

# Robust Sliding Mode Friction Control with Adaptive Friction Observer and Recurrent Fuzzy Neural Network

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**Abstract**—A robust friction compensation scheme is proposed in this paper. The recurrent fuzzy neural network and friction parameter observer are developed with sliding mode based controller in order to obtain precise position tracking performance. For a servo system with incomplete identified friction parameters, a proposed control scheme provides a satisfactory result via some experiment.

**Index Terms**—Dynamic friction, Sliding mode control, Friction Observer, Recurrent fuzzy neural network, Servo system.

## I. INTRODUCTION

FRICION is important thing that must be considered in high precision and fast motion of robot and machine tools, etc. In order to improve the transient performance and reduce the steady-state tracking errors, the friction influence on the control system must be compensated adequately. The main difficulties associated with friction compensation are that friction properties are varied according to several conditions such as time, temperature and relative velocity. Thus, through many researches on the friction phenomena, several friction properties have been investigated.

The LuGre friction model, suggested by Canudas de wit et al. [1], is the friction model which can represent several useful and important friction properties by only a few parameters. Until now, most of the friction compensation method using friction observer have tried to compensate the friction by combing basic controller with friction observer using fixed friction model [2-5]. Even though this method has provided many useful results, more improved friction

compensation performance can be achieved if a robust estimator is supplemented in control system in addition to the friction observer. Since it is so difficult to estimate error of each model parameter respectively in real system, compensating method to approximate total lumped friction error can provide more effective results with parallel use of friction observer. This uncertainty of the system can be approximated well by a fuzzy or neural networks scheme.

Recently, the concept of incorporating fuzzy logic into a neural network has grown into popular research topic. In contrast to the pure neural network or fuzzy system, the fuzzy neural network (FNN) possesses both their advantages; it combines the capability of fuzzy reasoning in handling uncertain information and the capability of artificial neural networks in learning from process. It has been proven that the FNN can approximate a wide range of nonlinear functions to any desired degree of accuracy under certain condition [6,7]. However the major drawback of the existing the FNN is that their application domain is limited to static problem due to feedforward network structure, which requires a large number of neuron or membership function. On the other hand, the recurrent FNN (RFNN) [8,9] naturally involves dynamic elements in the form of feedback connections used as internal memories. Thus, the RFNN is a dynamic mapping and demonstrates good control performance in the presence of uncertainty such as parameter variations of the system, external load, unmodeled dynamics compared to the feedforward FNN.

In this paper, we propose a friction compensation system. It consists of a sliding mode controller, a friction observer, a RFNN approximator and a reconstructed error compensator. The friction parameter observer can estimate immeasurable internal friction state variable and related parameters of the LuGre friction model. In addition, RFNN and reconstruction error compensators approximate and compensate a lumped friction uncertainty. The feasibility of a proposed control scheme will be verified by experiments for the servo system with nonlinear dynamic friction.

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## II. CONTROLLER AND OBSERVER DESIGN

The dynamic model of the servo system with nonlinear friction is

$$J\dot{\omega} + \tau_f + \tau_d = u \quad (1)$$

where  $J$  is a moment of inertia,  $\omega$  is the angular velocity, and  $\tau_f$  is the nonlinear dynamic friction and  $\tau_d$  is the uncertain torque. The dynamics of the LuGre friction model [1] is given as follows:

$$\dot{z} = \omega - \mu_0 f(\omega)z \quad (2)$$

where

$$f(\omega) = g(\omega)|\omega|. \quad (3)$$

The function  $g(\omega)$  is parameterized as follows:

$$g(\omega) = \frac{1}{\tau_c + (\tau_s - \tau_c)e^{-(\omega/\omega_s)^2}} \quad (4)$$

where  $\tau_c$  is Coulomb friction,  $\tau_s$  is stiction level,  $\omega_s$  is Stribeck angular velocity. The dynamic friction term is described by

$$\tau_f = \mu_0 z + \mu_1 \dot{z} + \mu_2 \omega \quad (5)$$

where  $\mu_0$  is the stiffness of the elastic bristle,  $\mu_1$  is a damping coefficient in elastic range and  $\mu_2$  is the viscous friction coefficient.

