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# An Experienced Teacher's Representations of Beliefs and Knowledge in Mathematics Instruction

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The purpose of this study is to explore how a mathematics teacher's beliefs about mathematics and teaching and learning and mathematics and how such beliefs are related to her knowledge manifested in her mathematics instruction. The study illustrates images of teaching practice of an American mathematics teacher in middle grades mathematics classrooms. Results suggest that the teacher seems consistent in teaching in terms of her beliefs about mathematics and learning and teaching mathematics in some degrees. In particular, the teacher's beliefs affected the ways in which mathematics teacher organized and structured her lessons.

## I. Introduction

Teaching, in a generic sense, refers to "actions undertaken with the intention of bringing about learning in another. In this way, teaching is different from mere telling or showing how" (Robertson, 1985, p. 15). Teaching in mathematics classrooms is not composed only of lecturing or telling to present or pass on information (Raths, 1999). Rather, mathematics teaching is an integrated process that includes lecturing, discussing, questioning, responding by teachers and students (Gage, 1984, National Council of Teachers of Mathematics, 1991, 2000). Mathematics teachers' mathematical knowledge plays a critical role in their teaching as such integrated process. It affects both what mathematics they teach and how they teach it (Ball, 2000; Ball & Bass, 2000; Ball & McDiarmid, 1990; Buchmann, 1984; Leinhardt & Smith, 1985; Ma, 1999).

Teachers' beliefs also critically affect their teaching. What teachers believe about mathematics teaching and learning shapes the way that they teach, make decisions and change actions in their instruction (Pajares, 1992; Philipp, 2007; Thompson, 1984, 1992). Moreover,

> Beliefs have both affective and evaluative functions, acting as information filters and impacting how knowledge is used, organized, and retrieved. Beliefs are also powerful predictors of behavior, in some cases reinforcing actions that are consistent with beliefs and in other cases allowing for belief compartmentalization, allowing for inconsistent behaviors to occur in different contexts. (Gess-Newsome, 1999, p. 55)

Teachers have many untested presumptions that

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influence how they think about teaching and learning.

This study aims to investigate teachers' beliefs about mathematics, about teaching, and about learning mathematics and how those beliefs are related to teacher knowledge manifested in their mathematics instruction. In particular, I explore an American middle school mathematics teacher's beliefs and knowledge manifested in her mathematics classrooms in order to broaden perspectives about mathematics teaching. In doing so, this study would provide opportunities to appreciate how an American mathematics teacher's beliefs and knowledge look like and thus, to get insights for mathematics teaching for understanding.

## II. Teacher Knowledge and Beliefs

Teaching is a very complex process that is influenced by many kinds of teacher knowledge (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Carpenter & Franke, 1996; Even, 1993; Even & Tirosh, 1995; Fernandez, 1997; Geddis & Wood, 1997; Leinhardt, 1986; Leinhardt & Greeno, 1986; Ma, 1999; Wilson, Shulman, & Richert, 1987). Although teaching is different from knowing, teaching any subject matter depends on knowing that subject matter.

Research studies have examined the mathematical knowledge of teachers and how it might influence how they structure learning activities. Many studies imply that teachers with greater subject matter knowledge tend to emphasize the conceptual, problem-solving, and inquiry aspects of their subject. Less knowledgeable teachers tend to emphasize fact, rules, and procedures and to stick closely to detailed lesson plans or the text, sometimes missing opportunities to focus on important ideas or connections among ideas. (Borko & Putnam, 1996)

Likewise, Ball and McDiarmid (1990) affirmed that knowledge of subject matter is an essential part of teacher knowledge: "If teaching entails helping others learn, then understanding what is to be taught is a central requirement of teaching" (p. 437). They also point out that prospective teachers' lack of expertise and confidence in subject matter might be a serious issue in teacher education. It is, however, extremely important to recognize that "mathematical knowledge alone does not translate into better teaching" (Cooney, 1999, p. 166).

