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A Note on Determination of Sample Size for a Likert Scale

Jinwoo Park[1,a], Misook Jung[a]

[a]Department of Applied Statistics, University of Suwon

Abstract

When a social scientist prepares to conduct a survey, he/she faces the problem of deciding an appropriate sample size. Sample size is closely connected with cost, time, and the precision of the sample estimate. It is thus important to choose a size appropriate for the survey, but this may be difficult for survey researchers not skilled in a sampling theory. In this study we propose a method to determine a sample size under certain assumptions when the quantity of interest is measured by a Likert scale.

Keywords: Sample survey, sample size, Likert scale, level of precision, coefficient of variation.

1. Introduction

When a social scientist prepares to conduct a research survey, he/she faces the problem of determining an appropriate sample size. A desirable sample represents the population and guarantees a desired level of precision for the statistic in consideration. To determine a sample size for a survey, one must first consider the most common sample survey estimates such as mean, proportion, total. A researcher must select a desired level of precision for the statistic in consideration. For a researcher who is not acquainted with specifying this, the most common method of determining on a sample size is using a generalized table of sampling errors for samples of various sizes and for various proportions, provided that samples are from a simple random sampling (Yamane, 1967). Fowler (1984) has pointed out the following mistakes in determining sample size for social surveys. One common misconception is that the adequacy of a sample depends heavily on the fraction of the population included in that sample. Another wrong approach is to decide sample size according to a "typical" sample size.

A survey researcher is seldom able to devise a single questionnaire items that adequately tap respondents' degrees of prejudice, religiosity, political orientations, alienation and the like. He/she frequently constructs some scales that are composite measures of several variables. One of the simplest and most widely used scale types is the Likert scale (Babbie, 2005). In the case that the quantity of interest is a Likert scale composed of several variables, a different method of determining a sample size is needed.

The objective of this study is to provide a way to determine a sample size when the quantity of interest is a Likert scale. We suggest some tables for determining sample sizes for various situations. In Section 2 we describe the theoretical background of determining a sample size using a Likert scale. We provide some tables for various situations in Section 3. Finally, some concluding remarks are given in Section 4.

[1] Corresponding author: Professor, Department of Applied Statistics, University of Suwon, Bongdam-Eup, Hwasung-Si, Kyunggi-Do 445-743, Korea. E-mail: jwpark@suwon.ac.kr

Table 1: Questions for the student satisfaction survey

Are you satisfied with your:	Variable
academic experiences	X_1
quality of instruction	X_2
faculty	X_3
sense of belonging	X_4
resource services	X_5

Table 2: Basic statistics of the study variables

Variable	X_1	X_2	X_3	X_4	X_5
Sample size	250	250	250	250	250
Sample mean	2.71	1.93	2.14	2.03	1.41
Sample SD	0.98	0.86	0.97	0.84	1.02

Table 3: Estimates of pairwise correlation coefficients

	X_1	X_2	X_3	X_4	X_5
X_1	1.00	0.34	0.30	0.19	0.17
X_2	0.34	1.00	0.61	0.33	0.34
X_3	0.30	0.61	1.00	0.49	0.30
X_4	0.19	0.33	0.49	1.00	0.27
X_5	0.17	0.34	0.30	0.27	1.00

2. Determining a Sample Size for Likert Scale

Likert scaling, the measurement method developed by Likert (1932), is one of the most widely used methods in contemporary survey questionnaires. A typical test item in a Likert scale is a statement; the respondent is asked to indicate his or her degree of agreement with the statement or a subjective or objective evaluation of the statement. Traditionally, a four-point or a five-point scale is used. Whereas identical response categories will have been used for k items intended to measure a given variable, each item might be scored in a uniform manner. In the case of a D-point scale, scores of 0 to $(D-1)$ may be assigned. Each respondent would then be assigned an overall score representing the total of the scores he or she received for responses to the individual items.

Denote $X_{i1}, X_{i2}, \ldots, X_{ik}$ by the Likert-item scores of the i^{th} respondent $(i = 1, 2, \ldots, n)$. Let Y_i be a Likert scale of the i^{th} respondent; then it may be denoted as $Y_i = X_{i1} + \cdots + X_{ik}$.

In general, Likert-items $X_{ij}\,(j = 1, 2, \ldots, k)$ are not independent but are correlated. In order to determine a sample size, we consider some simple statistical assumptions. We assume that each $X_{ij}(i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, k)$ has the same mean and variance (μ, σ^2). Furthermore, assume that each pairwise correlation coefficient between X_{ij} and $X_{ij'}$ for any $j \neq j'$ is the same, that is, $\mathrm{Corr}(X_{ij}, X_{ij'})$ is the same for all $j \neq j'$.

