An Analytical Model of the First Eigen Energy Level for MOSFETs Having Ultrathin Gate Oxides

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Abstract— **In this paper, we present an analytical model for the first eigen energy level (E_0) of the carriers in the inversion layer in present generation MOSFETs, having ultrathin gate oxides and high substrate doping concentrations. Commonly used approaches to evaluate E_0 make either or both of the following two assumptions: one is that the barrier height at the oxide-semiconductor interface is infinite (with the consequence that the wave function at this interface is forced to zero), while the other is the triangular potential well approximation within the semiconductor (resulting in a constant electric field throughout the semiconductor, equal to the surface electric field). Obviously, both these assumptions are wrong, however, in order to correctly account for these two effects, one needs to solve Schrödinger and Poisson equations simultaneously, with the approach turning numerical and computationally intensive. In this work, we have derived a closed-form analytical expression for E_0, with due considerations for both the assumptions mentioned above. In order to account for the finite barrier height at the oxide-semiconductor interface, we have used the asymptotic approximations of the Airy function integrals to find the wave functions at the oxide and the semiconductor. Then, by applying the boundary condition at the oxide-semiconductor interface, we developed the model for E_0. With regard to the second assumption, we proposed the inclusion of a fitting parameter in the well-known effective electric field model. The results matched very well with those obtained from Li's model. Another unique contribution of this work is to explicitly account for the finite oxide-semiconductor barrier height, which none of the reported works considered.**

Index Terms— **MOSFET, eigen energy level, ultrathin gate oxide**

I. INTRODUCTION

With aggressive scaling of dimensions of MOSFETs over the last decade or so, in order to keep the various short-channel effects under check, the substrate doping is continually increased, while the oxide thickness keeps on reducing. The net effect is a tremendous increase in the vertical electric field at the oxide-semiconductor interface (henceforth, referred to simply as the interface), which initiates quantum mechanical effects [1-3], causing carrier confinement within a narrow potential well close to the interface, and, consequently, giving rise to the phenomenon of energy quantization. The quantized energy levels can be obtained accurately by solving the Schrödinger and Poisson equations self-consistently [4-6], however, it cannot be done analytically, with the result that the solution procedure becomes numerical and computationally extremely intensive and expensive.

In [4], Janik and Majkusiak solved the set of equations self-consistently and showed that neglecting the phenomenon of the energy quantization leads to an underestimation of the threshold voltage for MOSFETs with highly doped substrates. Stern [5] came up with an approach of finding the eigen energy levels analytically by making two important assumptions: one is that of infinite potential barrier height at the oxide-semiconductor interface, while the other is the familiar triangular potential well approximation. However, rigorously, none of these two assumptions is valid.

Mudanai et al. [6] presented a comprehensive analysis of the effects of the wave function penetration into the

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oxide, by solving Schrödinger and Poisson equations self-consistently. Now, the triangular potential well approximation is invoked by the researchers, since the Schrödinger equation has an analytical solution under this condition. In [7], using this approximation, Mueller and Schulz proposed a simplified method, which accounts for the energy quantization of the inversion layer charge carriers through a self-consistent set of parameterized solutions of the Poisson and Schrödinger equations.

Ma et al. [8] showed that the results for the carrier sheet density and the surface potential using the triangular potential well approximation match with those obtained from the numerical simulations very well provided that the effective field is properly chosen. Liu et al. [9] proposed an approximate model to describe the effect of the inversion layer quantization for deep submicron MOSFETs, indicating that this effect should be considered when the device is scaled down to the sub-0.1 μm regime.

For MOS structures, to the best of our knowledge, there is no existing report in the literature regarding any explicit analytical expression of the first Eigen energy level, without invoking either of the two assumptions stated earlier in this section. In 2005, Li et al. [10] proposed a semi-empirical expression for the Eigen energy level with the coefficient and exponent terms (present in Stern's [5] eigen energy model) as fitting parameters. The values of these parameters were adjusted by fitting the results obtained from their semi-empirical model with the numerical results. Though this model is semi-empirical, however, the results obtained from the model match very well with the numerical results [10]. Therefore, in this work, we used the semi-empirical model proposed by Li et al. [10] as the reference, with which we compared the results obtained from our proposed model.