Teachers ought to know more than just the facts, terms, and concepts of a subject. Teachers' knowledge of the organizing ideas and knowledge growth within the subject is an important factor in how they will teach it (Borko & Putnam, 1996). Teachers of mathematics must have deep and highly structured content knowledge so that they can retrieve it flexibly, efficiently, and effectively for their students (Sternberg & Horvath, 1995).

The assumptions teachers have about their students and how their students learn are likely to direct how they approach teaching tasks and how they interact with their students (Calderhead, 1996). Beliefs about teaching may be closely related to beliefs about learning and the subject itself. For instance, "if a mathematics teacher believes mathematics to be about the application of techniques, this may itself imply certain beliefs about how the subject is most appropriately taught and learned and what the role of the teacher should be" (Calderhead, 1996, p. 719).

Research studies suggest that student teachers' conceptions about learning to teach affect how they approach their professional learning and the aspects of their preservice education programs (Calderhead & Robson, 1991; Mewborn, 2000). Teachers tend to keep many beliefs about the learning and teaching of mathematics that they had as prospective teachers (Borko, Eisenhart, Brown, Underhill, Jones, & Agard, 1992; Calderhead & Robson, 1991; NRC, 2001; Raymond, 1997; Thompson, 1992).

Teachers' beliefs have a profound influence on all aspects of their teaching practice (Hashweh, 1987; Nespor, 1987). In particular, teachers' beliefs about teaching and mathematics considerably influence their behavior because "the process of learning is fundamentally connected to how beliefs are structured, whether the beliefs are rooted in rationality or are the consequence of telling" (Cooney, 1999, p. 172). Mathematics teachers' beliefs and views about mathematics and its teaching, consciously or unconsciously, play a significant subtle role in shaping the teachers' characteristic patterns of instructional decisions (Cooney, 1985; Fennema & Franke, 1992; Ges-Newsom, 1999; NRC, 2001; Thompson, 1984, 1992). Therefore, teachers' professional development should include making their implicit belief systems explicit and thereby developing a language for talking and thinking about their own practice, questioning the sometimes contradictory beliefs underpinning their practice, and taking greater control over their own professional growth (Freeman, 1991).

This study attempts to explore and describe the beliefs of a middle school mathematics teacher about mathematics and learning and teaching mathematics. Further, the study explores how the mathematics teacher's beliefs are related to her knowledge manifested in the context of her teaching practice.

# III. Methods

I conducted a case study among qualitative research methods to investigate the research questions. Ms. Carey (pseudonym), an eighth-grade experience mathematics teacher in the U. S. agreed to participate in this study; the teacher was involved in a professional development program that was initiated by a research project at the time and was recommended for this study. Ms. Carey was the only eighth-grade mathematics teacher in the school system of a small city. She had about 18 years of teaching experience in elementary and middle grades; however, it was her first year of teaching mathematics to eighth graders. She had started her teaching career as an elementary teacher and gradually moved up to middle-grades teaching.

Ms. Carey taught Pre-Algebra to all of the eighth graders, who were heterogeneously grouped. I observed all of the groups. The school where Ms. Carey taught eighth-grade mathematics was a Learning-Focused School, which encourages teachers to organize lessons in a certain way, such as starting with "essential questions."

The Learning-Focused Schools Model, developed by Learning Concepts and Learning-Focused Solutions professionals (Thompson & Thompson, 2004), is a school improvement model that provides exemplary practice strategies for learning and instruction within the framework of learning. This learning framework connects exemplary practice teaching strategies to teacher planning and instruction. Teachers plan and teach differently in schools when the focus is on learning. Learning Concepts and Learning-Focused Solutions provides professional development and resources to school districts and schools.

To build a detailed case, I collected multiple forms of data. The major sources of data were semi-structured interviews, observations of mathematics classes, and classroom artifacts. I conducted seven pre- and post-observation interviews with Ms. Carey; each interview lasted 30-60 minutes. I observed 25 class periods of Ms. Carey's mathematics classroom during more than 4 weeks. I took descriptive field notes while observing the lessons. Also, the lessons that I observed were all audiotaped and were later transcribed into a form of expanded field notes. The topics that she taught were solving equations with an unknown variable, probability, geometry, reviews of fractions and decimals, and algebraic expressions.

