# Effects of thermal boundary conditions and microgravity environments on physical vapor transport of Hg,Cl,-Xe system

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Abstract For the effects of the nonlinear temperature profiles and reduced-gravity conditions we conduct a two-dimensional numerical modeling and simulations on the physical vapor transport processes of Hg<sub>2</sub>Cl<sub>2</sub>-Xe system in the horizontal orientation position. Our results reveal that: (1) A decrease in aspect ratio from 5 to 2 leads to an increasingly nonuniform interfacial distribution and enhances the growth rate by one-order magnitude for normal gravity and linear wall temperature conditions. (2) Increasing the molecular weight of component B, Xenon results in a reduction in the effect of solutal convection. (3) The effect of aspect ratio affects the interfacial growth rates significantly under normal gravity condition rather than under reduced gravitational environments. (4) The transition from the convection-dominated regime to the diffusion-dominated regime ranges arises near at 0.1g<sub>0</sub> for operation conditions under consideration in this study.

Key words Mercurous chloride, Convection, Xenon, Microgravity, Physical vapor transport

#### 1. Introduction

Physical vapor transport (PVT) has become an important crystal growth process for a variety of acousto-optic materials. The mechanism of the PVT process is simple: sublimation-condensation in closed silica glass ampoules in temperature gradient imposed between the source material and the growing crystal. It also requires minimal process control and monitoring, and transport results are easily interpreted. Of particular importance is the use of PVT for materials processing experiments in low and high gravity environments, which would provide a better and thorough understanding of transport phenomena occurring in the vapor phase and crystal growth phenomena.

Over the past 25 years many theoretical modeling studies [1-12] on transport phenomena in PVT and much quantitative experiments [13-20] have been extensively investigated. Most important theoretical works were achieved by Rosenberger [1-4, 6, 8] and, recently extended for transition to chaos flow fields in specialty materials of mercurous chloride in applications of microgravity experiments by Duval [21-25]. They have addressed the underlying phenomena in the PVT pro-

cesses on the relative importance and influencing parameters of diffusion-advection, thermal and/or solutal convection on mass transport. Our recent studies [26-33] are for PVT processes of specialty materials such as mercurous halides (Hg<sub>2</sub>Cl<sub>2</sub> and Hg<sub>2</sub>Br<sub>2</sub>) under normal and microgravity environments to investigate the role of convection on the mass transport rate and its transition from diffusion-dominated to circulatory convection-dominated regimes in relation to total pressure, temperatures of source and crystal ends, aspect ratio (transport lengthto-width), molecular weights, wall temperature profiles.

It is the purpose of this paper to study systematically the transport phenomena covering the various gravity accelerations in the PVT processes of Hg<sub>2</sub>Cl<sub>2</sub> crystal growth system. For this theoretical analysis of the PVT processes, a two-dimensional model is in horizontally oriented, cylindrical, closed ampoules in a two-zone furnace system. Diffusion-limited processes are considered in this paper, although Singh, Mazelsky and Glicksman [34] demonstrated that the interface kinetics plays an important role in the PVT system of Hg<sub>2</sub>Cl<sub>2</sub>. The effects of normal and high gravity accelerations on solutally and/or thermally buoyancy-driven convection will be considered at this point, primarily for a mixture of Hg<sub>2</sub>Cl<sub>2</sub> vapor and impurity of Xenon (Xe), although for gaining insights into the convection, the low gravity environments are more important than high gravity conditions in some cases.

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# 2. Model and Numerical Analysis

Mercurous chloride (Hg<sub>2</sub>Cl<sub>2</sub>) materials are important for applications in acousto-optic and opto-electronic devices such as Bragg cells, X-ray detectors operating at ambient temperature [35]. The equimolar Hg<sub>2</sub>Cl<sub>2</sub> compound decomposes to two liquids at a temperature near 525°C where the vapor pressure is above 20 atm [36, 37]. Because of this decomposition and high vapor pressure, Hg<sub>2</sub>Cl<sub>2</sub> cannot be solidified as a single crystal directly from the stoichiometric melt. However, very similar to the mercurous bromide, mercurous chloride exhibits sufficiently high vapor pressure at low temperatures so that these crystals are usually grown by the physical vapor transport (PVT) in closed silica glass ampoules.

Consider a rectangular enclosure of height H and transport length L, shown in Fig. 1. The source is maintained at a temperature T<sub>s</sub>, while the growing crystal is at a temperature  $T_c$ , with  $T_s > T_c$ . PVT of the transported component A (Hg<sub>2</sub>Cl<sub>2</sub>) occurs inevitably, due to presence of impurities, with the presence of a component B (Xe). The interfaces are assumed to be flat for simplicity. The finite normal velocities at the interfaces can be expressed by Stefan flow deduced from the onedimensional diffusion-limited model [38], which would provide the coupling between the fluid dynamics and species calculations. On the other hand, the tangential component of the mass average velocity of the vapor at the interfaces vanishes. Thermodynamic equilibria are assumed at the interfaces so that the mass fractions at the interfaces are kept constant at  $\omega_{A,s}$  and  $\omega_{A,c}$ . On the vertical non-reacting walls appropriate velocity boundary conditions are no-slip, the normal concentration gradients are zero, and wall temperatures are imposed as nonlinear temperature gradients. Thermo-physical properties of the fluid are assumed to be constant, except for the density. When the Boussinesq approximation is

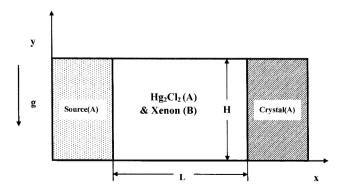


Fig. 1. Definition sketch for two-dimensional PVT model of  $Hg_2Cl_2$ -Xe system.

invoked, density is assumed constant except the buovancy body force term. The density is assumed to be a function of both temperature and concentration. The ideal gas law and Dalton's law of partial pressures are used. Viscous energy dissipation and the Soret-Dufour (thermo-diffusion) effects can be neglected, as their contributions remain relatively insignificant for the conditions encountered in our PVT crystal growth processes. The transport of fluid within a rectangular PVT crystal growth reactor is governed by a system of elliptic, coupled conservation equations for mass (continuity), momentum, energy and species (diffusion). They can be represented by Eqs. (1) through (7) [39] with their appropriate boundary conditions Eqs. (8) through (10). Let u<sub>x</sub>, u<sub>y</sub> denote the velocity components along the x- and y-coordinates in the x, y rectangular coordinate, and let T,  $\omega_{A}$ , p denote the temperature, mass fraction of species A (Hg<sub>2</sub>Cl<sub>2</sub>) and pressure, respectively, where the superscript of \* denotes the dimensionless [26-33].

