100$$

To compare two groups t test, which was applied to the dependent groups and Pearson product-moment correlation coefficient, were used.

3. FINDINGS

3.1 Students'achievement with regard to equation types

In this section, the findings about students' achievement to form and solve more equations with respect to equation types were presented. For this aim, the arithmetic averages and the absolute achievement points that were calculated from the equation tests of students are given separately for "the forming equation test" and "the solving equation test" in the Table 1 and in the in the Figure 1.

As seen in the table 3.1, averages of solving the equation points of 145 students vary between 3.27 and 10.39. Averages of forming the equation points vary between 3.18 and 8.94. As it is stated above the highest points which obtained from the equation types were different from each other. This is why absolute achievement points were used to determine the achievement with regard to the equation types.

In solving the equations absolute achievement points vary between 43.29 and 66.67;

while the lowest point was calculated in 6th type of equation and the highest point was calculated in 2nd type of equation. On the other hand, in forming the equations the absolute achievement points varied between 37.25 and 60.50; the lowest and highest points were calculated in 6th and 2nd type of equations respectively. As seen in the Table 1 and Figure 1, students were very successful at solving and forming the 2nd type of equations while they were less successful at the 6th type.

Table 1: Students' absolute achievement and averages points of solving and forming equations with regard to the equation types

	Type of equation	N	$\overline{X}$	S	ASP
Solving	1st type of equation	145	3.27	2.40	54.50
	2nd type of equation	145	4.00	2.26	66.67
	3rd type of equation	145	7.75	4.63	64.58
	4th type of equation	145	8.08	5.70	53.87
	5th type of equation	145	7.23	5.92	48.20
	6th type of equation	145	10.39	9.87	43.29
	The sum	145	40.72	27.99	52.21
Forming	1st type of equation	145	3.18	2.47	53.00
	2nd type of equation	145	3.63	2.65	60.50
	3rd type of equation	145	7.03	5.22	58.58
	4th type of equation	145	6.52	6.83	43.47
	5th type of equation	145	6.60	6.64	44.00
	6th type of equation	145	8.94	9.90	37.25
	The sum	145	35.90	30.08	46.03

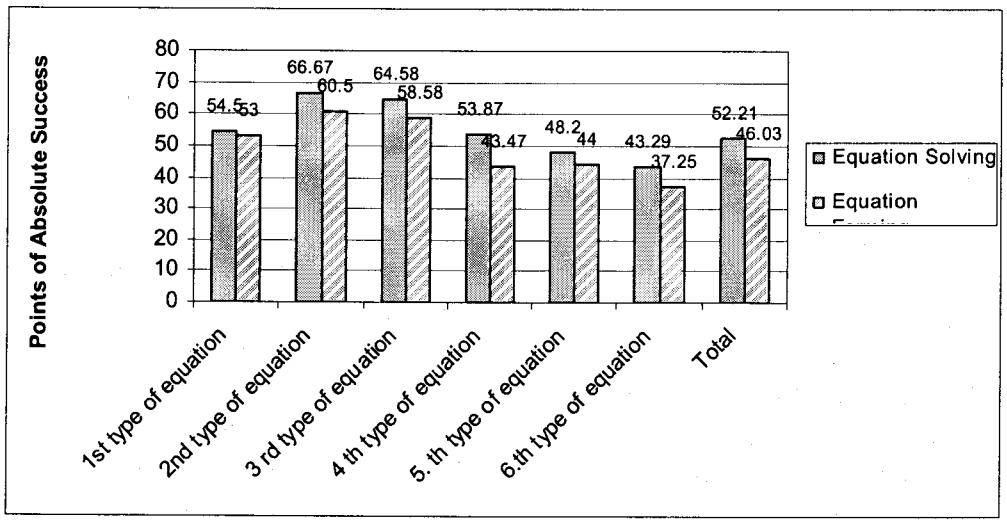

Figure 1. Solving and forming points of absolute achievement graph for the equation types

3.2 The relation between achievement of solving and forming the equations

In this section, the relation between achievement of solving and forming the equations was tried to be described. To reveal this relation, calculation of correlation coefficients between solving the equation point and forming the equation points and comparison of solving and forming the equation point averages by the statistical methods were used.