We define the sliding surface and the position tracking error  $e(t)$  as follows:

$$s = \dot{e} + k_1 e \quad (6)$$

$$e = \theta_d - \theta \quad (7)$$

where  $\theta$  is an angular velocity,  $k_1$  is a positive constant and  $\theta_d$  is the desired position trajectory.

The uncertain torque can be approximated as follows:

$$\tau_d = \mathbf{W}_{ko}^{*T} \mathbf{U}_{ko} + \varepsilon \quad (8)$$

where  $\mathbf{W}_{ko}^{*T}$  is the optimal weighting vector that can be approximated later via RFNN and a RFNN approximation error  $\varepsilon$  is bounded by a small positive constant  $E$ , i.e.,  $|\varepsilon| \leq E$ . Our objective is to choose control torque  $u(t)$  such that the system state is driven to the sliding surface  $s = 0$  regardless of the friction. Let us choose the control torque  $u(t)$  of the form with unified smooth control law as follows:

$$u(t) = J(\dot{\omega}_d + k_1 \dot{e}) + \alpha s + \hat{\tau}_f + \hat{\tau}_d + \hat{\tau}_{us} \quad (9)$$

where  $\alpha$  is a positive constant,  $\hat{\tau}_f$  is an estimation of  $\tau_f$ ,  $\hat{\tau}_d$  is an estimation of  $\tau_d$  and  $\hat{\tau}_{us}$  is an estimation of the reconstruction error.  $\hat{\tau}_f$  is given as follow:

$$\hat{\tau}_f = \hat{\mu}_0 \dot{z} + \hat{\mu}_1 \dot{z} + \hat{\mu}_2 \omega. \quad (10)$$

Introducing Eq. (8) and (9) into Eq. (1), the following equation can be obtained

$$Js = \mu_0 \tilde{z} + \hat{z} \tilde{\mu}_0 + \mu_1 \tilde{z} + \dot{z} \tilde{\mu}_1 + \omega \tilde{\mu}_2 - \alpha s + \tilde{\mathbf{W}}_{ko}^{*T} \mathbf{U}_{ko} + \varepsilon - \hat{\tau}_{us} \quad (11)$$

where  $\tilde{z} = z - \hat{z}$ ,  $\tilde{\mu}_0 = \mu_0 - \hat{\mu}_0$ ,  $\tilde{\mu}_1 = \mu_1 - \hat{\mu}_1$ ,  $\mu_2 = \mu_2 - \hat{\mu}_2$  and  $\tilde{\mathbf{W}}_{ko}^{*T} = \mathbf{W}_{ko}^{*T} - \hat{\mathbf{W}}_{ko}^{*T}$ .

Since, however, the state variable  $z$  cannot be measured directly, we suggest the friction state observer to estimate  $z$  as follows:

$$\dot{\hat{z}} = \omega - \hat{\mu}_0 f(\omega) \hat{z}. \quad (12)$$

Then, the estimation error of the friction state variable is given from Eq. (2) and (11) as follows:

$$\dot{\tilde{z}} = -\mu_0 f(\omega) \tilde{z} - f(\omega) \hat{z} \tilde{\mu}_0. \quad (13)$$

Next, the Lyapunov function is defined as

$$V_1 = \frac{1}{2} Js^2 + \frac{1}{2} \tilde{z}^2 + \frac{1}{2\eta_0} \tilde{\mu}_0^2 + \frac{1}{2\eta_1} \tilde{\mu}_1^2 + \frac{1}{2\eta_2} \tilde{\mu}_2^2 + \frac{1}{2\eta_w} \tilde{\mathbf{W}}_{ko}^T \tilde{\mathbf{W}}_{ko} \quad (14)$$

where  $\eta_0$ ,  $\eta_1$  and  $\eta_2$  are positive constants.