Consider the appropriateness of the above assumptions for the following real survey example.

Example 1. The 2008 Student Satisfaction Survey of University of Suwon.

► Students were asked to indicate their level of satisfaction in the following five categories at Table 1. Here, a five-point scale was used for each item. Two hundred fifty students were selected by stratified random sampling.

Table 2 lists the sample means and the sample standard deviations of the study variables. The sample means range from 1.41 to 2.71, and the sample standard deviations from 0.84 to 1.02. Since determination of sample size is done roughly and conservatively at the design stage, it seems tolerable to assume that each $X_{ij}(i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, k)$ has the same mean and variance (μ, σ^2).

Next, Table 3 shows estimates of the pairwise correlation coefficients of all five variables. Though they range from 0.17 to 0.61, most of them are near 0.3. The results show us that the assumption of identical pairwise correlation coefficients is allowable.

Since $Y_i = X_{i1} + \cdots + X_{ik}$, we derive the following asymptotic distribution of Y_i:

$$Y_i = \frac{1}{k}\sum_{j=1}^{k} X_{ij} \sim N\left(\mu, \sigma^{*2}\right), \tag{2.1}$$

where $\sigma^{*2} = \sigma^2/k[1 + (k-1)\rho]$.

Here, the estimator in consideration is the overall sample mean, $\bar{Y} = 1/n \sum_i^n Y_i$. Since $Y_1, Y_2, \ldots, Y_n$ are independent and identically distributed random variables, we get the following distribution of $\bar{Y}$:

$$\bar{Y}_i = \frac{1}{n}\sum_{i=1}^{n} Y_i = \frac{1}{nk}\sum_{i=1}^{n}\sum_{j=1}^{k} X_{ij} \sim N\left(\mu, \frac{\sigma^{*2}}{n}\right). \tag{2.2}$$

To estimate the sample size, one must specify a tolerable error. The desired precision is often expressed in absolute terms, B, the bound of error. The simplest equation relating the specified precision and sample size comes from the confidence intervals for μ, that is,

$$B = z_{\frac{\alpha}{2}} \cdot \sqrt{\mathrm{Var}(\bar{Y})} = z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\sigma}{nk}\{1 + (k-1)\rho\}}, \tag{2.3}$$

where $z_{\alpha/2}$ is the $100(1-\alpha/2)^{th}$ percentile of the standard normal distribution.

Solving the above equation for n, we derive the following equation. Since σ and ρ are the unknown model parameters, they must be guessed.

$$n = \frac{z_{\frac{\alpha}{2}}^2 \cdot \sigma^2}{kB^2}\{1 + (k-1)\rho\}. \tag{2.4}$$

In some situations it is useful to consider relative measures of the variation rather than absolute ones (Kish, 1965). Common relative measure is the coefficient of variation, in which the unit of measurement is cancelled by dividing with the mean.

Denote the relative tolerable error as $D, D = B/\mu$. Then,

$$n = \frac{z_{\frac{\alpha}{2}}^2 \cdot C^2}{kD^2}\{1 + (k-1)\rho\}, \tag{2.5}$$

where C is the coefficient of variation of each Likert-item score.

3. Tables for Sample Size Determination

To determine a sample size using of the Equation (2.5) in Section 2, we need to specify a confidence level, $(1-\alpha)$, a relative tolerable error, D, a number of items, k, used for Likert scale, a coefficient of variation of a population, C, and a pairwise correlation coefficient, ρ. Since it is not easy for a researcher to use the Equation (2.5) to determine a sample size, several tables of various situations are presented, and some simple guidelines for using the tables are suggested.

Table 4: Sample size with $C = 1.0$ and $\rho = 0.3$

k \ D	1%	2%	3%	4%	5%	10%
1	38416	9604	4268	2401	1537	384
2	24970	6243	2774	1561	999	250
3	20489	5122	2277	1281	820	205
4	18248	4562	2028	1140	730	182
5	16903	4226	1878	1056	676	169
6	16007	4002	1779	1000	640	160
7	15366	3842	1707	960	615	154
8	14886	3722	1654	930	595	149
9	14513	3628	1613	907	581	145
10	14214	3553	1579	888	569	142

Table 5: Sample size with $C = 1.0$ and $\rho = 0.5$

k \ D	1%	2%	3%	4%	5%	10%
1	38416	9604	4268	2401	1537	384
2	28812	7203	3201	1801	1152	288
3	25611	6403	2846	1601	1024	256
4	24010	6003	2668	1501	960	240
5	23050	5762	2561	1441	922	230
6	22409	5602	2490	1401	896	224
7	21952	5488	2439	1372	878	220
8	21609	5402	2401	1351	864	216
9	21342	5336	2371	1334	854	213
10	21129	5282	2348	1321	845	211

The number of Likert items, k, changes from 1 to 10. When the number of response categories is 2, it is the case of estimating a population proportion. Hence, a generally used table of sampling errors for samples of various sizes and for various proportions may be regarded as a special case of the currently proposed method. The coefficients of variation of a population, C, varies from 0.1 to 1.0, and ρ from 0.1 to 0.7. The listed bounds of the tolerable error, D, are 1%, 2%, 3%, 4%, 5% and 10%.