In another empirical model, Shams et al. [11] had made the exponent (present in the model of Li et al. [10]) an empirical function of the oxide-semiconductor barrier height and the substrate doping. However, the drawback of this model is that it uses too many fitting parameters, and most of them do not have any physical justification. From the foregoing discussion, it should be apparent that there exists a need for an analytical Eigen energy model, which does not use either of the two assumptions listed earlier, and, at the same time, is sufficiently accurate. Also, an accurate evaluation of the Eigen energy level is a must for an exact estimate of the gate tunneling current [12-16], which is so very important for today's MOSFETs.

The work has been presented in the following sequence. Section II presents the details of the development of the first Eigen energy model using the asymptotic approximations of the Airy function integrals, taking into account the finite barrier height at the oxide-semiconductor interface. The effective electric field present within the semiconductor is a required parameter for this model, for which we considered the widely used effective electric field model, however, with the inclusion of a fitting parameter, as presented in Section III. The results obtained from our model, which showed an excellent match with those predicted by the semi-empirical model of Li et al. [10], are presented in Section IV. Also, the proposed model for the first Eigen energy level is completely analytical, and has only one fitting parameter. The summary and conclusion of the work is presented in Section V.

II. Eigen Energy Model for Finite Oxide-Semiconductor Barrier Height

1. Background

With shrinking gate oxide thickness and increasing substrate doping concentration, penetration of the wave function into the oxide increases [6]. Accurate evaluation of the quantized eigen energy levels in the semiconductor region depends on the amount of this wave function penetration into the oxide region. Therefore, in this section, the eigen energy level is modeled without forcing the wave function to vanish at the oxide-semiconductor interface. The exact analysis results in the solution in terms of the Airy function integrals. Because the exact analytical expression for the Airy function integral cannot be formed, hence, we have invoked the approximation of the asymptotic Airy function integrals. The validity of this approximation is established, and the wave functions in the oxide and the semiconductor regions are formulated after applying proper boundary conditions at the interface. This also leads to the expression of the eigen energy level in terms of the surface electric field, as

will be shown shortly.

The solution of the Schrödinger wave equation, assuming infinite barrier height at the interface and under the triangular potential well approximation, takes the form of the Airy function, expressed as [5]:

$$\psi(x) = Ai\left(\left(\frac{2m_s qF_s}{\hbar^2}\right)^{1/3}\left[x - \left(\frac{E_i}{qF_s}\right)\right]\right) \quad (1)$$

where $\psi(x)$ is the wave function, with the position coordinate x defined to be positive into the substrate, with the origin at the interface, m_s is the effective mass of electrons in the semiconductor, q is the coulomb charge, F_s is the surface electric field, and E_i is the eigen energy level, expressed as [5]

$$E_i = \left(\frac{\hbar^2}{2m_s}\right)^{1/3}\left[\frac{3}{2}\pi qF_s\left(i + \frac{3}{4}\right)\right]^{2/3} \quad (2)$$

where substitution of i = 0 gives the ground state energy level E_0, which is of paramount importance in our work.

In Eq. (1), the Airy function integral Ai(p), with p being any general variable, is given by [2]:

$$Ai(p) = \frac{A}{2\pi}\int_{-\infty}^{+\infty} \exp\left[j\left(pu + \frac{u^3}{3}\right)\right]du \quad (3)$$

where A is the normalization factor, and u is an integration variable. Equation (3) is not solvable analytically, and the solution requires numerical integration. However, there are asymptotic approximations for the Airy function integral, which can be expressed as [3]:

$$Ai(p) \approx \frac{c_1 \cos\left(\frac{2}{3}|p|^{3/2} - \frac{\pi}{4}\right)}{\sqrt{\pi}\,|p|^{1/4}} \qquad (p < 0) \quad (4)$$