The dimensionless variables are defined as follows:

$$x^* = \frac{x}{H}, \quad y^* = \frac{y}{H}, \tag{1}$$

$$u^* = \frac{u_x}{U_c}, \quad v^* = \frac{u_y}{U_c}, \quad p^* = \frac{p}{\rho_c U_c^2},$$
 (2)

$$T^* = \frac{T - T_c}{T_s - T_c}, \ \omega_A^* = \frac{\omega_A - \omega_{A,c}}{\omega_{A,s} - \omega_{A,c}}.$$
 (3)

The dimensionless governing equations are given by:

$$\nabla^* \bullet \overrightarrow{V}^* = 0. \tag{4}$$

$$\overrightarrow{V}^* \bullet \overrightarrow{V}^* \overrightarrow{V}^* = - \overrightarrow{\nabla}^* \overrightarrow{p}^* + Pr \cdot Ar \overrightarrow{\nabla}^{*2} \overrightarrow{V}^* - \frac{Ra \cdot Pr}{Ar} \cdot \overrightarrow{T}^* \cdot e_g, \quad (5)$$

$$\overrightarrow{V}^* \bullet \nabla^* T^* = Ar \nabla^{*2} T^*$$
 (6)

$$\overrightarrow{V}^* \bullet \nabla^* \omega_A^* = \frac{Ar}{Le} \nabla^{*2} \omega_A^* \tag{7}$$

On the walls  $(0 < x^* < L/H, y^* = 0 \text{ and } 1)$ :

$$\begin{split} &u^{*}(x^{*},0)=u^{*}(x^{*},1)=v^{*}(x^{*},0)=v^{*}(x^{*},1)=0\\ &\frac{\partial \omega_{A}^{*}(x^{*},0)}{\partial y^{*}}=\frac{\partial \omega_{A}^{*}(x^{*},1)}{\partial y^{*}}=0,\\ &T^{*}(x^{*},0)=T^{*}(x^{*},1)=\frac{T-T_{c}}{T-T} \end{split} \tag{8}$$

On the source  $(x^* = 0, 0 < y^* < 1)$ :

$$\mathbf{u}^{*}(0,\mathbf{y}^{*}) = -\frac{1}{Le(1 - \omega_{A,s})} \frac{\partial \omega_{A}^{*}(0,\mathbf{y}^{*})}{\partial \mathbf{x}^{*}}, \tag{9}$$

$$v^*(0,y^*) = 0,$$
 $T^*(0,y^*) = 1,$ 
 $\omega_A^*(0,y^*) = 1.$ 

On the crystal  $(x^* = L/H, 0 < y^* < 1)$ :

$$\mathbf{u}^{*}(\mathrm{L/H},\mathbf{y}^{*}) = -\frac{1}{\mathrm{Le}(1 - \omega_{\mathrm{A,c}})} \frac{\partial \omega_{\mathrm{A}}^{*}(\mathrm{L/H},\mathbf{y}^{*})}{\partial \mathbf{x}^{*}}, \tag{9}$$

$$v^{*}(L/H, y^{*}) = 0,$$
 $T^{*}(L/H, y^{*}) = 0,$ 
 $\omega_{A}^{*}(L/H, y^{*}) = 0.$ 

The crystal growth rate  $V_c$  [7, 26-33] can be expressed as follows:

$$\int \rho_{\mathbf{v}} \vec{\mathbf{v}}_{\mathbf{v}} \cdot \vec{\mathbf{n}} dA = \int \rho_{\mathbf{c}} \vec{\mathbf{v}}_{\mathbf{c}} \cdot \vec{\mathbf{n}} dA, \tag{11}$$

$$V_{c} = \frac{\rho_{v} \vec{\nabla}_{v} \cdot \vec{n} dA}{\rho_{c} \int dA}.$$
 (12)

It should be noted that from a mass balance at the crystal vapor interface, assuming fast kinetics, i.e. all the vapor is incorporated into the crystal, see refs. [26, 27] in details.

There is only limited information available for the thermophysical properties of  $Hg_2Cl_2$ , with the exception of the vapor pressure. To estimate the properties of the mixture of  $Hg_2Cl_2$  and Xe the Chapman-Enskog relations [43] based on the Lennard-Jones potentials are used. The vapor pressure  $p_A$  of  $Hg_2Cl_2$  was estimated from the correlation [10]

$$p_A = e^{(a-b/T)},$$
 (13)

in which  $p_A$  is in Pascal units, and a = 29.75 and b = 11767.1.

The detailed numerical schemes in order to solve the discretization equations for the system of nonlinear, coupled governing partial differential equations are found in ref. [42]. From considerations of both spatial convergence and accuracy, a grid mesh 23 × 43 has been found satisfactory. In each calculation, depending upon the operating conditions such as the imposed boundary conditions and the molecular weights, it takes about 5,000~ 10,000 iterations to reach convergence.

