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3차원 회전축 대칭 물체 조각의 축 추정 방법

Fast Axis Estimation from 3D Axially-Symmetric Object's Fragment

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요 약

깨어진 항아리 조각들을 가상 공간에서 조립하기 위하여 조각 표면을 이용한 빠른 3차원 회전축 추정 방법을 제안한다. 물체의 원형성과 표면의 국지적 평면성을 이용하여 대칭축을 찾는 방법을 사용한다. 항아리 조각 같은 회전축 대칭 물체는 반지름이 다른 여러 원통의 중첩으로 생각될 수 있다. 각 원통의 원형성을 회전축 계산에 이용한다. 먼저, 표면 위 임의의 한 점을 지정하고 그 점을 통과하는 여러 개의 원통상의 궤도를 각각의 곡률의 변화를 측정 검사하여 조사한다. 올바른 원의 궤도는 곡률의 변화가 없을 것이므로 가장 작은 곡률의 변화가 원의 궤도로 선택된다. 또한 원의 중심점으로 축이 통과하는 경로가 되므로 원의 중심점이 축의 위치가 된다. 표면의 국지적 평면성과 프로파일 곡선 근사를 통한 축 위치 추정 방법 또한 연구 되었다. 제안된 방법은 기존의 방법에 비해 계산 속도가 향상되었고 조각의 부위에 영향을 받지 않는 강건성을 가짐을 실험적으로 입증하였다.

키워드 : 3D 조각 조립, 대칭축 추정, 고고학, 국부 곡률

Abstract

To reduce the computational cost required for assembling vessel fragments using surface geometry, this paper proposes a fast axis estimation method. Using circular constraint of pottery and local planar patch assumption, it finds the axis of the symmetry. First, the circular constraint on each cylinder is used. A circular symmetric pot can be thought of unions of many cylinders with different radii. It selects one arbitrary point on the pot fragment surface and searches a path where a circumference exists on that point. The variance of curvature will be calculated along the path and the path with the minimum variance will be selected. The symmetric axis will pass through the center of that circle. Second, the planar patch assumption and profile curve is used. The surface of fragment is divided into small patches and each patch is assumed as plane. The surface normal of each patch will intersects the axis in 3D space since each planar patch faces the center of the pot. A histogram method and minimization of the profile curve error are utilized to find the probability distribution of the axis location. Experimental results demonstrate the improvement in speed and robustness of the algorithms.

Key Words : 3D Sherds Assembly, Symmetric Axis Estimation, Archeology, Local Curvature

1. Introduction

Cultural heritage protection digitization has become a new research direction in cross-disciplinary of information technology and archaeology. Too much emphasis cannot be put on preserving and reconstruction of artifacts found on excavation site. Especially, a number of vessels are excavated usually in their fragments called sherds on the sites and should be re-assembled to be restored. The most common assembling practice is through professional experience by human operators to estimate the configuration of the broken fragments. Virtual assembly methods of sherds using computer vision techniques are developed to help such time consuming tasks.

Recently methods for automatic assembly of pot fragments have been introduced. Kampel [9] introduced a method for reconstruction of broken sherds by calculating the axis of rotation using the fact that surface normals of rotationally symmetric objects intersect their axis of rotation and using inner and outer surface, a profile curve is generated. Finally, they perform a surface matching procedure using a modified ICP(Iterative Closest Point) algorithm. It needs more complete sherd information which means that the axis should be easily obtained. The sherds dealt in this paper are more gen-

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eral and contain partial information of the entire vessel. Cooper et. al. [1][2] developed a method for fragment matching based on a Bayesian approach using break curves, estimated axis and profile curves. It is a bottom-up maximum likelihood performance-based search and is associated with sub assemblies of fragments via a likelihood of energy functions. Algorithms for the assembly of pot fragments using surface geometry utilize an axis estimation procedure which requires extensive computational cost due to the high dimensional non-linear solution space. The profile of sherd surface is modeled using implicit polynomial and the axis is estimated by minimizing implicit profile curve error. It is a high dimensional gradient descent problem requiring extensive search time. They also show very slow speed in estimating axes of sherds. Matching and alignment of fragments were based on matching (a) break-curves (curves on a pot surface separating fragments), (b) estimated axes and (c) profile curves for individual fragments and groups of matched fragments. Especially for (b) and (c), the search for the axes configuration made overall algorithm to require at least several hours of computation on PC.