The data were analyzed using a case study strategy (Yin, 1994) and the constant comparison method of data analysis (Glaser & Strauss, 1967). The process of data analysis began with finding key words from the interview transcripts and categorizing those words into themes emerging from the interviews.

# IV. Results

This section describes the eighth-grade mathematics teacher's beliefs and knowledge manifested in her mathematics instruction. I present her beliefs about mathematics and about learning and teaching mathematics as they emerged from the data. Then I describe in detail how Ms. Carey's knowledge was manifested in her mathematics teaching.

## 1. Mathematics

Ms. Carey viewed mathematics as "just so much a part of life," saying that we can survive without knowing science or social studies, but not without reading and mathematics. "Mathematics is working with numbers and is such an active part of our lives." To Ms. Carey, "mathematics is just out there," and being able to read and work with numbers was indispensable to surviving in the world. Mathematics also involved problem solving. Ms. Carey viewed problem solving as involving an "ability to take a problem, set it up, and follow the legal steps or whatever and come up with a correct answer." The more she got students to think, the more she helped them develop their problem-solving ability. "If we do all the thinking for them, then it is not teaching them to think for themselves." To Ms. Carey, a problem was one that people dealt with in daily life. She believed that problem solving forces the students to think and develop their creative powers, which, in turn, helps their problem solving in mathematics learning. She aimed to get them to learn how to master their multiplication, division, addition, and subtraction facts so that they could balance their checkbooks, handle money, and follow a recipe and cook. Ms. Carey's ultimate goal was for the students to develop problem-solving skills through working in groups, which could only help them in life; "life is full of problems," and there is one problem-solving episode after another.

In her instruction, Ms. Carey defined an algebraic equation as "a math problem, a mathematical sentence that contains an equal sign, but it also contains variables, numbers, and at least one operation." She understood that "formulas are really equations." When teaching algebraic expressions, she tied them in with formulas for perimeter and area. Ms. Carey explained, "Those formulas are just algebraic expressions or algebraic equations." Further, she said, "If the values of the variables were provided, then you [could] substitute them and find the answers" in the lesson. As a result, she seemed to believe that she "took away" the students' fear of the formula because they just understood it to be just an algebraic equation, and they knew how to deal with that.

Ms. Carey explained that in her instruction, "algebra is just solving for the unknown." Algebra is basic mathematics that is spread throughout the grades from K to 12. Even in kindergarten and first grade, children are "given a box or a question mark" and asked to solve "five plus question mark equals eight."

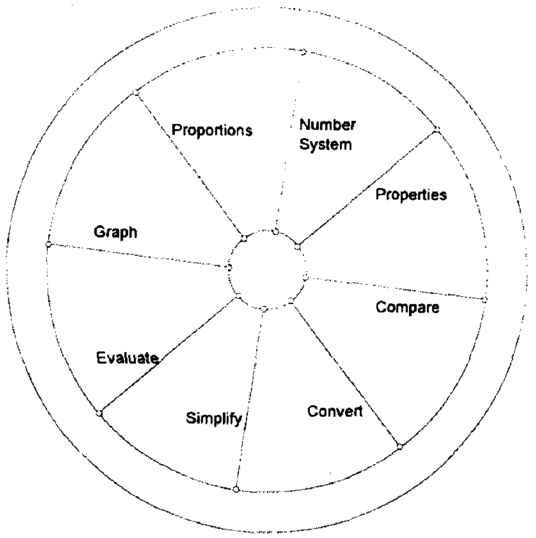
> Then it became five plus a box equals eight. They were taught to find the missing addend. Algebra just replaces that box or that question mark with a letter, and then it becomes five plus n equals eight. So, to me, algebra is no different than what they were really being taught in first and second grade, but it is just using variables. And for many students, all of a sudden, they think it is something they can't do. And I actually have taken it all the way back to kindergarten and first grade, and I showed them, "Do you remember where it was five plus a box equals eight?" "Yeah." And I took them step by step and showed them, "Well, all they've done is replace that box with a variable, and you are still doing the exact [same]thing." And I said, "What are you going to do find the value of n?" And they said, "I'm going to subtract eight minus five." And I said, "That's exactly right. That's all algebra is."