## 3. Results and Discussion

The purposes for this study is to investigate the effects of solutal convection (called as a concentration buoy-

Table 1 Typical thermo-physical properties used in this study ( $M_A = 472.086$ ,  $M_B = 131.3$ )

Transport length, L	10 cm
Height, H	2 cm
Source temperature, T <sub>s</sub>	612.5 K
Crystal temperature, T <sub>c</sub>	576.0 K
Thermal diffusivity, κ	$0.16 \text{ cm}^2/\text{s}$
Kinematic viscosity, μ	$0.13 \text{ cm}^2/\text{s}$
Diffusivity, D <sub>AB</sub>	$0.49 \text{ cm}^2/\text{s}$
Thermal expansion coefficient, β	$0.0016 \mathrm{K}^{-1}$
Prandtl number, Pr	0.82
Lewis number, Le	0.33
Peclet, Pe	3.74
Concentration number, Cv	1.02
Total system pressure, P <sub>T</sub>	293.54 Torr
Partial pressure of component B, P <sub>B</sub>	10 Torr
Thermal Grashof number, Grt	$2.65 \times 10^4$
Solutal Grashof number, Gr <sub>s</sub>	$2.99 \times 10^{5}$

ancy driven convection) with gravitational acceleration and nonlinear/linear temperature profiles on the growth rate and its interfacial distributions, and the convective intensities during physical vapor transport. Because the molecular weight of a noble element xenon (Xe) is not equal to that of the crystal component (Hg<sub>2</sub>Cl<sub>2</sub>) during the physical vapor transport, solutal effects should be considered mainly in this study. Typical dimensionless parameters and physical properties for the operating

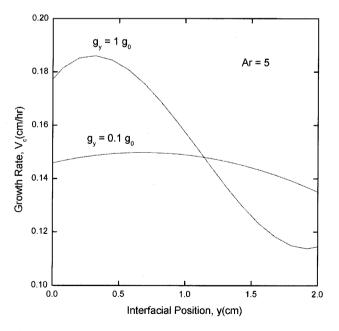


Fig. 2. Interfacial distribution of crystal growth rates for a horizontal ampoule of aspect ratio 5, a linear wall temperature profile between  $T_s=612.5~K$  and  $T_c=576.0~K,$  and  $g_v=1g_0$  and  $0.1g_0.$  Based on Ar = 5, Pr = 0.80, Le = 0.37,  $Gr_t=2.85\times 10^4,$   $Gr_s=3.05\times 10^5,$  Pe = 2.27,  $C_v=1.11.$ 

conditions of this study are shown in Table 1 on the basis of Fig. 9 with the nonlinear wall temperatures.

Fig. 2 shows our results for a horizontal system of aspect ratio of 5 (L = 10 cm, H = 2 cm), with the source temperature,  $T_s = 612.5 \text{ K}$  and the crystal temperature,  $T_c = 576.0 \text{ K}$ , and gravity accelerations of  $g_y = 1g_0$  and  $0.1g_0$  in the negative y-direction, where  $g_0$  denotes the

standard terrestrial gravitational acceleration 981 cm s<sup>-2</sup>. Interfacial distributions of the growth rate, indicative of nonuniformities, exhibits the effects of solutal convection. In this study thermally buoyancy driven convection (normally called as thermal convection) could be neglected because the solutal convection is one order of magnitude larger than the thermal convection. As shown

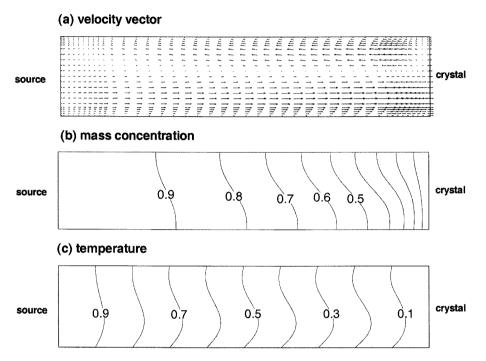


Fig. 3. Dimensionless velocity vector fields, temperature and mass concentration contours corresponding to the case of  $g_y = 1g_0$  in Fig. 2. The maximum magnitude of the velocity vector is 0.45 cm s<sup>-1</sup>.

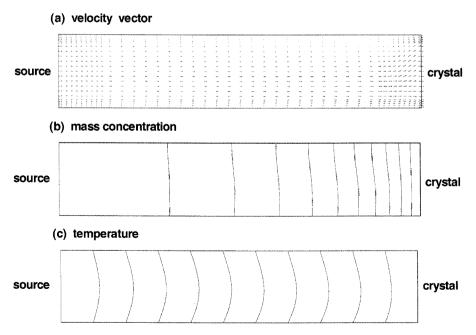


Fig. 4. Dimensionless velocity vector fields, temperature and mass concentration contours corresponding to the case of  $g_y = 0.1g_0$  in Fig. 2. The maximum magnitude of the velocity vector is 0.12 cm s<sup>-1</sup>.

in Fig. 2, the convection enhances the overall mass transport of component A (Hg<sub>2</sub>Cl<sub>2</sub>), and, then results in an increase in its growth rate, but pays for the expense of uniformity in the growth rate. Figs. 3 and 4 show dimensionless velocity vector fields, temperature and mass concentration contours corresponding to the case of  $g_v = 1g_0$  and  $g_v = 0.1g_0$  in Fig. 2, respectively. In Figs. 3 and 4, the maximum magnitude of the velocity vector is 0.45 cm s<sup>-1</sup>, 0.12 cm s<sup>-1</sup>, respectively. For the cases of  $g_v = 1g_0$  and  $0.1g_0$ , the same governing parameters of Ar, Pe, Pr, Le and C<sub>v</sub> are considered except for a Grashof numbers of  $2.85 \times 10^4$  and  $2.85 \times 10^3$ , respectively. As shown in Fig. 3, the flow field shows an asymmetrical unicellular flow structure. It can be seen that no secondary flow is predicted in the neighborhood of the crystal interface. The mass concentration contours show a sharp gradient in front of the crystal interface. The temperature contours show symmetrical at y = 1 and are in well linearly ordered arrangement. This reflects the importance of solutal convection rather than thermal convection for PVT operating conditions under consideration. As can be seen in Fig. 4, the dimensionless velocity vector field shows relatively asymmetrical uniflow profiles with velocities at bottom walls greater than at top walls. The corresponding mass concentration and temperature contours are symmetrical. It can be found that no cellular convective flow structure appears in the vapor phase during the PVT processes under 0.1g<sub>0</sub>. Therefore, the reduction in solutally buoyancyinduced convection results in improved uniformity of growth at the crystal interface. In other words, the recirculating flow have a significant effect on the distribution of growth rate at the crystal interface. The differences in flow fields obtained between under normal and one tenth-reduced gravitational environments are attributed to the effects of gravity on the vapor phase transport mechanism. Also, the transition from recirculating flow due to solutal buoyancy convection to diffusion-dominant mode is reflected in the interfacial velocity. Note diffusion-dominant mode is not necessarily attained in buoyancy-free environments under PVT processes with thermal creep phenomena [41].