and

$$Ai(p) \approx \frac{c_2 \exp\left(-\frac{2}{3}p^{3/2}\right)}{2\sqrt{\pi}p^{1/4}} \qquad (p > 0) \quad (5)$$

where c_1 and c_2 are normalization constants. Assuming all the constants (c_1, c_2, and A) to be equal to unity, a qualitative plot of the results using these asymptotic approximations along with the one obtained through numerical integration of Eq. (3) is shown in Fig.1. It is to be noted that the results obtained from the asymptotic approximations diverge for p = 0, and hence, are not good estimates of the actual Airy function in this region. However, barring this small region around p = 0, the results of the asymptotic approximations match very well with the numerical results. Hence, the next task is to find the validity of these approximations for our work, which we describe in the next section.

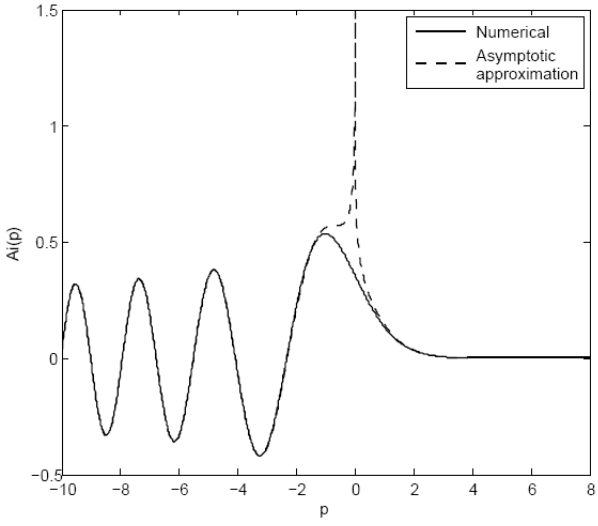

Fig. 1. Comparison of the results obtained from numerical simulation of the exact Airy function [Eq. (3)] (shown by the solid line) with those obtained using the asymptotic approximations [Eqs. (4) and (5)] (shown by the dashed line). It can be observed that except near a small region around p = 0, the results of the asymptotic approximations match the numerical result accurately.

2. Validity of the Approximations

To simplify the problem at hand, we denote all the variables pertaining to the semiconductor region by suffix 1, and those for the oxide by suffix 2. In order to find the wave function in the semiconductor region, it is required to solve the one-dimensional Schrödinger equation, with the potential function $\phi(x) = xF_s$ (assuming triangular potential). The resulting wave function is identical to that given by Eq. (1), which now gets expressed as

$$\psi_1(x) = Ai(p_1) \quad (6)$$

with

$$p_1 = \left(\frac{2m_s qF_s}{\hbar^2}\right)^{1/3}\left[x - \left(\frac{E_i}{qF_s}\right)\right] \quad (7)$$

with x = 0 corresponding to the interface. From Fig.1, it can be observed that for negative values of p_1, Eq. (4) is a very good estimate of Eq. (3) for p_1 less than −1.35 (approximately). Now, if we substitute x = 0 and use the ground state eigen energy E_0 predicted by Stern [Eq. (2) with i = 0] in Eq. (7), the result turns out to be $-(9\pi/8)^{2/3}$, which equals −2.32, and is sufficiently less than −1.35. Thus, the accuracy of the Airy function asymptote for $p_1 < 0$ [Eq. (4)] is established. We later checked that this accuracy is maintained even when we used the result for E_0 obtained from our model.

Now, for the oxide region, the one-dimensional Schrödinger equation can be solved with the potential now given by $\phi(x) = \phi_b + xF_{ox}$, where ϕ_b is the oxide-semiconductor barrier height (in V), having a value roughly equal to 3.2 V for the Si-SiO_2 system, and F_{ox} is the oxide electric field. Note that in the oxide region, x is negative, and, at the interface, i.e., for x = 0, $\phi(x) = \phi_b$, as expected. The solution again takes the form of Airy function, and can be expressed as