She would also use tables in which the students could begin to see patterns as a way to help them develop algebraic concepts. Those tables yielded xs and ys, and students could plot them on a coordinate plane, which led to ideas of slope and linear equations.

Regarding rational numbers, Ms. Carey understood that the concepts of number system, properties, compare, convert, simplify, evaluate, graph, and proportions are all related to rational numbers. That insight guided her to create "a wheel of rational numbers" (Figure IV-1). She used the wheel to help the students see "where the proportions came in, the graphing, evaluating the equations and inequalities, simplifying fractions, and converting fractions to decimals to percents."

After teaching rational numbers according the wheel of rational numbers, Ms. Carey moved on to probability. She said, "Probability is a ratio." "It is expressed as a ratio, and a ratio is basically a fraction." She thought that it would be a smooth transition into teaching probability. "Just like decimals and percents are equivalent expressions of fractions," she was inclined to teach "the connectivity" of fractions and probability. To help students understand the concept of probability, she sought to show that it was to "something they use more often than they think they do."

Ms. Carey sought to associate integers with realistic situations. She gave an example:




[Figure IV-1] Ms. Carey's Wheel of Rational Numbers

> We talked about negatives, and we talked about the three basic places where we actually saw negatives were. In football, the boys could definitely understand loss of yardage—they understood that to be a negative. And then we talked about thermometers—temperature—they could easily understand that one. [And] being in debt.

She tried to connect the students' real lives to the concept of negative integers so that the students could appreciate that "it is not an isolated concept but real" and that "they are using it" in their lives.

## 2. Learning Mathematics

Learning mathematics, according to Ms. Carey, is "a culmination of so much." She always attempted to tell her students that "learning is in a form of a spiral." In other words, what the students had learned in kindergarten was built upon in first grade, second grade, third grade, and so on. Thus, if students do not learn and understand basic concepts of mathematics in certain grades, then they come to have "learning gaps." In addition, Ms. Carey thought that students should be not passive, but actively participating and discussing while learning. Learning occurs more when the students get actively involved than when they passively sit and "soak up [learning] like a sponge." The more the students dynamically engaged in learning activities in class, the greater the chances of their learning and remembering would be.

Ms. Carey believed that students are different in the way they learn; in any class there are "concrete learners" and "abstract learners." Teaching mathematics may well involve the abstract, and some students might not be able to grasp it yet. To motivate them, she used various manipulatives, visuals, and hands-on activities. Some of the students were prepared to move on; so they could "picture the abstract."

Considering the many different learning styles of her students, Ms. Carey sought to have a variety of ways for her students to actively participate in her classroom:

> Basically, they can use their learning style. Because you've got some who are talkers; they learn by hearing, so maybe they are my talkers in the group. And then you have those who are writing it down; they are your kinesthetic, or they could also be your visual learners, who like to see it in writing. I don't limit.

Ms. Carey did not force those students who were very quiet and diligent in their note taking to make responses. She recognized such students and respected their style of learning. At the same time, she did not mind calling on them to see whether they understood. Also, when putting the students in groups, she did not force students who do not want to play a part to work in a group. Instead, she let them work alone provided that they made an effort.

Similarly, Ms. Carey realized that there existed motivational differences among the students; students who like mathematics, are good at mathematics, and like to work with numbers will learn anything that teachers put out in front of them. Others, who accomplish "on a minimum scale only just to get by," have little or no desire to learn mathematics. Ms. Carey explained that if a student likes mathematics, then he or she is willing to learn mathematics that teachers present. Moreover, in some cases, it is like a challenge for them. Those who have developed a good number sense will take a given problem as a challenge and attempt to solve it; they enjoy being challenged. They are more interested in word problems.