Taking the time derivative of Eq. (14), it follows that

$$\begin{aligned} \dot{V}_1 &= Js\dot{s} + \tilde{z}\dot{\tilde{z}} + \frac{1}{\eta_0} \tilde{\mu}_0 \dot{\tilde{\mu}}_0 + \frac{1}{\eta_1} \tilde{\mu}_1 \dot{\tilde{\mu}}_1 + \frac{1}{\eta_2} \tilde{\mu}_2 \dot{\tilde{\mu}}_2 \\ &\quad + \tilde{\mathbf{W}}_{ko}^T (\mathbf{U}_{ko} s + \frac{1}{\eta_w} \dot{\tilde{\mathbf{W}}}_{ko}) + (\varepsilon - \hat{\tau}_{us}) s \\ &= -\alpha s^2 - \mu_0 f(\omega) \tilde{z}^2 \\ &\quad + \tilde{\mu}_0 (\dot{z} s - \frac{1}{\eta_0} \dot{\tilde{\mu}}_0) + \tilde{\mu}_1 (\dot{z} s - \frac{1}{\eta_1} \dot{\tilde{\mu}}_1) \\ &\quad + \tilde{\mu}_2 (\alpha s - \frac{1}{\eta_2} \dot{\tilde{\mu}}_2) + \mu_0 (1 - \mu_1 f(\omega)) s \tilde{z} \\ &\quad - \dot{z} f(\omega) \tilde{z} \tilde{\mu}_0 - \mu_1 \dot{z} f(\omega) s \tilde{\mu}_0 \\ &\quad + \tilde{\mathbf{W}}_{ko}^T (\mathbf{U}_{ko} s + \frac{1}{\eta_w} \dot{\tilde{\mathbf{W}}}_{ko}) + (\varepsilon - \hat{\tau}_{us}) s \end{aligned} \quad (15)$$

In order to estimate the friction parameters of LuGre friction model, the following friction parameter observers are proposed

$$\dot{\hat{\mu}}_0 = \eta_0 \dot{z} s, \quad (16)$$

$$\dot{\hat{\mu}}_1 = \eta_1 \dot{z} s, \quad (17)$$

$$\dot{\hat{\mu}}_2 = \eta_2 \alpha s. \quad (18)$$

Since  $\mathbf{W}_{ko}^*$  is constant vector, the following adaptation law and compensation law are chosen as

$$\dot{\hat{\mathbf{W}}}_{ko} = -\dot{\tilde{\mathbf{W}}}_{ko} = \eta_w \mathbf{U}_{ko} s \quad (19)$$

$$\hat{\tau}_{us} = E \operatorname{sgn}(s) \quad (20)$$

where  $\text{sgn}(\cdot)$  is a sign function, then Eq. (15) can be rewritten as

$$\begin{aligned} \dot{V}_2 &= -\Phi^T M \Phi + \varepsilon s - E|s| \leq |\varepsilon||s| - E|s| \\ &= -(E - |\varepsilon|)|s| = -\delta|s| \leq 0 \end{aligned} \quad (21)$$

where  $\Phi = [s \quad \tilde{z} \quad \tilde{\mu}_0]^T$ ,

$$M = \begin{bmatrix} \alpha & \mu_0(\mu_1 f(\omega) - 1) & \mu_1 \hat{z} f(\omega) \\ 0 & \mu_0 f(\omega) & \hat{z} f(\omega) \\ 0 & 0 & 0 \end{bmatrix} \geq 0$$

and  $\delta = E - |\varepsilon| \geq 0$  is a small positive constant.

However, the sign function of the control law leads to chattering in control input. Thus, the following equation can be chosen by replacing  $E$  by  $\hat{E}$  in Eq. (20)

$$\tau_{us} = \hat{E} \text{sgn}(s) \quad (22)$$

where  $\hat{E}$  is the estimation bound value of the approximation error. Then, the estimation error is defined as

$$\tilde{E} = E - \hat{E}.$$

Define the following Lyapunov function

$$V_2 = V_1 + \frac{1}{2\eta_E} \tilde{E}^2 \quad (23)$$

where  $\eta_E$  is a positive constant. Differentiating Eq. (23) with respect to time and using the previous results, it can be obtained that

$$\begin{aligned} \dot{V}_3 &= -\Phi^T M \Phi + \tilde{W}_{ko}^T (U_{ko} s + \frac{1}{\eta_w} \dot{\tilde{W}}_{ko}) \\ &\quad + (\varepsilon - \hat{T}_{us})s + \frac{1}{\eta_E} (E - \hat{E}) \dot{\tilde{E}} \\ &= -\Phi^T M \Phi + \varepsilon s - \hat{E}|s| + \frac{1}{\eta_E} (E - \hat{E}) \dot{\tilde{E}}. \end{aligned} \quad (24)$$