Table 4 shows the sample sizes based on (2.5) with $C = 1.0$ and $\rho = 0.3$. In the case that $C = 1.0$, the standard deviation is the same as the mean. Here, if $k = 1$, this is the same problem as determining a sample size for estimate when the population proportion is $\pi = 0.5$.

In Table 4, we find that sample sizes are small for large k under a given tolerable error bound. When $k = 5$, the sample sizes are smaller than half those under $k = 1$. That is, when one determines a sample size in a social survey containing 5 survey variables for a Likert scale, using the formula to determining the sample size for estimates of the population proportion yields an inefficient sample size, more than twice that is needed. However, using (2.5) does not increase a sample size unnecessarily.

Table 5 shows the sample size for $C = 1.0$ and $\rho = 0.5$. By comparing Tables 4 and 5, one can find that the required sample size to meet the given restrictions is directly proportional to the magnitude of the correlation coefficient. Sample sizes for larger ρ are greater than those for smaller ones.

Table 6 displays the sample size for $C = 0.5$ and $\rho = 0.5$. In Equation (2.5), we find that the required sample sizes are directly proportional to the squares of the coefficients of variation.

Generally, when a 4-point or 5-point scale is used for each Likert-item, the CV of a population, C, is much smaller than 1.0 ($C < 1$) . Since respondents tend to avoid choosing extreme response categories, the proportion of choosing the middle option of 'Neither agree nor disagree' is larger than those of extreme options, so that the CV is smaller than 1.0. As illustrated by the example in Section

Table 6: Sample size with $C = 0.5$ and $\rho = 0.5$

k \ D	1%	2%	3%	4%	5%	10%
1	9604	2401	1068	601	385	97
2	7203	1801	801	451	289	73
3	6403	1601	712	401	257	65
4	6003	1501	667	376	241	61
5	5763	1441	641	361	231	58
6	5603	1401	623	351	225	57
7	5488	1372	610	343	220	55
8	5403	1351	601	338	217	55
9	5336	1334	593	334	214	54
10	5283	1321	587	331	212	53

2, the CV generally ranges from 0.3 to 0.5.

$\rho(= \mathrm{Corr}(X_{ij}, X_{ij'}))$ is the pairwise correlation coefficient between X_{ij} and $X_{ij'}$ for $j \neq j'$. Typically, ρ assumes a value between 0.2 and 0.5. For larger ρ and C, larger sample sizes are required.

When we are to determine an appropriate sample size for a research survey, it is desirable to choose a conservative one. We thus suggest the choice $C = 0.5$ and $\rho = 0.5$, since this seems to be enough to achieve the expected sampling errors in typical surveys.

4. Concluding Remarks

When a social scientist prepares a research survey, he/she must decide an appropriate sample size. Because the sample size is closely connected with survey cost, time, and the precision of the sample estimate, a desirable sample size must fit the purpose of the survey. Doing this may be difficult, however, for survey researchers not skilled in a sampling theory.

In this study we have proposed a method to determine a sample size when the quantity of interest is measured on a Likert scale. We have suggested a method of determining an appropriate sample size under some assumptions. The results show that such parameters as the pairwise correlation coefficient (ρ) of Likert items and the coefficient of variation (C) influence the required sample size. We also provided some tables for determining a sample size for various ρ and C. We also proposed determination of sample sizes using parameter values $C = 0.5$ and $\rho = 0.5$.

The results show that in a research survey using a Likert scale, using a method to determining the sample size for estimates of the population proportion may yield an unnecessarily large sample size. If we use the proposed method, however, we can reduce the sample size by about 30% to 40%, compared with the other conventional method.

References

Babbie, E. R. (2005). *The Basics of Social Research,* Thomson Wadsworth.
Cochran, W. G. (1977). *Sampling Techniques,* John Wiley & Sons, New York.
Fowler, F. J. (1984). *Survey Research Methods,* Beverly Hills, California.
Kish, L. (1965). *Survey Sampling,* John Wiley & Sons, New York.
Likert, R. (1932). A technique for the measurement of attitudes, *Archives of Psychology,* **140**, 1–55.
Yamane, T. (1967). *Elementary Sampling Theory,* Prentice-Hall.

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