In this paper, two sub-optimal but fast independent axis estimation methods are proposed which can be combined and used in the original problem of sherds assembly with considerable reduction of computational cost. The first method uses local geometrical constraint which is circle along the circumference and the second method uses the local planar patch assumption. On a circle with radius R, the curvature is defined as $1/R$. By calculating three separate points on the circle, the radius can be found. By computing such curvature along a path, the variance of curvature is computed. Even though a brutal force search is used, the speed of computation is fast since the search space is very small.

However, the proposed method can not process the fragment which is part of sphere, because the difference of curvatures among different directions will be too small to estimate the symmetric axis.

The rest of this paper is organized as follows: In section 2, an axis/curve model is established and axis estimation problem is introduced. In section 3, the proposed methods are described in detail. A problem encountered when using local curvature constraint is discussed in section 4. The experiment results are presented in section 5. Finally, the conclusion and future work will be discussed in section 6.

2. Axis/Profile Curve Model

The curve model of this paper follows the notation in [1]. The pose of a rotationally symmetric object is defined by its axis of symmetry. The axis of symmetry has parameterized using a standard parametric equation of a 3D line

$$\begin{cases} x = m_x z + b_x \\ y = m_y z + b_y \end{cases} \tag{1}$$

These equations contain 4 unknown parameters which describe the 3D axis of symmetry, $l = \{m_x; m_y; b_x; b_y\}$.

Two of these parameters, m_x and m_y, describe the slope of the line when it is projected into the $X-Z$ plane and the $Y-Z$ plane respectively. The remaining two parameters, b_x and b_y, specify the x and y coordinates where the axis line intercepts the $X-Y$ plane at $Z=0$.

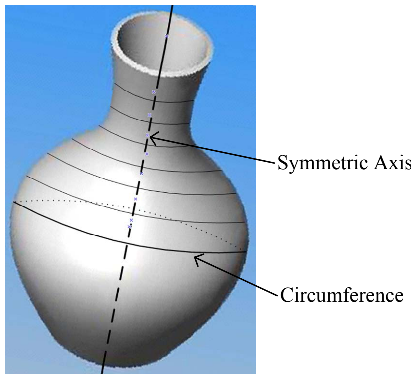

Fig. 1. Surface of Axial Symmetry

As shown in Fig. 1, many circles exist on a pottery, which have a common center on the axis. By observing local structure of its fragments, an axis can be found or cannot be found especially when the fragments are from spherical shape part. However, in most cases, the axis can be found as shown in Fig. 2.

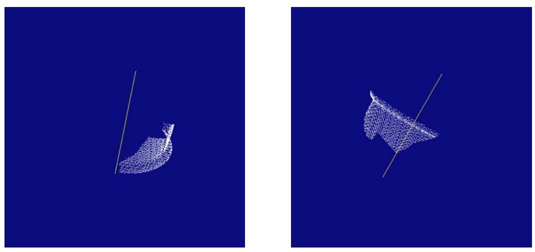

Fig. 2. An example of a sherd and its axis

For the axis and profile curve estimation, it requires non-linear minimization technique. Minimization algorithm has to search 4 unknown parameters $\{m_x; m_y; b_x; b_y\}$ with 2D profile curve parameters of the 3D surface (high order implicit polynomial). Fig. 2 shows an example of sherd and its estimated axis respectively, and the circumference on the sherd is corresponding with its symmetric axis. If the circumference curve is found, then the symmetric axis can be estimated easily.

3. Axis Estimation from the Fragment

The geometry of the sherd surface will be used for the estimation of the axis: the circular constraint of the surface is used and the local surface patch (plane) is used for the axis estimation. Those newly developed methods will be described.