If the estimation law is chosen as

$$\dot{\hat{E}} = -\dot{\tilde{E}} = \eta_E |s| \quad (25)$$

then Eq. (24) can be rewritten as

$$\begin{aligned} \dot{V}_3 &= -\Phi^T M \Phi + \varepsilon s - \hat{E}|s| - (E - \hat{E})|s| \\ &\leq |\varepsilon||s| - E|s| = -(E - |\varepsilon|)|s| = -\delta|s| \leq 0 \end{aligned} \quad (26)$$

Therefore, similar to the previous statements, it can be also concluded that the asymptotic stability of  $s = 0$ ,  $\tilde{z} = 0$  follows. Also  $s \rightarrow 0$ ,  $\tilde{z} \rightarrow 0$  as  $t \rightarrow \infty$  by Barbalat lemma [10].

A four-layer RFNN, as shown in Fig. 1, which comprises the input, membership, rule, and output layer is adopted to implement the function of online gain tuning. The signal propagation and the basic function in each layer of the RFNN are introduced as follows.:

**Layer 1- Input layer:** Each node  $i$  in this layer is denoted by  $\Pi$ , which multiplies the input signals and

outputs the result of the product. The net inputs and the net output are represented as

$$\text{net}_i^1(N) = \prod_o u_o^1 w_i O_i^1(N-1) \quad (27)$$

$$O_i^1(N) = f_i^1(\text{net}_i^1) = \text{net}_i^1(N), \quad i = 1, 2 \quad (28)$$

where  $u_i^1$  represent the inputs;  $N$  denotes the number of iterations,  $w_i^1$  are the recurrent weights for the units in the input layer, and  $O_i^1$  is the output of the input layer.

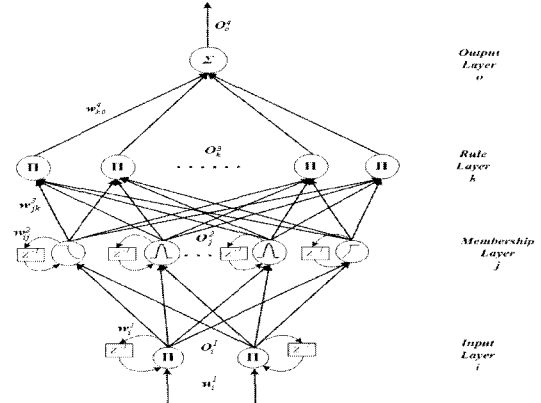


Fig. 1 A structure of the proposed RFNN

**Layer 2-Membership Layer:** The  $j$ th node input and output are represented as

$$\text{net}_j^2(N) = -\frac{[u_{ij}^2(N) - m_{ij}]^2}{(\sigma_{ij})^2} \quad (29)$$

$$O_j^2 = \exp(\text{net}_j^2(N)), \quad j = 1, \dots, n \quad (30)$$

where  $m_{ij}$  and  $\sigma_{ij}$  are, respectively, the mean and the standard deviation of the Gaussian function in the  $j$ th term of the  $i$ th term input linguistic variable  $u_{ij}^2$  to the node of the layer 2, and  $n$  is the total number of the linguistic variables with respect to the input nodes.