Fig. 5 shows the results for a system with an aspect ratio 2 (L = 4 cm),  $T_s = 612.5$  K and  $T_c = 576.0$  K, the gravity accelerations of  $g_y = 1g_0$  and  $0.1g_0$ ,  $g_x = 1g_0$  and  $0.1g_0$ , where  $g_y = 1g_0$  and  $g_x = 1g_0$  correspond to the horizontal and the vertical orientations against the gravity vector, respectively. Note Figs. 2 and 5 are based on  $P_B = 50$  Torr, and linear wall temperature profiles. As can be seen in Figs. 2 and 5, the rates for Ar = 2 are

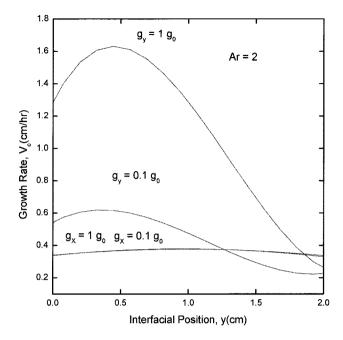


Fig. 5. Interfacial distribution of crystal growth rates for a horizontal ampoule of aspect ratio 2, a linear wall temperature profile between  $T_s=612.5~K$  and  $T_c=576.0~K,$  and  $g_y=1g_0$  and  $0.1g_0.$  Based on Ar = 2, Pr = 0.86, Le = 0.23,  $Gr_t=2.14\times10^4,$   $Gr_s=2.78\times10^5,$  Pe = 2.74,  $C_v=1.07.$ 

much greater than for Ar = 5 as well as the nonuniformities in interfacial distributions because of the effects of convection. The maximum rates for both Ar = 2 and 5 occur in the neighborhood of interfacial position, y = 0.5 cm. For Ar = 2, the reduction in gravitational level from  $g_v = 1g_0$  to  $0.1g_0$  in the horizontal orientation position weakens the effect of convection significantly. For example, the maximum rate of 1.61 cm/hr at  $g_v = 1g_0$  is reduced to 0.61 cm/hr. It is clear that a factor of ten reduction in the gravitational level is enough to suppress the convective effects on the growth rate. In reduced gravity environments the growth rates can be also controlled by altering the aspect ratio of ampoules, i.e., the smaller the aspect ratio is, the larger the growth rate is, under otherwise unchanged operating conditions. Under normal gravitational conditions, as the aspect ratio is reduced from 5 down to 2, i.e., by a factor of 0.4, the maximum rate for Ar = 2 is found to be one order of magnitude larger than for Ar = 5. On the other hand, under one tenth-reduced gravitational conditions, the former is four times greater than for the latter. Under reduced-gravity environments a factor of 0.4 reduction in an aspect ratio enhances the maximum rate by a factor of 4. As can be seen in Fig. 5, the convection under the horizontal orientation affects more significantly the growth rate than under the vertical orientation. It should be emphasized that the buoyancy driven convection due

to density gradients always occurs in the horizontal orientation. For the system under consideration in this study solutally buoyancy driven convection is much more dominant by one order magnitude than thermally buoyancy driven convection. The growth rate in the vertical orientation with the bottom-heated source, top-positioned crystal (referred as thermally convective-destabilizing orientation) is nearly invariant with the gravity levels of both  $g_x = 1g_0$  and  $0.1g_0$ . This indicates the mass transport is mainly governed by the diffusion rather convection mode, at least, in the vertical orientation under consideration in this study. The diffusive effects are reflected through governing parameters of Lewis and Peclet number. As seen in Figs. 2 and 5, small non-uniformities in the growth rates appear even in the ten reduction of gravitation level, which is likely to stem from the recirculation of the inert gas. It is consistent with the results of [1] which addressed, the nonuniformity can occur even for the condition of zero gravity. Note even for purely diffusive transport the growth rate is not strictly uniform, with growth being greater near the center than near the walls. In addition, the pattern of the interfacial distribution for Ar = 2 is more axisymmetrical against y = 0.5 than for Ar = 5, which reflects the competition between the intensity of convection and the effects of side walls [26, 42, 43].