3.1 Constant Curvature Constraint

In this proposed method, it is not necessary to rotate the fragment or curve and it just needs to detect the variance of the curvature along the curve selected. The principle of the algorithm is that the variance of the curvature will be constant along the curve if the curve is circle parts. A rotational symmetric object can be thought of being constructed with many local cylindrical objects. As is shown in Fig. 1, each cylindrical part has a constant curvature of radius. If a path is on that circle, it will have a constant curvature on the path. The procedures to compute the variance of curvature is as follows:

1. Let S be the set of the surface point of the fragment. A start point P is selected randomly. Then a sphere is generated centering on P with an arbitrary radius R_{arb}. Let a 3D curve I_c be the intersection of the surface S and the sphere. This procedure is necessary only for finding paths C_j's in step 2.

2. I_c is divided into sub-intervals as shown in Fig. 3. It will generate node points I_{p1} to I_{pN} on the curve I_c. C_j is defined as an intersection of S and the triangle plane Π_{tri} of three points P, I_{pj} and I_{pj*} (I_{pj*} is the conjugate point of I_{pj} which is on the opposite side). The variance will be computed on the path C_j.

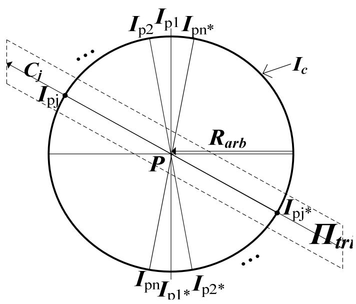

Fig. 3. I_c is divided into sub-intervals and node points I_{pj}'s are determined. Each conjugate pair I_{pj} and I_{pj*} will give one search path C_j.

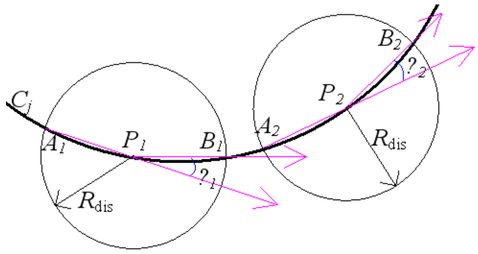

Fig. 4. Curve C_j on the plane Π_{tri} and the radius of small circle for computing curvature will be R_{dis}. The angle θ_1 and θ_2 will be the same if the curvature does not vary.

3. For the computation of curvature along C_j, a fixed distance R_{dis} is chosen via

$$R_{dis} = \min_{C_j}(length(C_j))/M,\ j=1,\cdots,n \tag{2}$$

In Eq. (2), the constant M guarantees the reliable number of points along the path C_j. As shown in Fig. 4, the θ will be computed as an angle between AP and PB. If the curve C_j is on the correct circumference, then the angle θ_i is fixed with moving the center P_i.

4. The variance of curvature δ_i is calculated along every curve C_j. The path close to circumference will be chosen using the minimum variance among them as shown in Fig. 5.

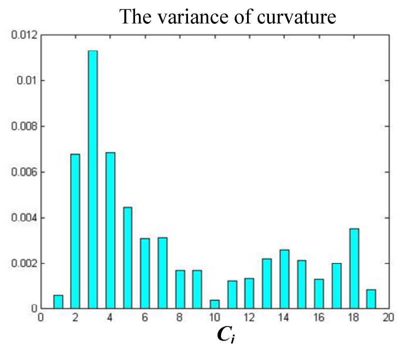

Fig. 5. An example showing variance of each C_j: x-axis in the graph shows the searching path index and index 10 has the minimum value.

5. In order to get the reliable result, the procedure can be repeated on other points by moving the point P in step 1 randomly and the result can be combined.

It calculates the curvature on the path, and then the circumference can be estimated using variance of the curvature.

3.2 Normal of Local Planar Patches

Local planar patch assumption can help estimating the location of the axis. Surface of a fragment can be regarded as being consisted of many small planar patches. Especially when a fragment is big piece compared to the entire shape, this method can help to estimate the axis fast. The procedure using local planar patch assumption is as follows:

1. Local surface normal of the sherd are found via small patches of the sherd which contains less than several hundred points.

2. A histogram method is used to find the location in 3D where the axis may pass through. Usually, a small piece of sherd is more like a sphere and hard to estimate the axis. Cylindrical shape is beneficial to axis estimation.