**Layer 3 – Rule Layer:** Each node  $k$  in this layer is also denoted by  $\Pi$ . The  $k$ th rule node is represented as

$$\text{net}_k^3(N) = \prod_j w_{jk} u_j^3(N) \quad (31)$$

$$O_k^3(N) = \text{net}_k^3(N), \quad k = 1, \dots, l \quad (34)$$

where  $u_j^3$  represents the  $j$ th input to the node of layer 3,  $w_{jk}^3$ , the weights between the membership layer and the rule layer, are assumed to be unity;  $l$  is the number of rules with complete rule connection if each input node has the same linguistic variables.

**Layer 4, Output layer:** The single node  $o$  in this layer is labeled as  $\Sigma$ , which computes the overall output as the summation of all input signals:

$$\mathbf{net}_o^4(N) = \sum_k w_{ko}^4 u_k^4(N), \quad (35)$$

$$O_o^4(N) = \mathbf{net}_o^4(N), \quad o = 1 \quad (36)$$

where the connecting weight  $w_{ko}^4$  is the output action strength of the  $o$ th output associated with the  $k$ th rule  $u_k^4$  represents the  $k$ th input to the node of layer 4, and  $O_o^4$  is the output of the fuzzy neural network and

$$W_{ko} = [w_{1o}^4, w_{2o}^4, \dots, w_{no}^4]^T \quad (37)$$

$$U_{ko} = [u_{1o}^4, u_{2o}^4, \dots, u_{no}^4]. \quad (38)$$

We omit the online learning algorithm of the above RFNN technique and similar explanations can be obtained in several literatures [6-9].

### III. EXPERIMENT WITH DISCUSSION

Some experiments are executed for the precision tracking control of a servo system in order to verify the performance of the proposed control scheme. A schematic diagram of the servo system is shown in Fig.2. The identified system parameters through experiments are given in Table 1. Three control schemes are designed for comparing with each result as sliding mode controller (SMC), sliding mode controller with friction observer (SMC\_OB) and sliding mode controller using friction state observer and RFNN (SMC\_RFNS).

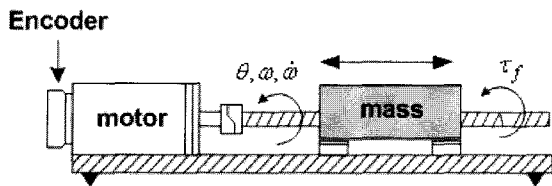


Fig.2 Schematic diagram of the servo system

Table 1. Parameters of ball-screw and friction model

Symbol	Value
$J$	$0.000246 \text{ kgm}^2$
$T_c$	$0.0088 \text{ Nm}$
$T_s$	$0.0011 \text{ Nm}$
$\omega_s$	$0.025 \text{ m/sec}$
$\sigma_0$	$0.225 \text{ N/rad}$
$\sigma_1$	$0.0055 \text{ N sec/rad}$
$\sigma_2$	$0.00023 \text{ N mrad/sec}$
torque constant	$0.24 \text{ Nm/A}$
amplifier gain	$2.17 \text{ A/V}$

The control algorithms are programmed in 'Turbo-C' language in DOS-mode and the control signals are transmitted into the DC motor drive through the DR8330 data acquisition board. The position information of the servo mechanical system is transmitted into the computer through the PCL-833 encoder counter board. The sampling rate is set to be 4ms in order to consider the calculation burden on the online training and updating parameters of the RFNN, calculation of SMC\_RFNS controller, calculation of the friction state observer and adaptive error estimator.  $\theta_d = 0.1 \sin(2\pi \times 0.2t)$  is chosen as a command input. The RFNN has also two, five, five, and one neuron at the input, membership, rule and output layers, respectively.

The command input and obtained position and control input of the SMC system is shown in Fig. 3.