Fig. 6 shows the influence of molecular weight M<sub>B</sub> on interfacial distribution of crystal growth rates for a hori-

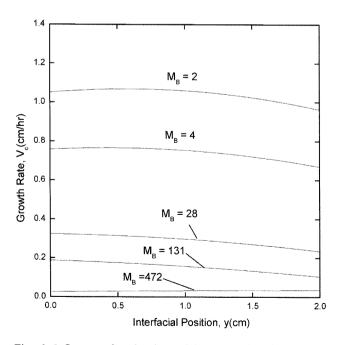


Fig. 6. Influence of molecular weight  $M_B$  on interfacial distribution of crystal growth rates for a horizontal ampoule of aspect ratio 5, with a linear wall temperature profile between  $T_s = 612.5~K$  and  $T_c = 576.0~K$ ,  $g_v = 1g_o$ ,  $4 \le M_B \le 472$ .

zontal ampoule of aspect ratio 5, with a linear wall temperature profile between  $T_s = 612.5 \text{ K}$  and  $T_c = 576.0 \text{ K}$ ,  $g_v = 1g_0$ ,  $4 \le M_B \le 472$ . As the molecular weight of component B, Xenon increases from 2 up to 472, i.e. closer to that of the growing mercurous chloride crystals, the effect of solutal convection decreases, which is indicative of the transition from convection-dominated mode to diffusion-dominated. Also, increasing of the inert gas with the molecular weight M<sub>B</sub> could enhance the intensity of thermal convection in comparison with solutal convection, i.e., thermal convection becomes much dominant in the convection mode. To be specific, as the  $M_B \rightarrow M_A$  (= 472 g/gmole), the rate for  $M_B = 2$  is reduced to the rate for  $M_A = 472$  by a factor of ten reduction in the rate, which implies the solutal convection-dominated regime is switched over the thermal regime. Note thermally buoyancy driven convection only is governed when the molecular weights of A and B are equal, in other words thermal convection overcomes solutal convection. The density dependence on the solute and temperature can be expressed by the following Boussinesq approximation:

$$\rho = \rho_0 (1 - \beta \Delta T + \gamma \Delta \varpi), \tag{14}$$

where  $\beta$  is a thermal volume expansion coefficient and  $\gamma$  is a pressure-dependence coefficient. It can be well

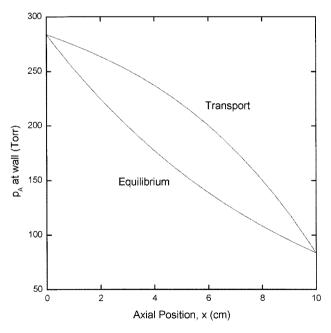


Fig. 7. Axial distribution of partial pressure of component A at the walls resulting from purely diffusive transport for Ar=5 and the linear wall temperature profile between  $T_s=612.5\,\mathrm{K}$  and  $T_c=576.0\,\mathrm{K}$ . The equilibrium vapor pressure profile corresponding to the wall temperature distribution is shown for the purpose of comparison. Based on Ar=5.0, Pr=0.8, Le=0.37,  $Gr_t=0$ ,  $Gr_s=0$ , Pe=2.27, Cv=1.1, Pe=0.8

understood from Eq. (14) the situations of thermal convection only caused by the interactions between the thermal density gradient and the gravity acceleration.

Fig. 7 shows the axial distribution of partial pressure of component A at the walls resulting from purely diffusive transport for Ar = 5, the linear wall temperature profile between  $T_s = 612.5 \text{ K}$  and  $T_c = 576.0 \text{ K}$ , and  $M_B =$ 131.3 g/gmole. Also, the equilibrium vapor pressure profile corresponding to the wall temperature distribution is shown for the purpose of comparison. The equilibrium vapor pressures of component A at the wall temperatures are calculated from Eq. (13). The partial pressures of component A at the walls are obtained from purely diffusive transport phenomena. Fig. 7 are based on Ar = 5.0, Pr = 0.8, Le = 0.37,  $Gr_t = 0$ ,  $Gr_s = 0$ , Pe = 2.27, Cv = 01.1, and  $g_v = 0$ . Fig. 8 shows the axial distribution of partial pressure of component A at the walls resulting from diffusive-convective transport at  $g_v = 1g_0$ , with the same system as for Fig. 7 except with  $Gr_t = 2.8 \times 10^4$ ,  $Gr_s = 3.05 \times 10^5$ ,  $g_v = 1g_0$ . As shown in Figs. 7 and 8, the pressures of Hg<sub>2</sub>Cl<sub>2</sub> (component A) vapors in the top and the bottom walls are above the corresponding equilibrium pressures so that the vapors of are in a supersaturated state throughout the vapor phase in the ampoule. Such a vapor supersaturation could cause undesirable parasitic nucleations at the walls and, thus in actual

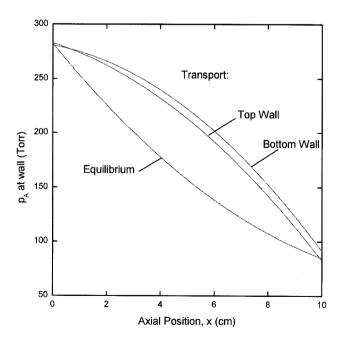


Fig. 8. Axial distribution of partial pressure of component A at the walls resulting from diffusive-convective transport at  $1g_y = 1g_0$ , and equilibrium vapor pressure for Ar = 5 and linear wall temperature profile between  $T_s = 612.5~K$  and  $T_c = 576.0~K$ . Based on Ar = 5.0, Pr = 0.8, Le = 0.37,  $Gr_t = 2.8 \times 10^4$ ,  $Gr_s = 3.05 \times 10^5$ , Pe = 2.27, Cv = 1.11,  $g_v = 1g_0$ .

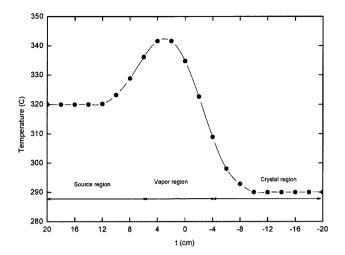


Fig. 9. The nonlinear temperature profile along the ampoule [11].

experiments a linear wall temperature condition is not employed frequently [6].