$$\psi_2(x) = Ai(p_2) \quad (8)$$

with

$$p_2 = \left(\frac{2m_{ox} qF_{ox}}{\hbar^2}\right)^{1/3}\left[x - \left(\frac{E_i - q\phi_b}{qF_{ox}}\right)\right] \quad (9)$$

where m_{ox} is the effective mass for electrons in the oxide. From Eq. (9), note that for x = 0 (i.e., at the interface), p_2 is positive, since $q\phi_b$ is invariably greater than E_0 (because of the fact that we are considering only the ground state eigen energy level corresponding to i = 0). Also, from Fig.1, we observe that for positive values of p_2, Eq. (5) is a very good estimate of Eq. (3) for p_2 more than 2 (approximately). Again, substituting the ground state eigen energy from Stern's model [Eq. (2) with i = 0] in the expression for p_2 [Eq. (9)], and evaluating it at the interface (i.e., at x = 0), we get

$$p_2\big|_{x=0} = \left(q\phi_b - \left(\frac{\hbar^2}{2m_s}\right)^{1/3}\left(\frac{9}{8}\pi qF_s\right)^{2/3}\right)\left(\frac{2m_{ox}}{\hbar^2}\right)^{1/3}(qF_{ox})^{-2/3} \quad (10)$$

where we have used the principle of continuity of electric displacement across the interface (i.e., $\varepsilon_s F_s = \varepsilon_{ox} F_{ox}$).

Equation (10) clearly shows that $p_2\big|_{x=0}$ has $F_{ox}^{-2/3}$ dependence. Substitution of appropriate values in this equation predicts that $p_2\big|_{x=0}$ decreases monotonically from roughly 55 (for F_{ox} = 500 kV/cm) to about 6 (for F_{ox} = 10 MV/cm, which, incidentally, is larger than the breakdown field for SiO_2). Thus, even the lowest value of $p_2\big|_{x=0}$ is larger than 2, which is the minimum required value of p_2 for the results obtained from the asymptotic approximation [Eq. (5)] to match with those obtained from numerical simulations of the exact Airy function integral [Eq. (3)]. Therefore, for positive values of p_2, and for practical values of the oxide field, it can be safely concluded that Eq. (5) is an excellent approximation of Eq. (3). This would be true even if the exact value of E_0 obtained from this work were substituted in Eq. (9). Thus, both the approximations have been justified. In the next section, we look at the boundary conditions.

3. The Boundary Conditions

Let the wave functions in the semiconductor and the oxide regions (given by $\psi_1(x)$ and $\psi_2(x)$ respectively) be denoted in the p-domain by $\phi_1(p_1)$ and $\phi_2(p_2)$ respectively. By denoting the variables p_1 and p_2 at x = 0 (i.e., the interface) by p_1' and p_2' respectively, from the continuity of the wave function across the interface, and using the expressions given by Eqs. (4) and (5), we get

$$c_2 \exp\left[-(2/3)(p_2')^{3/2}\right] = 2(p_2')^{1/4}\left(\frac{c_1 \cos\left[(2/3)|p_1'|^{3/2} - \pi/4\right]}{|p_1'|^{1/4}}\right) \quad (11)$$

Now, with the effective mass of the carrier being different on the two sides of the interface, the second

boundary condition, i.e., that of the continuity of the derivative of the wave function at the interface, takes the following form:

$$\frac{1}{m_s}\left[\frac{d\phi_1(p_1)}{dx}\right]\Bigg|_{x=0} = \frac{1}{m_{ox}}\left[\frac{d\phi_2(p_2)}{dx}\right]\Bigg|_{x=0} \quad (12)$$

Using Eqs. (7) and (9), Eq. (12) can be written as

$$\left[\frac{d\phi_1(p_1)}{dp_1}\right]\Bigg|_{x=0} = f_1\left[\frac{d\phi_2(p_2)}{dp_2}\right]\Bigg|_{x=0} \quad (13)$$

where f_1 is a physical constant, and is expressed as

$$f_1 = \left(\frac{m_s}{m_{ox}}\right)^{2/3}\left(\frac{\varepsilon_s}{\varepsilon_{ox}}\right)^{1/3} \quad (14)$$

Now, taking the derivatives of $\phi_1(p_1)$ and $\phi_2(p_2)$ (expressed by Eqs. (4) and (5) respectively) with respect to p_1 and p_2 respectively, Eq. (13) gets expressed as

$$\left[\frac{1}{4|p_1'|} + |p_1'|^{1/2}\tan\left(\frac{2}{3}|p_1'|^{3/2} - \frac{\pi}{4}\right)\right] = f_1\left[\frac{1}{4p_2'} + (p_2')^{1/2}\right] \quad (15)$$

where we have used Eq. (11).