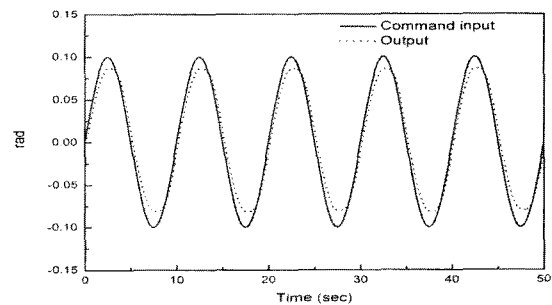
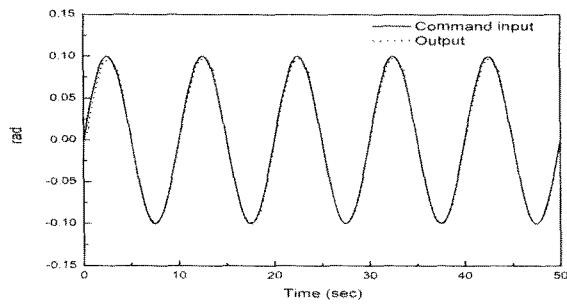


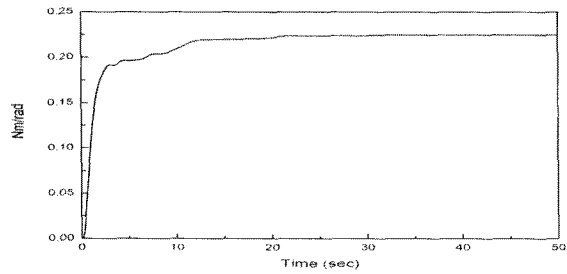
Fig. 3 Position tracking of the SMC system

The position tracking performance of the Back control system is very poor like simulation. If we increase the controller's gains of the Back control system to improve the tracking performance, the chattering and noise amplification due to the friction dynamics will be greatly increased. Also the stability of the entire control system can be worsened and the actuator saturation or uncontrollable dangerous situation that often appears in feedback control system is possible to be occurred. Since this trial is not desirable to real implementations, it can be concluded that the Back control system without friction observer is not appropriate to be used to control the precise friction dynamics.

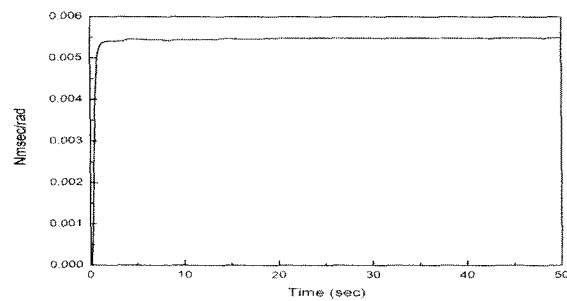
In case of the SMC\_OB, the tracking error is greatly decreased by virtue of the friction state observer like the results in simulation as shown in Fig. 4 and 5. And the estimated friction parameters are well adapted to each identified value in Fig. 4. However, since the measured friction parameters in Table 1 can be varied as operating conditions and can be different to the real values, the chosen friction parameters of each controller cannot be also said to the optimal values.