As shown in Fig. 9, such a hump region corresponding to the location of the vapor component A and B inside the ampoule is used in actual experiments to avoid parasitic nucleations at the walls resulting from the supersaturations along the transport path. The following temperature profile was used as a boundary condition along the ampoule (y = 0 and y = H): this equation [11] is expressed in reference to an approximate fit of experimental data, see Fig. 9.

$$T(t) = \begin{cases} 563.16 & \text{for } -20 \le t \le -10 \text{cm} \\ 608 + 4.97t - 0.70t^{2} \\ -5.91 \times 10^{-2}t^{3} + 6.67 \\ \times 10^{-3}t^{4} + 2.60 \times 10^{-4}t^{5} \\ -2.49 \times 10^{-5}t^{6} \\ 593.16 & \text{for } 12 \le t \le 20 \text{cm} \end{cases}$$
(15)

During the crystal growth the ampoule is placed in the nonlinear thermal profile as shown in Fig. 9. The hump region corresponds to the location of the vapor component A and B inside the ampoule. The source material lies in the region with the larger temperature near  $t \geq 8$  cm. Whereas crystal growth occurs in the region corresponding to  $t \leq -4$  cm. In the experiments one positions the ampoule in the growth region with a temperature less than the source in order to drive the process. In addition, the length of the hump region can also be adjusted so that we have a much larger source region. With respect to Fig. 9, the following transformation is used to relate the laboratory reference to the ampoule: where  $K_i$  is the position of the source and vapor inter-

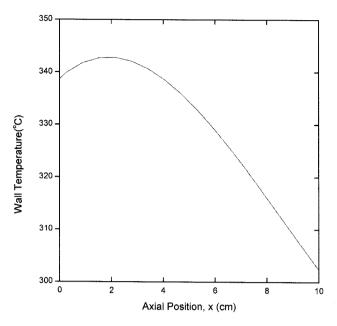


Fig. 10. A nonlinear axial wall temperature profile given by Eq. (5) with maximum ("hump") for Ar = 5,  $T_s = 612.5 \text{ K}$  and  $T_c = 576.0 \text{ K}$ .

face in the laboratory reference frame.

Fig. 10 shows an axially nonlinear wall temperature profile given by Eq. (15) with maximum ("hump") for Ar = 5,  $T_s = 612.5$  K and  $T_c = 576.0$  K. Because the aspect ratio of 5 with L = 10 cm and H = 2 cm, and the nonlinear wall temperature profile in Fig. 9 are set, the corresponding nonlinear wall temperature profile with T<sub>s</sub> = 612.5 K and  $T_c = 576.0 \text{ K}$  is obtained like Fig. 10. The source region is positioned at x = +5 cm, and the crystal is at x = -5 cm. Fig. 11 shows the axial distribution of partial pressure of component A at the walls resulting from diffusive-convective transport at  $g_v = 1g_0$ , and equilibrium vapor pressure for Ar = 5 and the nonlinear wall temperature profile of Fig. 9. For the transport path ranged from axial position x = 0 throughout near x = 5. the equilibrium vapor pressures are above the vapor partial pressures of component A, i.e. an undersaturation state, and since at x = 5, the supersaturation along the path occurs. Thus, for our system under consideration in this investigation, the problem of the supersaturation inevitably occur. Fig. 11 is based on Ar = 5,  $D_{AB} = 0.49$ cm<sup>2</sup>s<sup>-1</sup>,  $P_B = 10$  Torr,  $g_v = 1g_0$ ,  $P_T = 0.82$ , Le = 0.33,  $Gr_t =$  $2.65 \times 10^4$ ,  $Gr_s = 2.99 \times 10^5$ , Pe = 3.74,  $C_v = 1.02$ . Fig. 12 shows the axial distribution of partial pressure of component A at the walls in the same system as in Fig. 9, except for  $D_{AB} = 0.1 \text{ cm}^2 \text{s}^{-1}$ : based on  $D_{AB} = 0.1 \text{ cm}^2 \text{s}^{-1}$ , Le = 1.59,  $Gr_t = 2.85 \times 10^4$ ,  $Gr_s = 3.05 \times 10^5$ , Pe = 2.27,  $C_v = 1.11$ . To avoid parasitic nucleation near the crystal region, the temperature near the crystal region should be

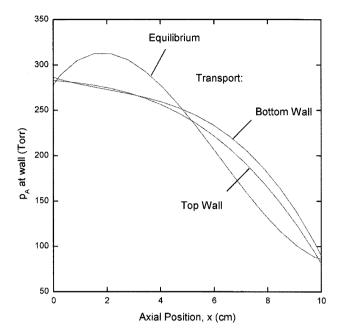


Fig. 11. Axial distribution of partial pressure of component A resulting from diffusive-convective transport and equilibrium vapor pressure at nonlinear wall temperatures. Based on Ar = 5,  $D_{AB} = 0.49 \text{ cm}^2 \text{s}^{-1}$ ,  $P_B = 10 \text{ Torr}$ ,  $g_v = 1g_0$ ,  $P_T = 0.82$ , Le = 0.33,  $Gr_t = 2.65 \times 10^4$ ,  $Gr_s = 2.99 \times 10^5$ ,  $P_B = 3.74$ ,  $C_v = 1.02$ ,  $T_s = 612.5$  K, and  $T_c = 576.0$  K.

