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CM 방법을 활용한 설비 수명분석

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A Study on the Computed Mortality Method in Life Analysis

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공공사업이나 자동화 생산시스템 등에 있어서 설비 자산이 차지하는 비중은 매우 크며 이러한 설비들의 경제적 설비 대체 분석을 위하여 설비에 대한 수명분석이 이루어져야 한다. 본 논문에서는 설비들의 폐기 자료가 불충분한 경우의 수명분석에서 Iowa형 생존곡선을 기초로 한 CM 방법을 제시하였다. 기존의 SPR 방법이 평균 수명만을 추정하는데 비하여 제시된 CM 방법은 설비의 수명분포를 추정함으로써 평균 수명뿐만 아니라 폐기 자료의 폐기곡선과 폐기율을 추정할 수 있어 설비 교체 분석 시 유용하게 사용될 수 있다.

Keywords : Life analysis, Iowa type survivor curves, CM method

1. Introduction

Life analysis for physical property has developed procedurally along two different processes, actuarial and non-actuarial life analysis procedures. The two procedures depend upon the type of records required for implementation. The actuarial life analysis procedure is similar to that used for human morality studies and requires a complete knowledge of property. The installation and retirement date must be known for individual units of property in order to determine the life span of the retired units and the age of those remaining service [2, 3]. The non-actuarial life analysis procedure has been developed for situation where complete property records are not available and require only a record of gross additions and annual volume of plant remaining in service [7].

Actuarial life analysis of the physical property can best be described as a two parameter problem : 1) the estimation of an average service life for the property, and 2) the calculation of a dispersion pattern showing the manner in which the total lives are distributed on either side of the average service life. The average service life is simply the sum of the service lives of all units, divided by the number of units. The three basic methods for actuarial life analysis are used : 1) the annual rate or retirement rate method, 2) the original

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group method, and 3) the individual unit method [9-13].

The actuarial methods of life analysis all require that records be maintained on each individual unit or like group of units. These records must show the date of installation and current age or age at retirement if actuarial life is to be performed. The impracticality of maintaining extensive records for many property groups in order to use the actuarial methods has encouraged the development of non-actuarial methods of life analysis. Two non-actuarial methods are the turnover and Simulated Plant Record (referred to herein as SPR) method [5, 15]. Both methods require only a record of gross additions and the ending volume of plant in service for the accounting period.

The turnover method may be applied by the plotting curves of the accumulated additions and accumulated retirements on the same graph. With time plotted on the abscissa, the turnover period is then the horizontal distance measured from the retirement curve to the additions curve. This method allows a determination of the turnover period for any time period without additional calculation; if the graph is maintained, comparisons may be made at the end of each accounting period. A disadvantages to this type of analysis is that no indication of the dispersion pattern is obtainable with the indicated service life.

The SPR method was first introduced by Hill [4]. He proposed a trial and error procedure which attempted to construct mathematically the total property balance in a given year by spreading each of the annual gross additions according to an assumed dispersion pattern. The reconstructed balance was determined by accumulating the product of each gross annual addition multiplied by the portion surviving from Hill's assumed life table at its attained age. A serious problem with this procedure was that every survivor curve has an average service life that will produce a single balance equal in magnitude to the actual plant balance. Dispersion was therefore indeterminate by this method, resulting in the possibility of gross error in the estimated average service life.

When applied to unaged property records, the SPR model indicates one survivor curve to represent the life characteristics of vintage. This model is appropriate, then, only if the property in each vintage has the same life characteristics. Application of the model to accounts with property lives that vary by vintage frequently results in anomalies [8]. With these accounts, it is more appropriate to develop separate life estimates for each vintage.

In order to make these life estimates, aged data can be generated and then analyzed by actuarial methods. The computed mortality (referred to herein as CM) model may be used for generation of aged data. If the aged data are in units(instead of dollars), the computed mortality model becomes a tool for pricing retirements. The data may then be adjusted for transactions that are not additions and retirements. The object of this study is to analyze the ability of the computed mortality model to describe industrial property mortality characteristics.