3. The sherd is rotated and the error of projection is computed for each angle. Since search pyramid is used, computational cost is not significant. Here, profile curves are modeled non-parametrically.

4. As a result, we can find 3D area where axis lies in. The full 3D probability distribution of axis location is generated and can be used sherd assembly.

4. Error on Search Path

Given a fragment of sherd, the circumferences which exist on the sherd will be searched by minimizing the variance of the curvature as explained before. However, the assumption is that the axis and the tangent plane at the point A are parallel to each as in Fig. 6(a). That is not true in general. When there is an angle between tangent plane and the axis, the path of computing angle is not correct. It has to be along the real circumference $O-P$ in Fig. 6(a) (Not $O'-P$). There will be error in searching path C_j as follows.

In Fig. 6(a), the tangent plane π on the point P has an acute angle θ with the symmetric axis. The pink circle in the plane Π_1 is the circumference which must be used for the search path. The red plane(Π_2) from triangle DPE is perpendicular with the plane π, two planar angles between Π_1 and Π_2 is θ. Unfortunately, θ is not known before hand and the path on Π_2 will be used for the C_j. This will bring the path error in the variance calculation.

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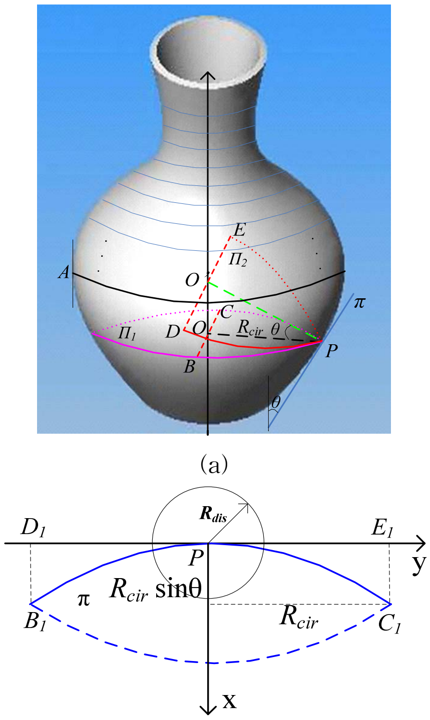


Fig. 6. (a) A 3D object (b) The correct circumference with the radius R_{cir} will be projected as B_1PC_1 on the plane π.

Clearly, there is limitation using this method when the angle θ is big since the search path will be much off from the correct one. However, when θ is small, D_1B_1 is also small and the overall path will not be different much. Also, by using the short path C_j near P, the error can be reduced.

5. Experiments & Analysis

The fragment with smooth surface are used in the experiment because the surface curvature features are used for estimate the symmetric axis. The data set is composed of real pot sherds of 3D points of laser scanning.

5.1 Constant Curvature Constraint

First method using constant curvature constraint is tested; the most probable circumference is detected. The estimated circle path's center gives axis estimation.

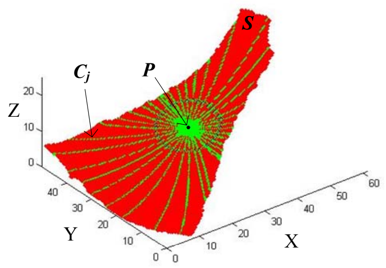


Fig. 7. The intersection curve I_c between the sphere with radius of R_{arb} and the sherd surface S. Green curves are search paths C_j's.

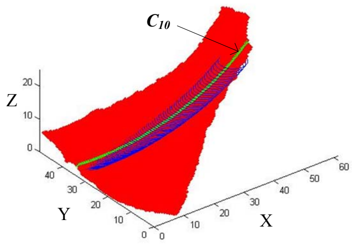


Fig. 8. Green curve C_j is chosen as the minimum variance curve. A series of curvature values were computed using R_{dis} along C_j, and then variance δ_j is computed.