The calculated correlation coefficients between solving the equation and forming the equation are as follows:

$r_1 = 0.59^{**}$ for 1st type of equations;
$r_1 = 0.48^{**}$ for 2nd type;
$r_1 = 0.44^{**}$ for 3rd type;
$r_1 = 0.56^{**}$ for 4th type;
$r_1 = 0.52^{**}$ for 5th type and
$r_1 = 0.48^{**}$ for 6th type.

In summary, these coefficients varied between 0.44 and 0.60 with regard to and α=at a meaningful level of 0.01. This shows that there is a positive meaningful relation between solving and forming points for equation types.

In determining whether students are more successful in forming or solving the equations, the averages of solving and forming points for equation types were compared with the t test which was applied to the dependent groups and the obtained results were presented in the Table 2.

Table 2. Comparison of the students' solving and forming equation points

	1st type	2nd type	3rd type	4th type	5th type	6th type	Sum
Solving	3.27	4.00	7.75	8.08	7.23	10.39	40.72
Forming	3.18	3.63	7.03	6.52	6.60	8.94	35.90
Difference	0.09	0.37	0.72	1.56	0.63	1.45	4.82
Std. Deviation	2.22	2.53	5.26	5.97	6.18	10.07	26.11
t value	0.486	1.770	1.643	3.141*	1.236	1.740	2.227*

As based on the obtained results; the difference between the averages of solving and forming the equation points were meaningfully different for solving the equations. ($t = 2.227$; $p > 0.05$). This is a sign of that students are generally more successful at solving the equation. When this examination was made by the equation types, the meaningful difference was only at the 4th type and again for the solving of equation ($t = 2.539$; $p > 0.05$). Moreover it was observed that no meaningful differences were

present in other equation types. Consequently, it can be said that students are more successful at solving the equations in

$$\frac{ax+b}{c}=d$$

type than forming the equations, additionally, there were no similar results obtained in the other types.

4. DISCUSSION AND RESULTS

The results obtained from the equation test revealed that the absolute achievement points of equation solving of the 145 students was 52.21 while absolute achievement points of equation forming equation was 46.03. This result shows that students had achievement in solving and forming equation about 50%. However this is less than the expected ratio of general achievement in the class which was 70%.

In the elementary mathematics curriculum, one of the main strategies of solving problems was described as "writing the equation", which is described as "writing the mathematical sentence" by Baykul (2005). In this strategy, writing an appropriate equation of the problem and then solving the equation is essential. Therefore, forming the equation and solving the equation is primarily important in solving a problem. The ratio of achievement in solving and forming of the equation which was under the expected ratio that mentioned above can affect students' achievement in solving problems. Trying to increase solving and forming of the equation achievement may cause a rise in achievement of solving a problem.

It is meaningful that the highest achievement level was at the 2nd type of equations and the lowest achievement level was at the 6th type of equations in terms of solving and forming the equations likewise the ones in the curriculum. When moving to 6th type of equation from 1st type of equation, it can be thought that the question become more difficult as the aspect of the structures and this might explain the reason of the decrease in achievement in terms of solving and forming the equations. But, the reason for the highest achievement level being at the 2nd type of equation instead of 1st type of equation can be based on the mistakes generally made by the students while forming and solving the equations in first type of equation (especially in "replacing to the other side of the equality"). Moreover there is an interesting result revealed that the achievement in 3rd type of equations is higher than the achievement in 1st type of equations despite 1st type of equations are involved in 3rd type of equations. Additionally, even, the achievement in the 4th type of equations is nearly the same as in the 1st type. It is thought-provoking that the achievement of students at the type of

$$\frac{ax+b}{c}=d$$

is nearly the same at the type of $x+a+b$.

It was mentioned before that there was a meaningful correlation coefficients between the solving and forming of the equation are positive at middle level and at the value of nearly $\alpha=0.01$ for the types of equations. This shows the positive relation of forming and solving the equations and generally it is an expected event. In the light of this relation, it is expected that a student who is successful at forming of equations will have be successful at solving of equations and vice versa. But according to the results, students are more successful at solving the equations than forming the equations.

Since forming the equations needs higher order mental skill and solving the equations need less high order mental skill, so it can be already expected that the achievement at solving the equations will be higher. But, importance should be given to the studies of forming equations as they are parts of strategies of solving problems. In addition to the routine strategies, other strategies such as the usage of number knowledge, the usage of counting techniques, comparison, studying the backwards, heuristic approach, transferring, applying the same operation to both two sidesof solving equations should be taught.

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