Ms. Carey recognized that some of her students were not capable of working on equations with fractions and decimals. Many of them apparently had not worked enough with fractions and decimals, which caused them to feel uncomfortable with those concepts. Moreover, she was aware that they did not understand that fractions are part of a whole. She assumed that enough time was not spent in helping them develop the concept of fractions in the previous grades. Ms. Carey noticed the common mistakes students made when they worked with mixed numbers. In particular, the students tended to leave mixed numbers as they were and then try to subtract a fraction from the fractional part even when they needed to regroup. They appeared not understand why they needed to take one of those whole number units and by regrouping rewrite it as a fraction, which, she recognized, was confusing to them. In contrast, there was not much confusion with the addition of mixed numbers. The students were aware that if they got an improper fraction, then they could easily rewrite it as a mixed number, adding the whole number parts together and the fractional parts, respectively. However, she thought that approach was likely to confuse them. Instead, she used the approach of turning all mixed numbers into improper fractions to assist the students who could not grasp the concept. For instance, she had her students convert $4\frac{1}{3}-2\frac{4}{7}$ into the form $\frac{13}{3}-\frac{18}{7}$ rather than to do subtraction of whole numbers first, 2 from 4, and then solve $\frac{1}{3}-\frac{4}{7}$. The result of subtraction of fractions is negative, but the students tended to ignore the negative sign. Indeed, they just combined 2 and $-\frac{5}{21}$, and thus, got $2\frac{5}{21}$, instead of $1\frac{16}{21}$.

## 3. Teaching Mathematics

Ms. Carey considered teaching mathematics to be similar to being a coach in that both involve teaching skills. Once students learn skills, they can put them all together, see what they have created, and apply them to new situations. Just as coaches teach by breaking skills into steps for skill building, in teaching mathematics, teachers "present the whole problem, and then you break it down." Having served as a softball coach at the school, Ms. Carey recognized that breaking skills down was very important. Further, she viewed teaching mathematics as a challenge as well and said that teachers should be entertainers:

> Teaching school is a challenge because of all the different personalities [they] are seeing. Some [students] who are aggressive in their learning, they desire to learn, and you see that. The hands go up. They are going to ask, and they are going to keep asking until they understand. Those are the kind of students [teachers] delight in. And then [there are] those who are sitting back, bored, could care less, misbehaving—how do you reach them? [Teachers] can try to catch them with an interesting question. [A teacher] can only be an entertainer so much.

For Ms. Carey, teaching mathematics was letting students see what mathematics is like. Thus, she tried to think of interesting ways to involve the students and make them want to participate actively.

Ms. Carey saw her role as a compound of a "facilitator," "teacher or instructor," and "guidance counselor," depending on the teaching situation. When introducing a concept, she tried to present it in the form of a problem and let the students try to figure it out through some strategy of their own. At that point, she played the role of facilitator. When she let the students engage in an activity, for instance, she wanted them to actively think, talk, and create. Her "job was to be there to assist," "advise," and "give" them some direction when they got confused. Sometimes, as a teacher, she might know a method that would work better; their method might have worked only for that particular problem. Then she would regard it as her role as to present methods, possibly more than one method, for finding the solution. In doing so, she emphasized not only that there is more than one way to arrive at the same answer, but also that "it is not that somebody else is wrong and somebody else is right." Also, she regarded herself as a guide in that she led the students in their explorations.

Importantly, Ms. Carey believed that it was important to know the "why" behind skills, procedures, formulas, and algorithms. If students have been taught only a formula, they can easily forget it and not know what to do. She emphasized the importance of knowing the why by asking students to prove their answers. Furthermore, she believed that if the students were able to explain "why," then they had an understanding. The reason that she encouraged proving was to "reinforce" how the students arrived at their answers.

> All of that is ⋯ to reinforce. And as they teach it back, any time you are the teacher, you learn it better. So by saying "prove it," that puts them in the teacher position. And they go back and say how they arrived at it. So it now only helps them, it helps those who maybe didn't get the answer. And they can see how they should have arrived at it.