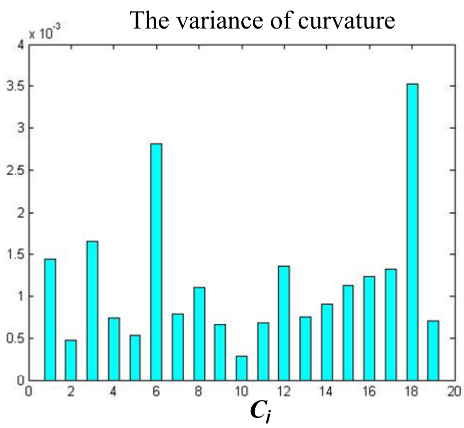
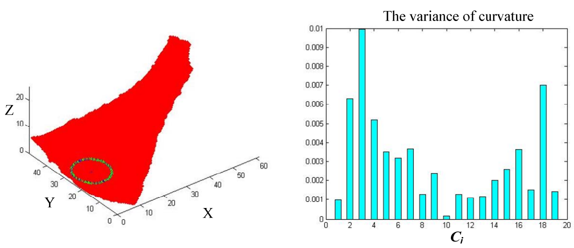
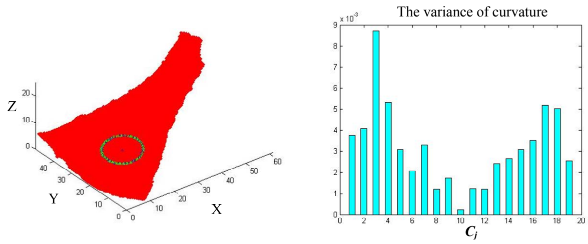


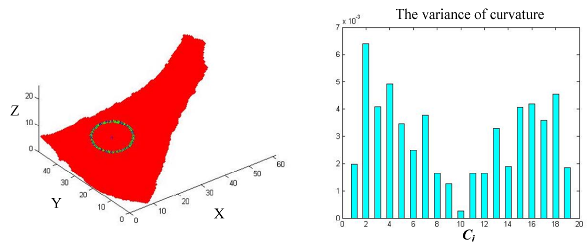
Fig. 9. the variance of the curvature for each curve



(a)



(b)



(c)

Fig. 10. Examples showing the variance of curvature with different starting point P.

Fig. 7 shows I_c with the radius R_{arb} and the intersection curves C_j. For each curve, the variance of curvature is calculated based on moving center point P_i, so each curve path C_j will have its own variance. In Fig. 8, circles with radius R_{dis} based on different center points P_i are shown (On a given path C_j).

In this way, there are 19 variances found on 19 paths and plotted in Fig. 9. If a curve C_j is on the circumference of the pot then the variance set will be smallest. In Fig.9, the 10th curve path has minimum variance and chosen as the circumference of the object.

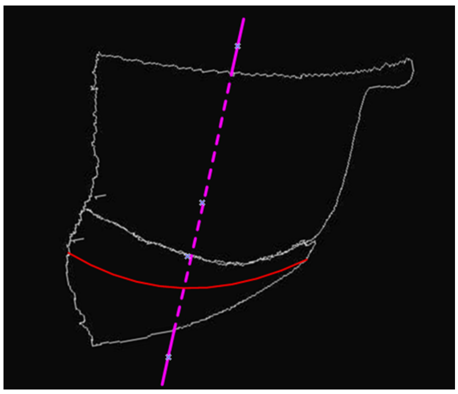


Fig. 11. The bottom sherd is used in the test, and the red curve is the circumference chosen as circumference. The pink line is the symmetric axis.

In order to show that the independence of the start point P, several center points are selected randomly and tested. The result is shown in Fig. 10, at which the minimum variance gives the same axis. In Fig. 11, the bottom sherd is used piece in the experiment, and the red curve is the chosen circumference path, the pink line is the symmetric axis. It shows the method works regardless of the choice of starting point P.

For this method, the experiment result shows fast computation performance requiring only 10 or so seconds for the estimation of axis of one sherd. Compared to the previous method [1][2], it shows big reduction in computation time since the method in [1][2] employs non-linear gradient decent procedure in high dimensional parameter space and needs huge amount of computational cost.