Upon algebraic simplification of Eq. (15) and using Eq. (14), we get

$$\tan\left(\frac{2}{3}|p_1'|^{3/2} - \frac{\pi}{4}\right) = \left[\frac{m_s}{m_{ox}}\left(\frac{q\phi_b}{E_i} - 1\right)\right]^{1/2} + \left[\left(\frac{E_i}{q\phi_b - E_i}\right)\left(\frac{\varepsilon_s m_s}{\varepsilon_{ox} m_{ox}}\right) - 1\right]\frac{1}{4|p_1'|^{3/2}} \quad (16)$$

Note that Eq. (16) shows the dependence of E_i on only one parameter, and that is the surface electric field F_S [through the term p_1' – refer to Eq. (7)]. Now, Eq. (16) can also be expressed as $S = T_1 + T_2$, where S is the only term on the left-hand-side of Eq. (16), while T_1 and T_2 are the first and second terms respectively in the same equation. Thus, the eigen energy E_i can be obtained by solving this expression using numerical techniques. In the next section, we present the result of such an analysis which led to a further simplification of the expression given by Eq. (16), resulting in an analytical expression for the eigen energy level, function only of the surface electric field F_s.

4. Further Simplification

It can be observed that Eq. (16) has only one unknown variable E_i for a given electric field F_s, however, from Eq. (16), it should be obvious that it is not possible to solve for E_i analytically. In order to understand the relative contributions of the terms S, T_1, and T_2, Eq. (16) is numerically solved as a function of the oxide electric field F_{ox}, with the result shown in Fig. 2. In plotting these functions, E_i is taken to be the ground state energy level E_0, obtained by substituting i = 0 in Eq. (2). From the figure, it is obvious that the relative contribution of the term T_2 in the overall sum S is negligible as compared to that of T_1, even for oxide fields as high as 10 MV/cm. Hence, it can be prudently approximated that $S \approx T_1$, which yields [from Eq. (16)]:

$$\tan\left(\frac{2}{3}|p_1'|^{3/2} - \frac{\pi}{4}\right) \approx \left[\frac{m_s}{m_{ox}}\left(\frac{q\phi_b}{E_i} - 1\right)\right]^{1/2} \quad (17)$$

Unfortunately, even this simplification failed to make

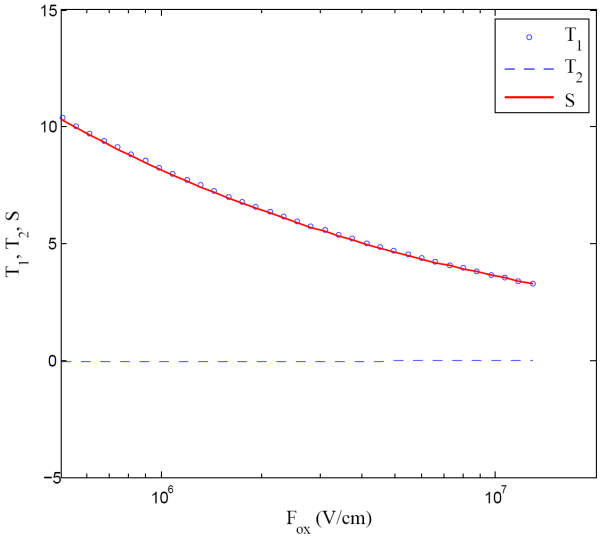

Fig. 2. The variations of the terms S, T_1, and T_2 as a function of the oxide electric field F_{ox}. Note that the contribution of the term T_1 is much more dominant than that of T_2 in dictating the overall sum S, and this holds for oxide fields even beyond 10 MV/cm.