2. The computed mortality model

The computed mortality model is used when the property records do not reflect the age of property upon its retirement; i.e., aged data are not available. For the explanation and illustration of computed mortality model provided below, the model has been applied to data that consists solely of gross annual additions and retirements. Adjustments for transactions other than additions and retirements are discussed in next section. Because a curve type must be supplied by the analyst when the computed mortality model is used, the selection of curve type is discussed following the explanation of the model.

2.1 Use of units in the CM model

Units (instead of dollars) are used in the simulation because units are unaffected by the pricing policies of the company, which may distort the life characteristics of the property as reflected in dollar data. Since units are not always available, the CM model will be capable of making simulation calculations in dollars as well as in units. A lower limit will be placed on the number of units (or dollars) used in the simulation.

2.2 Explanation of the model

The CM model is used to simulate aged data when actual aged data are not available. The data can then be analyzed as if the data were actual data. To begin the simulation of property records, each vintage's survivors at the beginning of an early experience year (as can be seen in <Table 1> are needed. This age distribution of survivors may be specified by the analyst or obtained from an Iowa type survivor curve[14].

<Table 1> Beginning age distribution in units.

year	Balance (Jan. 1, 2008)
2000	120
2001	140
2002	160
2003	210
2004	285
2005	360
2006	510
2007	635
2008	810
Total	3,230

To calculate vintage survivors at the end of this first year (and all subsequent years), the retirements from each vintage during the year must be calculated. To calculate retirements, retirement ratios from an Iowa type survivor curve are multiplied by each vintage's survivors at the beginning of the year.

2.2.1 The Iowa type survivor curve

The Iowa type curves are a family of curves which were developed empirically and conform to modified Pearson type frequency. Research by Winfrey and Kurtz [16] on the development of empirical curves was begun about 1921. In 1930, Kurtz [6] published his findings which combined 52 curves into 7 types. These 7 types were forerunners of the 13 types which were derived from a grouping of 65 curves and published in 1931 as Bulletin 103 by Winfrey and Kurtz[16]. In 1935, Winfrey [17] published further findings which expanded the 13 types to 18 types. The 18 types were derived from 176 curves which included the 65 curves from earlier research. In 1957, Couch [1] classified 4 more types from 24 curves. Thus 22 Iowa type curves now exit.

The curves of this system, which has acquired the name Iowa type survivor curves, are represented by a letter and a number. The possible letters are O, L, S and R and they respectively refer to the position of the modal frequency being at the origin, to the left of average life, symmetrical around average life, and to the right of average life. The numbers associated with the type curves range from 0 to 6 and indicate relative height of the mode with a large number indicating a high mode. These numbers are not on an absolute scale and are relative within a modal position classification only. The basic 22 Iowa type curves as they exit today are as follows: O_1, O_2, O_3, O_4, L_0, L_1, L_2, L_3, L_4, L_5, S_0, S_1, S_2 S_3, S_4, S_5, S_6, R_1, R_2, R_3, R_4, and R_5.

2.2.2 The retirement rate method

The retirement rate method is a procedure for generating a life table, utilizing the retirement experience of a maximum number of vintages over a relatively short period of time. A retirement rate for a given age interval is an indication of the probability of retirement of property during that age interval. Thus, if we have the number of data unit in service at a given age, and an estimate of the probability of retirement during the ensuing year for the type of property, we can use a mathematical expression for the expectation to forecast the number of unit which will be retired in the year.

$$R_x = S_x \times RR_x$$

where, R_x = number expected to be retired during age x interval,

S_x = number surviving at beginning of age x interval,

RR_x = retirement rate for that age x interval.