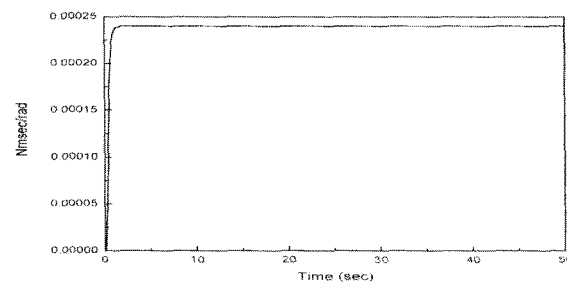
(a) Position tracking result



(b) Estimated result of  $\mu_0$



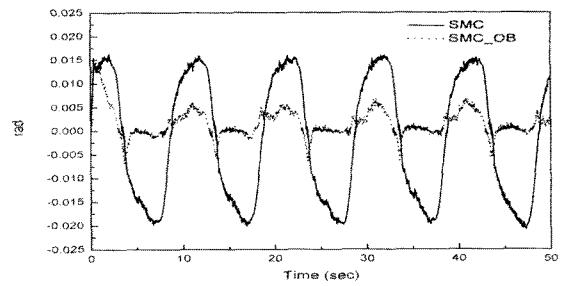
(c) Estimated result of  $\mu_1$



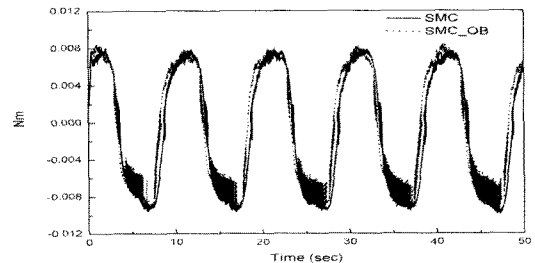
(d) Estimated result of  $\mu_2$

Fig. 4 Experimental results of the SMC\_OB system

Thus, the robust control using the RFNN estimator and adaptive approximation error estimator have to be considered. The experiment to examine the robustness to these uncertainty shows that the tracking error of the proposed SMC\_RFNS system is more decreased when compared with SMC\_OB system as shown in Fig. 7.

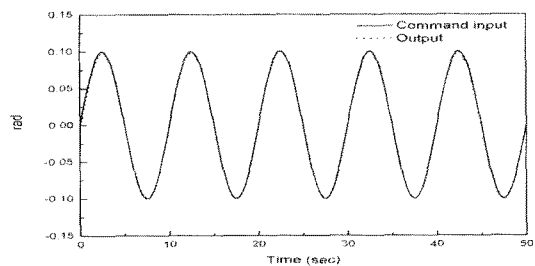


(a) Position tracking errors

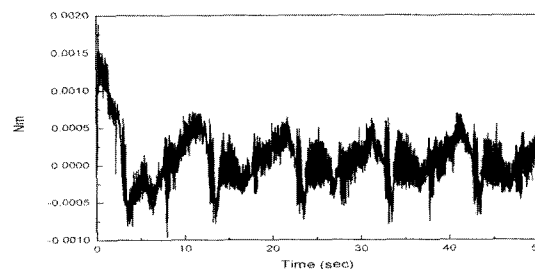


(b) Control inputs

Fig. 5 Experimental results of the SMC and the SMC\_OB system



(a) Position tracking result



(b)  $\hat{\tau}_{dm} + \hat{\tau}_{us}$

Fig. 6 Experimental results of the SMC\_RFNS system

Therefore, it is known that the proposed control system is very robust to uncertainty caused by the variation of the friction torque

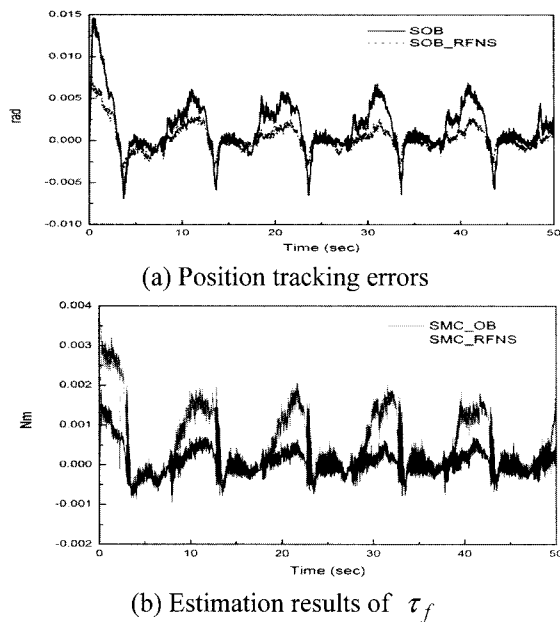


Fig. 7 Experimental results of the SMC\_OB and the SMC\_RFNS system

#### IV. CONCLUSIONS

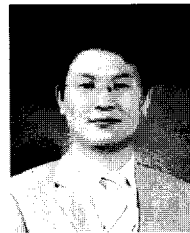
In this paper, the intelligent friction compensation scheme is introduced with the sliding mode controller and adaptive friction observer for position tracking control of the servo system with dynamic friction. The dynamic friction is modeled as the LuGre friction model. Parameters of the friction are estimated via adaptive friction observers. Online RFNN algorithm and reconstructed error compensator are also designed to provide additional robustness property to the friction control system. Some experiments show the feasibility of compensating friction of our proposed control scheme.

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