E_i solvable analytically from Eq. (17). Thus, we look for the next level of simplification, where we assume that the barrier height ϕ_b at the interface is infinite, which leads to the first order estimate of E_i, as given by Eq. (2) [it can be very easily shown that Eq. (17), under the assumption that $\phi_b \rightarrow \infty$ leads to Eq. (2)].

The rest of the procedure is algebraically slightly more tedious, however, it leads to an analytical expression of E_i, as will be shown immediately. Substituting Eq. (2) for E_i only in the right-hand-side of Eq. (17), we get

$$\tan\left(\frac{2}{3}\left|p_1'\right|^{3/2}-\frac{\pi}{4}\right)=f_i\left(F_s\right) \tag{18}$$

where $f_i\left(F_s\right)$ is a function of F_s (and, of course, ϕ_b, however, for a given semiconductor-insulator system, it is a constant), and is expressed as

$$f_i\left(F_s\right)=\left[\left(\frac{m_s}{m_{ox}}\right)\left(\frac{q\phi_b}{\left[\hbar^2/\left(2m_s\right)\right]^{1/3}\left[\left(3/2\right)\pi qF_s\left(i+3/4\right)\right]^{2/3}}-1\right)\right]^{1/2} \tag{19}$$

Equation (18) also implies that

$$\left(\frac{2}{3}\left|p_1'\right|^{3/2}-\frac{\pi}{4}\right)=i\pi+\tan^{-1}\left[f_i\left(F_s\right)\right] \tag{20}$$

where i = 0, 1, 2, ⋯, etc.

Now, by substituting p_1' (a function of E_i) in Eq. (20) and upon further simplification, we arrive at

$$E_i=\left(\frac{\hbar^2}{2m_s}\right)^{1/3}\left(\frac{3q}{2}\right)^{2/3}\left[\left(i+\frac{1}{4}\right)\pi+\tan^{-1}\left[f_i\left(F_s\right)\right]\right]^{2/3}F_s^{2/3} \tag{21}$$

Equation (21) gives an explicit analytical expression for the subband energy levels as a function of the electric field F_s. The ground state energy level E_0 can be obtained by substituting i = 0 in Eq. (21), with $f_0(F_s)$ obtained from Eq. (19), by substituting i = 0. Note that this expression for E_0 takes into account the finite semiconductor-insulator barrier height (ϕ_b), which other papers in this area hitherto had not taken into account. In the next section, we develop a model for effective electric field in the semiconductor region, which should account for the triangular potential well approximation.

III. Effective Electric Field Model in the Semiconductor Region

The model for E_0 developed in this work so far [Eq. (21) with i = 0] still assumes that the electric field throughout the semiconductor is constant and is equal to the surface electric field F_s. However, in reality, the electric field in the semiconductor is not at all a constant rather varies from F_s at the interface to zero deep into the bulk. It is well known that the one-dimensional Schrödinger's equation has an analytical solution when the electric field F_s in the semiconductor is constant, with the potential function $\phi(x)$ given by $\phi(x) = xF_s$ (assuming triangular potential). Though there is no analytical solution for a varying electric field profile in the semiconductor region, however, attempts have been made [4, 5] to express this varying field by an effective field $F_{s,eff,}$ given by

$$F_{s,eff}=\frac{1}{\varepsilon_s}\left(Q_d'+\eta Q_n'\right) \tag{22}$$

where Q_d' is the depletion layer charge density, Q_n' is the inversion layer charge density, ε_s is the permittivity of Si, and η is an empirical fitting parameter, which has been taken to be equal to 11/32 in [5], and 0.5 in [4]. The primary goal of introducing this parameter is to account for the overestimation of the Eigen energy value, resulting from the assumption of the triangular potential well in the semiconductor region.