5.2 Surface Normal of Local Planar Patches

Using planar surface normal, the area is found where the axis may exist. Top piece in Fig. 11 is tested. In Fig. 12, surface normal of local planes are calculated via SVD(Singular Value Decomposition) and Eigen-value. Each normal vector must intersect the axis. Most probable area is plotted as red in Fig. 13. 3D lines come from surface normal and collected in 3D histogram bin. Red dots are most popular are where those lines go through.

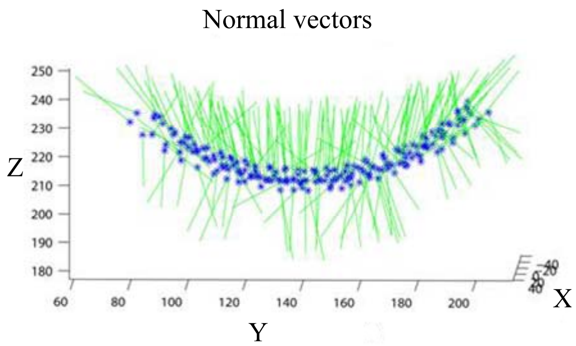


Fig. 12. Local surface normal vectors

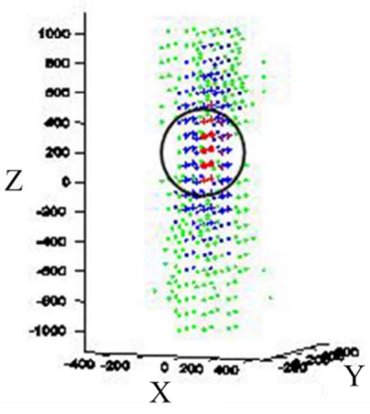


Fig. 13. Red dots are most probable area where axis may pass through. The original sherd surface lies in $X-Y$ plane as in Fig. 12.

Fig. 14 shows an example of the axis estimation which minimizes the surface profile curve projection error. In top-left image, the axis is along the Z-axis. Note that the configuration of the sherd is not correct (90 degree off), but the correct configuration also has a low profile curve fitting error, too.

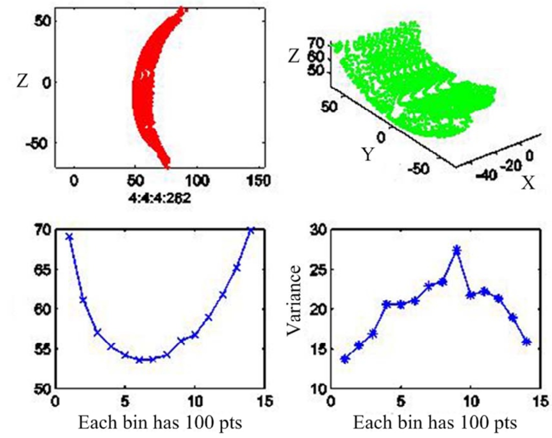


Fig. 14. Example of projection of 3D points onto $Y-Z$ plane (Top-left) and its original sherd (Top-right). Bottom-left shows the non-parametric curve. Bottom-right is the residual error.

This example shows that the possible configurations which minimize profile curve error can be spatially apart. It also shows that just using surface normal and profile curve error minimization to estimate symmetric axis can give incorrect estimation.

6. Conclusion

Fast sub-optimal axis estimation algorithms have been proposed. Two methods are developed and tested. First, constant curvature constraint is used for the detection of the circumference of the object. The search path with minimum variance will be selected and used for the circumference. The center of the circle will be on the axis of symmetry. Second, by using local planar patch assumption and non-parametric profile curve error minimization, the probable axis configuration is found. Both methods can be used combined. They can also be used in multi-scale set-up and it will reduce significant amount of computational cost in addition. For a reliable axis estimation of a sherd with only partial information, it requires the search on high dimensional parameter space. However, the proposed method does not search in high dimensional parameter space, the computation time is minimal. The fast performance of the proposed algorithm will improve the speed of the original virtual pot assembly problem.

However, the proposed method will fail in certain cases. For example, when the fragment is part of sphere or similar shape, the difference of curvatures among different directions will be too small to estimate the symmetric axis using the proposed method. In addition, the detected result will be distorted when the surface of the fragment is rough. Therefore, fragments with smooth surface are used in the experiment.

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