If we obtain, in some fashion, a table of retirement rates for a given type of property for every age interval from zero to maximum life, we can start with 100% surviving at age zero and apply each retirement rate in turn to those which survive to that age. When the routine is completed we will find that we have generated a life table. We can put the repetitive routine into a general formula,

$$PS_{N+1} = PS_N \times (1\text{-}RR_N)$$

where, N = the number of the age interval considered,

PS_N = the percent surviving at the beginning of N^{th} age interval,

RR_N = the retirement rate for the N^{th} age interval.

The mean of the curve is varied until the sum of retirements calculated by vintage equals the actual retirements that were recorded for the experience year as can be seen in <Table 2> and <Table 3>.

The calculated retirements for each vintage are then subtracted from the vintage's survivors at beginning of the year to give its survivors at the end of year as can be seen in <Table 4>. These end of year survivors will be the basis for calculating retirements for the next experience year. The survivors that were simulated in previous experience years.

<Table 2> Simulating units retired using S_0-10 curve

year	Balance	retirement rate	simulated retirement
2000	120	0.1142	14
2001	140	0.0880	12
2002	160	0.0765	12
2003	210	0.0656	14
2004	285	0.0551	16
2005	360	0.0444	16
2006	510	0.0330	17
2007	635	0.0199	13
2008	810	0.0035	3
Total simulated retirements			117
Actual retirements			117

<Table 3> Simulating units retired using S_1-10 curve

year	Balance	retirement rate	simulated retirement
2000	120	0.1171	14
2001	140	0.0960	13
2002	160	0.0766	12
2003	210	0.0586	12
2004	285	0.0421	12
2005	360	0.0271	10
2006	510	0.0142	7
2007	635	0.0046	3
2008	810	0.0002	0
Total simulated retirements			83
Actual retirements			117

<Table 4> Calculating end of year survivors

year	Balance (Jan. 1)	simulated retirement	Balance (Dec. 31)
2000	120	14	106
2001	140	12	128
2002	160	12	148
2003	210	14	196
2004	285	16	269
2005	360	16	344
2006	510	17	493
2007	635	13	622
2008	810	3	807
Total	3,230	117	3,113

2.3 Selection of curve type

The curve specified by the analyst for use in the simulation should represent the probability of retirement for each age interval as the property in the accounts passes through the experience year. If each vintage follows the same survivor curve, the retirement rate in any experience year can be represented by this common vintage curve, which makes the curve appropriate for use by the CM model to simulate vintage retirements. If the life characteristics vary by vintage, however, no single vintage curve may represent the collection of retirement rates in an experience year. For example, if all vintages follow an S_0 type survivor curve but the average life of the property is decreasing by vintage, the retirement rates in an experience year may be represented by an S_1 or S_2 Iowa type survivor curve. The curve type may change from one experience year to the next if vintage life characteristics are changing.

3. Results of the computed mortality model

The CM model will simulate aged data as described above. In addition, this model will contain the features described below.

3.1 Genearte beginning age distribution

At the end of each year the CM model will be used to simulate the year's retirements by vintage. The basis for this calculation is the vintage's survivors at the beginning of the year. After the CM model has been used with an account, the vintage retirements simulated by the CM model may be subtracted from the beginning of the vintage survivors to simulate end of year vintage survivors for use in calculating the following year's retirements. But for the initial application of the CM model to an account, a beginning age distribution is retired.

The CM model will allow the user to either enter the beginning age distribution or specify a file in which vintage survivors have been stored. Alternatively, the CM model can age the initial annual balance using one of the following methods : 1) allocate the balance equally among the vintage, 2) spread vintage installations using an Iowa type survivor curve. The first method is the lump sum allocation, to age an initial balance so that vintage installations may be calculated. As with the CM model, the analyst may specify the range of vintages among which the annual balance will be allocated. The analyst may direct that this set of vintages be divided into two groups and specify the percent of the annual balance to be divided equally among the vintages in each group. Instead of allocating all of the balance to the specified set of vintages, the analyst may specify that the remainder be lumped into a vintage prior to the earliest vintage.