It has also been established in [5] that the triangular potential well (with $\eta = 1$) is an excellent approximation in the evaluation of the Eigen energy level under the weak inversion condition, i.e., when the inversion charge density is much less than the depletion charge density. However, beyond weak inversion, the triangular potential well approximation overestimates the carrier confine-

ment in the 2DEG at the oxide-semiconductor interface [4], and, hence, also overestimates the Eigen energy level.

In our work, we propose that using the effective electric field model $F_{s,eff}$ [Eq. (22)], a simple way to account for such overestimation of carrier confinement is to decrease the value of η. From extensive simulations, we have observed that the value of $F_{s,eff}$ obtained by choosing $\eta = 0.85$ makes the calculation of the Eigen energy level accurate. When we substituted $F_{s,eff}$ [Eq. (22)] (with $\eta = 0.85$) in place of F_s in the expression for the eigen energy level [Eq. (21) in association with Eq. (19)], it was observed that the results obtained from our model of the Eigen energy level matched very closely with that of Li et al. [10].

Now, the value of η (= 0.85) that we propose is quite higher than 0.5 and 11/32 reported in [4, 5] respectively. Note that in Section II, using physics based methodology to include the effect of finite semiconductor-insulator barrier height, we have already accounted for the overestimation in E_0 resulting from infinite barrier height approximation at the oxide-semiconductor interface, thus, part of the correction has already been achieved. Hence, to account for the assumption of constant electric field throughout the semiconductor region, the amount of necessary reduction in the value of η (from its nominal value of unity) became correspondingly smaller. In the next section, we present our results.

IV. RESULTS

In Fig. 3, we plot the values of E_0 obtained from our model as a function of the oxide electric field F_{ox}, along with the results of Stern's [5] as well as those obtained from the semi-empirical model proposed by Li et al. [10]. In plotting these results, we have used the same data as reported in [5, 10]. Also, as a general norm, these characteristics are shown as a function of the oxide electric field F_{ox} (instead of F_s), keeping the fact of oxide breakdown in mind. As expected, with an increase in the electric field F_s (and, consequently, F_{ox}), the triangular potential well in the substrate becomes steeper, and the confinement of the charge carrier increases, causing an increase in the ground state eigen energy level. The same trend has been shown by the results obtained from both the models of [5, 10]. It is interesting to note that our

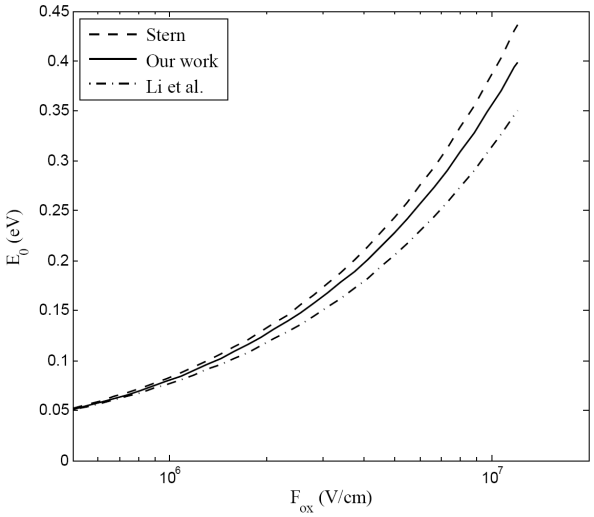

Fig.3. The ground state energy level E_0 as a function of the oxide electric field F_{ox}, obtained from our model (shown by the solid line), incorporating only the effect of finite semiconductor-insulator barrier height at the interface. The results obtained from Stern's [5] model (shown by the dashed line) as well as from the model of Li et al. [10] (shown by the dash-dotted line) are also shown.

results are closer to that of [10], which actually showed a very good match with the results obtained from an exact numerical simulation of the coupled Schrödinger-Poisson equations [10]. Note that by taking the results reported in [10] as the reference, at oxide fields of 0.7 MV/cm and 7 MV/cm, the percent errors corresponding to the results of [5] are 6% and 20.8% respectively, while the same with respect to our model are 2.7% and 12.3% respectively, which is a non-trivial improvement.