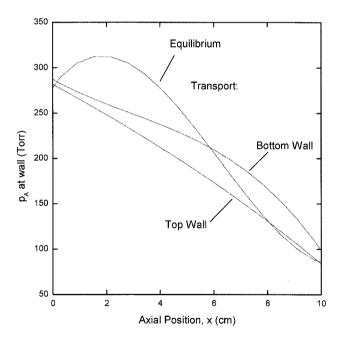


Fig. 12. Axial distribution of partial pressure of component A for the same system as in Fig. 11, except for  $D_{AB}=0.1~\text{cm}^2\text{s}^{-1}$ . Based on Ar = 5,  $D_{AB}=0.1~\text{cm}^2\text{s}^{-1}$ ,  $P_B=10~\text{Torr}$ ,  $g_y=1g_0$ ,  $P_B=0.8$ ,

enhanced as maxima as possible, which requires a temperature profile with a larger hump. If the binary diffusion coefficient,  $D_{AB}$  is reduced too much small, the smaller temperature profile is needed in the crystal

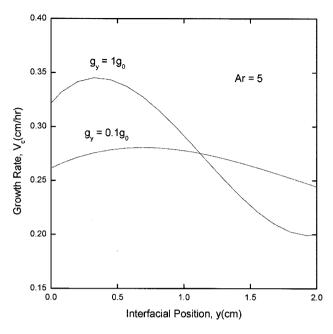


Fig. 13. Interfacial distribution of crystal growth rates for a horizontal ampoule of aspect ratio 5, a nonlinear wall temperature profile between  $T_s = 612.5 \text{ K}$  and  $T_c = 576.0 \text{ K}$ , and  $g_y = 1g_0$  and  $0.1g_0$ . For the case of  $g_y = 1g_0$ , the corresponding parameters and operating conditions are Ar = 5,  $D_{AB} = 0.49 \text{ cm}^2 \text{ s}^{-1}$ , Pr = 0.82, Le = 0.33,  $Gr_t = 2.65 \times 10^4$ ,  $Gr_s = 2.99 \times 10^5$ , Pe = 3.74,  $C_v = 1.02$ .

region [6]. A reduction of the diffusivity of  $0.49 \, \mathrm{cm}^2 \mathrm{s}^{-1}$  in Fig. 11 to  $0.1 \, \mathrm{cm}^2 \mathrm{s}^{-1}$  in Fig. 12 results in a slight shift of supersaturation range into near  $\mathrm{x}=6$ , and an decrease of at least 20 Torr in partial pressure of  $\mathrm{Hg_2Cl_2}$  at the top and bottom walls. It would provide a good insight into the microgravity experiments where the smaller humps would be appropriate to suppress the occurrence of supersaturation along the transport path.

Fig. 13 shows the effects of interfacial distribution of crystal growth rates for a horizontal ampoule of aspect ratio 5, a nonlinear wall temperature profile between  $T_s = 612.5 \text{ K}$  and  $T_c = 576.0 \text{ K}$ ,  $g_v = 1g_0$  and  $0.1g_0$ ,  $P_B = 10.0 \text{ K}$ 10 Torr. For the case of  $g_v = 1g_0$ , the corresponding parameters and operating conditions are as follows: Ar = 5,  $D_{AB} = 0.49 \text{ cm}^2 \text{s}^{-1}$ ,  $P_B = 10 \text{ Torr}$ , Pr = 0.82, Le = 0.33,  $Gr_t = 2.65 \times 10^4$ ,  $Gr_s = 2.99 \times 10^5$ , Pe = 3.74,  $C_v = 1.02$ . Fig. 13 are based on the same system as in Fig. 2 except for the nonlinear wall temperature profiles and  $P_B = 10$  Torr. In comparison of Fig. 13 with Fig. 2, the profiles of the growth rates for  $g_v = 1g_0$  and  $0.1g_0$  are the same profile each other, but the magnitudes of the growth rate shown in Fig. 13 are much larger than in Fig. 2 by a factor of 2. This discrepancy is likely to be due to not temperature profiles at walls but the effects of partial pressure of component B, i.e.  $P_B = 50$  Torr for Fig. 2 and 10 Torr for Fig. 13. It is reported that when

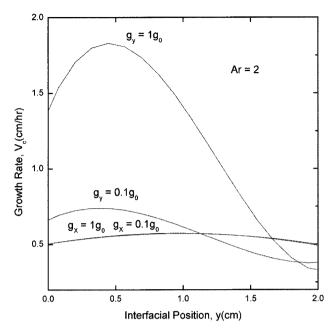


Fig. 14. Interfacial distribution of crystal growth rates for an ampoule of aspect ratio 2, a nonlinear wall temperature profile between  $T_s = 615.6 \, \text{K}$  and  $T_c = 576.0 \, \text{K}$ , and  $g_y = 1 g_0$  and  $0.1 g_0$ . Vertical orientation  $g_x = 1 g_0$ .

solutal convection is dominated, it is found that the imposed temperature profile has little effect on the growth rate compared with thermal convection [6]. These selections of two different partial pressures of component B are made to avoid oscillatory convection flow fields which result in the problem of convergence of our computer code. Fig. 14 shows the results for a system with an aspect ratio 2 (L = 4 cm),  $T_s = 612.5 \text{ K}$  and  $T_c =$ 576.0 K, the gravity accelerations of  $g_v = 1g_0$  and  $0.1g_0$ ,  $g_x = 1g_0$  and  $0.1g_0$ . In comparisons of Ar of 2 with 5, the maximum rate for Ar = 2 are much greater than for Ar = 5 by a factor of 5, but the corresponding nonuniformities in interfacial distributions are also much larger because of the effects of convection. Like the case of Fig. 5, the increase in gravitational level from  $g_v = .1g_0$ to 1g in the horizontal orientation position enhances the convection significantly. In comparisons of Fig. 14 with 5, it is found that the maximum growth rates are nearly invariant at  $g_v = 1g_0$  at the aspect ratio of 2, except for the partial pressures of component B and temperature profiles at walls. Also, the same trend is obtained for  $g_v = 0.1g_0$  and Ar = 2. For Ar = 5 and  $P_B = 10$  Torr, the maximum growth rate for the nonlinear wall temperatures, 0.34 cmhr<sup>-1</sup> in Fig. 13 is found to be greater than that for the linear wall temperatures, 0.21 cmhr<sup>-1</sup> (not shown in figures) by a factor of 1.6, which is likely to be due to the thermal boundary conditions imposed under consideration. Note for the cases of Ar = 5, increasing the partial pressure of component, B from 10 Torr to 50 Torr results in decreasing the corresponding maximum rate from  $0.21 \, \text{cmhr}^{-1}$  (not shown in figures) down to  $0.18 \, \text{cmhr}^{-1}$  shown in Fig. 2, which is reflected in solutal convection through the binary diffusion coefficient. For the system of  $g_x = 1g_0$  and  $0.1g_0$ , when the ampoule is positioned in the thermally buoyancy driven

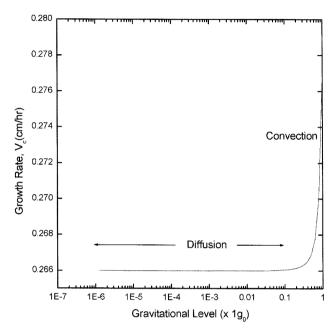


Fig. 15. The effect of the gravitational level on the growth rate of  $Hg_2Cl_2$ , for  $10^{-6}g_0 \le g \le l\,g_0$ .