In the second method, the percents surviving from an Iowa type survivor curve are multiplied by the vintage's installations and the curve mean varied until the simulated survivors match the annual balance of the beginning experience year. This method is used to generate an age distribution for use in calculating depreciation and to forecast renewals.

3.2 Simulate aged retirements

Under the normal flow through, the CM model will be used to age the annual retirements for one year only. But for the initial applications of the CM model to an account, an option will be provided to enable the user to simulate aged retirements for several years using Iowa type survivor curve. For example, the analyst may simulate retirements from the beginning experience year to the current year using a specified Iowa type survivor curve. Alternatively, the analyst may segment this experience year range and apply a different Iowa type survivor curve to each segment. The analyst may view the calculated results for each segment and either store or recalculation.

3.3 Specify Iowa curve by placement band

To age annual retirements, the analyst may divide the vintage into placement bands and specify, for each band, the total retirements to be allocated among the vintages in band. Each band's total retirements may be specified as a percent of the annual retirements. The allocation among the vintages in each band is made using an Iowa type survivor curve specified for the band. If placement bands are not specified, the entire set of vintages will be considered as one band.

3.4 Convert between units and dollars

Survivors in dollars are needed for depreciation and other calculations, so simulated units must be converted to dollars. Conversion is accomplished by multiplying each vintage's simulated units by the vintage's unit cost. <Table 5> illustrates the conversion of retired units to retired dollars.

<Table 5> Converting units retired to dollars retired

year	simulated retirements (units)	unit price	simulated retirements (dollars)
2000	14	63	882
2001	12	73	876
2002	12	75	900
2003	14	79	1,106
2004	16	85	1,360
2005	16	87	1,392
2006	17	92	1,564
2007	13	91	1,183
2008	3	92	276
Total	117		9,539

In <Table 6>, these retired dollars are subtracted from the beginning of year surviving dollars to give end of year surviving dollars. Alternatively, surviving units may be converted directly to dollars. The dollar survivors are appended to a file of survivors that were simulated in previous experience years.

<Table 6> Simulating vintage survivors in dollars

year	Balance (Jan. 1)	simulated retirement	Balance (Dec. 31)
2000	7,560	882	6,678
2001	10,220	876	9,344
2002	12,000	900	11,100
2003	16,590	1,106	15,484
2004	24,225	1,360	22,865
2005	31,320	1,392	29,928
2006	46,920	1,564	45,356
2007	57,785	1,183	56,602
2008	74,520	276	74,244
Total	281,140	9,539	271,601

Conversion from dollars to units may be required if an age distribution in units at the beginning of an early experience year is not available. In this case, the dollars surviving for each vintage may be simulated and then converted to units by dividing the surviving dollars by each vintage's unit cost.

3.5 Adjustment for other than additions and retirements

Provision will be made to permit the direct adjustment of calculated survivors for type of transactions not conforming to the assumption of the model that the data consist solely of additions and retirements. Before the adjustment can be made, any gross annual amounts must be broken down by vintage. This breakdown may be specified by the analyst or annual amounts can be aged using percentages either supplied by the analyst or obtained from an Iowa type survivor curve.

4. Conclution

For some categories of depreciable property, it is uneconomical and impractical to maintain actual retirements and plant in service by age. Additions, total retirements and total plant in service are normally known. One of the non-actuarial methods used to analyse such partial mortality data is the CM method. Based on this study, the CM model has several advantages that are not available with SPR method. For example, CM model permits pricing of retirements because an age distribution of retirements is generated each year. Subtracting this age distribution from the beginning of the year survivors gives an end of year survivor distribution for use in actuarial life analysis and as a basis for depreciation calculations. The display of a calculated retirement matrix rather than just the survivor curves indicated from the SPR analysis enables the user to evaluate the generated data based on a sampling of retirements. The use of a matrix also permits a more accurate reflection of transactions other than additions and retirements that is possible under the general allocation technique used with SPR method.

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