Next, we included the model for the effective electric field $F_{s,eff}$ [Eq. (22)] in the original model for E_0 [Eq. (21)]. The value of the coefficient η used in the expression for $F_{s,eff}$ was found to equal 0.85 in order to obtain the best possible accuracy in our model of E_0. Thus, we arrive at an analytical evaluation of E_0 as a function of V_{GB}, considering both the finite barrier height at the oxide-semiconductor interface, as well as accounting for the variation of the electric field within the semiconductor region, by using the effective electric field $F_{s,eff}$, with a modified value of η.

In Fig. 4, we plot the variation of E_0, obtained from our complete model, as a function of V_{GB}. The results obtained from the models of [5, 10] are also shown in the figure. The substrate doping concentration N_A is taken to be 10^{17} cm^{-3} and the oxide thickness t_{ox} is equal to 2 nm. Note the superior matching of our results (after account-

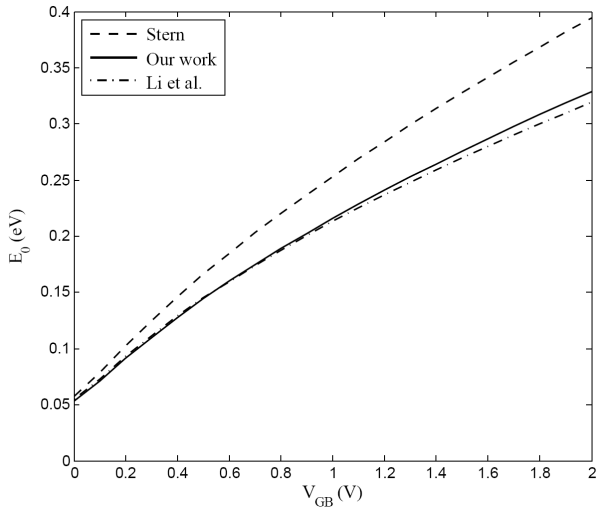

Fig. 4. E_0 as a function of V_{GB}, obtained from our complete model (solid line), and the models of Stern [5] (dashed line) and Li et al. [10] (dot-dashed line). Note that our results emulate those of Li et al. [10] very closely.

ing for both the effects) with those predicted by the semi-empirical model of [10]. In particular, by taking the result of [10] as the reference, at applied V_{GB} of 0.5 V and 1.5 V, the percent errors corresponding to the model of [5] are 14.7% and 21.5% respectively, while those of our model dropped to only −0.3% and 2.2% respectively.

V. Summary and Conclusions

In this work, we have presented a completely analytical model for the ground state eigen energy level E0 for the quantized carriers within the inversion layer of contemporary MOSFETs, having high substrate doping and ultrathin gate oxide. It has accounted for the two important (but false) assumptions commonly used in the literature in this area, namely infinite barrier height at the oxide-semiconductor interface and the triangular potential well within the semiconductor. We took into account the effect of finite barrier height at the oxide-semiconductor interface by invoking the asymptotic approximations of the Airy function integrals, and by applying the boundary conditions at the interface, obtained the wave functions in the semiconductor as well as within the oxide – note that a nonzero wave function in the oxide is an important contribution of this work, which resulted from the barrier at the interface being finite.

From this procedure, we also obtained the expression of the ground state eigen energy level. The results matched better with those obtained from the semi-empirical model of Li et al. [10] (which, in turn, match very well with the numerical simulations using the self-consistent approach of solving Schrödinger and Poisson equations) than that of Stern [5], however, the error was still pretty high (~ 12%). Hence, in the next part of our work, we attempted to remove the assumption of constant electric field throughout the semiconductor region, a fallout of the triangular well approximation) by modeling it as an effective electric field, using a widely used effective electric field model, with a modified value for the parameter η. The end result was almost accurate, where the percent error in the values of E0 dropped to a meager 2% with respect to the model of Li et al. [10]. Also, as has been mentioned earlier, the model is purely analytical and completely based on physical principles (with prudent assumptions made and proper justifications for the same presented)

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