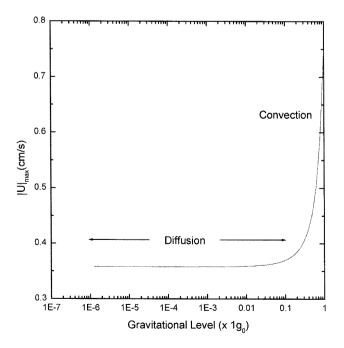


Fig. 16. The effect of the gravitational level on the maximum velocity magnitude,  $|U|_{max}$ , for  $10^{-6}g_0 \le g \le 1g_0$ , corresponding to Fig. 15.

convectively destabilizing orientation, the growth rates are nearly invariant with the gravity levels of  $1g_0$  and  $0.1g_0$  and with the conditions of two temperature profiles at walls and with aspect ratios of Ar = 2 and 5.

Figs. 15 and 16 show the effects of gravitational level on the growth rate of  $Hg_2Cl_2$ , and the maximum magnitude of velocity vector, for  $10^{-6}g_0 \le g \le 1g_0$ , Ar = 5, and nonlinear wall temperatures. As shown in Figs. 15 and 16, the transition from the recirculating convective flow to diffusion-dominated regime occurs near at  $g = 0.1g_0$  with tolerable limits. As can be seen in Figs. 3 and 4, the transition appears at  $g = 0.1g_0$  and linear wall temperatures.

#### 4. Conclusions

The results of this investigation reveal that, for thermal boundary conditions and reduced gravitational conditions of Hg<sub>2</sub>Cl<sub>2</sub>-Xe system, the gravity levels have a significant effect on the distribution of growth rate at the crystal interface and convective flow structures. A decrease in aspect ratio from 5 to 2 leads to an increasingly nonuniform interfacial distribution and enhances the growth rate by one-order magnitude for normal gravity and linear wall temperature conditions. The maximum rates under normal and one tenth-reduced gravitational conditions occur in the neighborhood of interfacial position, y = 0.5 cm. It is found that the interfacial growth rate profiles are asymmetrical at y = 1.0 cm. A factor of ten reduction in the gravitational level and an increase in aspect ratio suppress the convective effects on the growth rate. The effect of aspect ratio affects the interfacial growth rates significantly under normal gravity condition rather than under reduced gravitational environments. The transition from the convection-dominated regime to the diffusion-dominated regime ranges near at 0.1g<sub>0</sub> for both linear wall temperatures and specific nonlinear wall temperatures under consideration in this study. Increasing the molecular weight of component B, Xenon from 2 up to 472, i.e. closer to that of the growing mercurous chloride crystals, results in a reduction in the effect of solutal convection.

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# Nomenclature

- A component Hg<sub>2</sub>Cl<sub>2</sub>
- B component Xe
- D<sub>AB</sub> diffusivity of A and B
- eg unit vector of gravity acceleration
- g<sub>0</sub> standard terrestrial acceleration constant, 980.665 cm/s<sup>2</sup>
- H width (cm)
- L transport length (cm)
- p pressure
- P<sub>T</sub> total pressure
- T temperature
- $\Delta T$  temperature difference between source and crystal,  $T_s T_c$
- $\Delta\omega$  —mass fraction difference between source and crystal,  $\omega_{A,s}-\omega_{A,c}$
- x x-coordinate
- y y-coordinate
- u dimensionless x-component velocity
- U<sub>c</sub> characteristic velocity, k/L
- $|U|_{max}$  dimensional maximum magnitude of velocity vector (cm s<sup>-1</sup>)
- v dimensionless y-component velocity
- v<sub>x</sub> x-component velocity
- v<sub>v</sub> y-component velocity
- V velocity vector

### Dimensionless Governing Parameters

- Ar aspect ratio, L/H
- $C_v$  concentration parameter,  $C_v = (1 \omega_{Ac})/\Delta\omega$
- Le Lewis number, k/D<sub>AB</sub>
- Pe Peclet number, U<sub>adv</sub>L/D<sub>AB</sub>
- Pr Prandtl number,  $v/\kappa$
- Gr<sub>t</sub> thermal Grashof number,  $g\beta\Delta TH^3/v^2$
- Gr<sub>s</sub> solutal Grashof number,  $g\beta\Delta wH^3/v^2$

# Subscripts and Superscripts

- A component A, Hg<sub>2</sub>Cl<sub>2</sub>
- B component B, Xe

- c crystal
- H horizontal
- s source
- T total vapor pressure
- \* dimensionless

#### Greek Letters

- β thermal volume expansion
- κ thermal diffusivity
- v kinematic viscosity
- $\nabla^* \qquad (\partial/\partial x^*) + (\partial/\partial y^*)$
- $\nabla^{*2} \qquad (\partial^2/\partial x^{*2}) + (\partial^2/\partial y^{*2})$
- $\phi$  variable (u, v,  $T^*$ ,  $\omega_A^*$ )
- ψ dimensionless streamline
- mass fraction meaning dimensionless mass concentration density of fluid with component A and B

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