Ms. Carey said that she enjoyed teaching the mathematics in which she felt confident. The more she taught, the stronger she felt about her teaching. With experience, she came to learn new strategies. When she taught a concept repeatedly, she might realize that it had not gone over too well and would make an effort to come up with a different method or another style, which would result in her feeling stronger and better about her teaching. Also, her active involvement in teaching on a regular basis contributed to her confidence in mathematics. She expanded that view as follows:

> If you don't use it, you lose it. Well, I'm not going to lose it, because I'm using it. And in talking to my other team members, they don't feel confident in math. And I think the reason is, is they don't teach it. They don't use it on a daily basis, and so they are not working with those numbers. And so they begin to think they can't do math. The more you use it, the better you get at it.

She did not, however, consider herself an expert in mathematics. She said she would never teach advanced mathematics courses such as calculus or trigonometry, because she "would not feel comfortable" teaching those courses. She rationalized, "I don't use that enough in daily life. I would have to teach myself and re-teach myself that stuff. I could do it. I mean, I took trig in college and made an A, but I had to study." Therefore, she would not want to teach high school mathematics. Further, she did not think that her expertise lay in teaching high school mathematics; she judged that her expertise lay in working with middle-grades students.

In her mathematics classrooms, Ms. Carey had various ways of representing mathematical ideas and concepts during the lessons. The representations varied in explanations, examples, counter examples, and demonstrations conveyed by symbols, words, and pictorial forms. The representations were combined with one another most of the time. In addition, Ms. Carey presented more than one example for a concept. For a sample space of probability, Ms. Carey asked the students to list all the possibilities when rolling a die, choosing a letter from the alphabet, selecting a student from among those in the fourth-period class, and flipping a coin. While teaching the Pythagorean theorem, she showed several ways in which it can be applied to find the third side of a triangle.

During her lessons, Ms. Carey employed activities and games through which the students might come to understand a concept. She said that she tried to acquire "a packet" of activities that she could use for teaching a particular objective. In particular, she loved activities and games because those made learning fun, which influenced the students to enjoy learning mathematics. If the lesson was boring, the students would not be motivated or pay attention, which is why she tried to create activities and worksheets, to make it interesting and to help and reach every child. However, she admitted that there was "a time for paper and pencil" as well during her lesson; there were needs to balance. By doing games and activities, the students would accomplish the goal, which was to understand a concept and be able to apply it to different situations. Then she would go to paper and pencil. Further, she affirmed that she was unusual in that she was "not like some teachers who just open a book and assign pages"; she also attempted to design worksheets for both in-class tasks and homework.

For the lesson on probability, Ms. Carey prepared eight tasks in plastic bags for the students to work on in groups. She chose a couple of activities from the textbook and invented the rest of them. Each package contained an index card on which a problem was written and some material to use in solving a problem. For example, the Deck of Cards instructions said, "Use the given deck of cards – 36 cards, 18 red and 18 black, 6 face cards of each color." The Coins instructions said, "Use the given coins - 24 coins of 2 quarters, 5 dimes, 10 nickels, and 7 pennies." An example of each task was shown on each index card; for instance, P (choosing a quarter) = $\frac{2}{24}$. Each group had to create 10 probability questions that should include all the probability of "an impossible," "ascertain," and "a 50-50chance," that is, 0, 1, and $\frac{1}{2}$, and find answers to those 10questions. Later, after this group activity was done, each group had a presentation for the following two class periods that was called "student teaching or presentation." In the student presentation, a group of two or three students stood in front of the whole class and presented the questions about probability that they created for their activity in the previous class period. While one of the students read their questions, the other students wrote both a sample space for the activity and their probability questions. After reading all of the questions, the presenters played a role of teacher by going over the questions and interacting with their classmates to get an answer. The student teachers encouraged their classmates to come up with answers.

While preparing those materials, Ms. Carey tried to get enough probability items so that she could have small groups to foster the students' active involvement in the activity. If the students were to work in larger groups, then one or two of them might never do anything. If they were in a group of two or three, however, then they would be forced to participate. In making groups, she purposefully assigned the students to insure that she had a high-level child mixed with a low-level child in each group. She pursued such combinations because she wanted to keep the low-level students from only trying to come up with an answer. Varied abilities in each group allowed students to benefit from one another. For that purpose, Ms. Carey designed and made use of a seating chart. She also tried to bring about discussion in each group. She believed that the discussion would help those students who had trouble analyzing to develop that skill by listening to their peers analyzing.

Ms. Carey appeared to try to associate mathematics with realistic applications, which reflected her beliefs about mathematics and mathematics learning and teaching. She thought learning how to work with fractions, decimals, percents, negative integers, positive integers, and solving word problems was "real." She gave examples of how certain mathematical concepts could be applied to the world; for instance, weather forecasting and sports for probability, carpeting the kitchen floor for finding areas and perimeters, and shapes in real life for geometric figures and terms. Such efforts also resulted in frequent work on word problems. When teaching the Pythagorean theorem, she wrote a word problem on the board before the class began and kept it up during the lesson: "A baseball diamond is actually square. The distance between bases is 90 feet. When a runner on first base tries to steal second, a catcher has to throw from home to second base. How far must the catcher throw pick off the runner?" Along with the problem, she included a picture of a baseball diamond. As an application of the Pythagorean theorem, she let the students go to the field outside the classroom with protractors and measuring tapes to figure out how the Pythagorean theorem works in the real world.

The use of a mathematics journal was significant for the students' learning in Ms. Carey's class. Every student had to have his or her own mathematics journal as a resource and future reference. Her motto was "If I put it on the board, you put it in your book." The mathematics journal was a manual that the students could use. The students took their mathematics journals out to take notes as needed in every lesson. What is more, the students were encouraged to consult their mathematics journals when taking tests and exams, which was another purpose Ms. Carey used to encourage the students to keep their journals.

## V. Conclusions and Implications

This study illustrates images of teaching practice of an American middle-school mathematics teacher in middle grades mathematics classrooms; it shows that the teacher's beliefs and knowledge are related and intertwined in mathematics instruction . The teacher seems consistent in teaching in terms of her beliefs about mathematics and learning and teaching mathematics in some degrees. The experienced mathematics teacher made various manipulatives for hands-on activities for her students, which seems to come from her beliefs about mathematic teaching and learning. She emphasized of the importance of usability and applications of mathematical ideas in everyday life situations. This, also, leads her to highlight the problem solving skills in her lessons. Such beliefs was evident as her employment of activities and games to foster students' understanding of a concept in most of her lessons.

The results suggest that the experienced teacher's beliefs about how students learn affected the ways in which she structured her lessons in terms of students' learning styles, common misconceptions, difficulties and mistakes (Philipp, Ambrose, Lamb, Sowder, Schappelle, Sowder, Thanheiser, & Chauvot, 2007; Vacc & Bright, 1999). She seriously considered of the importance of different learning styles of students and it was obvious that she planned and organized her lessons accordingly. In addition, the teacher presented various representations for the same concept or topic, which provided students with more opportunities to revisit and revise their understanding it. This implies her beliefs about the spiral learning are emerged in her lessons. Her views and teaching, however, kept her from teaching more advanced knowledge than basic understanding of ideas and concepts.

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# 수학 수업에 표현된 수학 교사의 신념과 지식

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이 논문은 수학 교사의 신념과 지식이 수학 수업에 어떻게 표현되는 지 미국 중학교 수학 교사의 수업 분석을 통한 사례연구 보고서이다. 미국 중학교 수학 교실에서 이루어지는 수업에 대한 정보를 제공할 뿐만 아니라, 수학 교사의 신념과 지식이 수업에 어떻게 반영되고 적용되는지 상세하게 분석 설명한다. 사례 분석 연구 결과는 참여 수학 교사의 수학 교수 학습에 대한 신념체계가 수업에 일정 정도 일관성 있게 표현됨을 보여 준다. 특히, 그 교사의 신념체계가 수학 수업을 조직하고 구성하는 데 큰 영향을 미치는 것으로 나타났다.

* key words : Teacher beliefs(교사 신념), Teacher knowledge(교사 지식), Mathematics teaching(수학 교수), Middle school teacher(중학교 교사)

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