Communications of the Korean Statistical Society
2010, Vol. 17, No. 1, 89–98

# A Time Truncated Two-Stage Group Sampling Plan for Weibull Distribution

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**Abstract**

In this paper, a two-stage group sampling plan based on the time truncated life test is proposed for the Weibull distribution. The design parameters such as the number of groups and the acceptance number in each stage are determined by satisfying the producer's and consumer's risks simultaneously when the group size and the test duration are specified. The acceptable reliability level is expressed by the ratio of the true mean life to the specified life. It was demonstrated from the comparison with single-stage group sampling plans that the proposed plan can reduce the average sample number or improve the operating characteristics.

Keywords: Acceptable reliability level, average sample number, consumer's risk, group sampling plan, life test, producer's risk.

## 1. Introduction

As Pearn and Wu (2006) pointed out, an acceptance sampling plan involves quality contracting on product orders between the producers and customers. In a time-truncated sampling plan, a random sample is selected from a lot of products and put on the test where the number of failures is recorded until the pre-specified time. If the number of failures observed is not greater than the specified acceptance number, then the lot will be accepted. Two risks are always attached to an acceptance sampling plan. The probability of rejecting a good lot is known as the producer's risk and the probability of accepting a bad lot is called the consumer's risk. An acceptance sampling plan should be designed so that both the risks are smaller than the required values. Time-truncated single sampling plans have been studied for a variety of life distributions by many authors, including Sobel and Tischendrof (1959), Goode and Kao (1961), Gupta and Groll (1961), Gupta (1962), Kantam *et al.* (2001), Tsai and Wu (2006), Balakrishnan *et al.* (2007) and Aslam and Kantam (2008).

The usual acceptance sampling plans are used to test a single item in a tester. In practice, there is a type of testers that may accommodate the multiple items simultaneously. A sampling scheme called the group sampling plan comes to field for this purpose of using multiple-item testers. The main advantage of using a group sampling plan is that it reduces the cost and the time of the experiment than the ordinary sampling plan. Jun *et al.* (2006) suggested that a group sampling plan is useful for the sudden death testing. They determined the number of groups for the single and the double sampling plans under the sudden death testing. Pascual and Meeker (1998) proposed the modified sudden death test plans with a limited number of test positions. More recently, Aslam and Jun (2009a) introduced the group sampling plan based on the truncated life tests when the lifetime follows the inverse

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Rayleigh or log-logistic distribution. Aslam and Jun (2009b) developed the group sampling plan for the Weibull distribution with known shape parameter, where they determined the number of groups and the acceptance number by considering the producer's and the consumer's risks simultaneously. In this study, these group sampling plans will be called as single-stage group sampling plans to be compared with the proposed two-stage group sampling plan.

It has been well known that a double sampling plan performs better than a single sampling plan in terms of the sample size. So, there may be a need of developing a version of double sampling plan for a life test using groups, which will be called a two-stage group sampling plan in this paper. That is, in order to further reduce the sample size required, we propose a two-stage group sampling plan under the assumption that the lifetime follows the Weibull distribution with known shape parameter. The design parameters for the proposed plan will be determined so as to minimize the average sample number under the constraints of satisfying the producer's and consumer's risks simultaneously.

## 2. Two-Stage Group Sampling Plans

We propose the following two-stage group sampling plan when using the type of testers with the group size of $r$:

1. (First Stage) Draw the first random sample of size $n_1$ from a lot, allocate $r$ items to each of $g_1$ groups (or testers) so that $n_1 = rg_1$ and put them on test for the duration of $t_0$. Accept the lot if the number of failures observed from each group is $c_1$ or smaller. Truncate the test and reject the lot as soon as the number of failures in any group is larger than $c_2$ before $t_0$. Otherwise, go to the second stage.

2. (Second Stage) Draw the second random sample of size $n_2$ from a lot, allocate $r$ items to each of $g_2$ groups so that $n_2 = rg_2$ and put them on test for $t_0$. Accept the lot if the number of failures in each group is $c_1$ or smaller. Truncate the test and reject the lot if the number of failures in any group is larger than $c_1$ before $t_0$.

The above two-stage group sampling plan involves the design parameters of $g_1$, $g_2$, $c_1$ and $c_2$ when the test duration $t_0$ is specified. Note that the group size $r$ will be given by the type of testers to be used. The acceptance number in the second stage may be different from that in the first stage, but in this study it is assumed to be equal to reduce the number of parameters.

Suppose that the life of a product follows the Weibull distribution with known shape parameter. If the shape parameter is unknown in practice, the proposed sampling plan can be used for the estimated value of shape parameter from the past failure data. Normally, producers keep the estimated parameters of the life distribution for their products. The cumulative distribution function(cdf) of the Weibull distribution is given by

$$F(t; \lambda, m) = 1 - \exp\left(-\left(\frac{t}{\lambda}\right)^m\right), \quad t \geq 0, \tag{2.1}$$

where $\lambda > 0$ is a scale parameter and $m$ is a shape parameter. Then, the mean life under the Weibull distribution is obtained by

$$\mu = \left(\frac{\lambda}{m}\right)\Gamma\left(\frac{1}{m}\right). \tag{2.2}$$

It would be convenient to express the test duration or termination time as a multiple of the specified life $\mu_0$. So, we will consider $t_0 = a\mu_0$ for a constant $a$. Then, the probability that a failure occurs before the termination time $t_0$ is obtained as follows when the true mean life is $\mu$:

$$p = 1 - \exp\left(-\left(\frac{t_0}{\lambda}\right)^m\right) = 1 - \exp\left(-a^m\left(\frac{\mu}{\mu_0}\right)^{-m}\left(\frac{\Gamma(1/m)}{m}\right)^m\right). \tag{2.3}$$

This indicates that the failure probability or unreliability is obtained if the ratio of the mean life to the specified life is given.

The lot acceptance probability at the first stage under the proposed two-stage sampling plan is given by

$$P_a^{(1)} = \left[\sum_{i=0}^{c_1}\binom{r}{i}p^i(1-p)^{r-i}\right]^{g_1}. \tag{2.4}$$

The lot rejection probability at the first stage is given by

$$P_r^{(1)} = 1 - \left[\sum_{i=0}^{c_2}\binom{r}{i}p^i(1-p)^{r-i}\right]^{g_1}. \tag{2.5}$$

Now, the lot acceptance probability from the second stage is

$$P_a^{(2)} = \left[1 - \left(P_a^{(1)} + P_r^{(1)}\right)\right]\left[\sum_{i=0}^{c_1}\binom{r}{i}p^i(1-p)^{r-i}\right]^{g_2}. \tag{2.6}$$

Therefore, the lot acceptance probability for the proposed two-stage group sampling plan is given by

$$L(p) = P_a^{(1)} + P_a^{(2)}. \tag{2.7}$$

## 3. Determination of Plan Parameters

The two-point approach has been usually adopted in designing acceptance sampling plans, which considers the producer's risk at the acceptable reliability level(ARL) and the consumer's risk at the lot tolerance reliability level(LTRL). See for example Fertig and Mann (1980).

We also use the two-point approach for determining the design parameters of the proposed two-stage group sampling plan. Here, we express the ARL and LTRL as the ratio of the true mean life to the specified life. The consumer demands that the lot acceptance probability should be less than the specified consumer's risk, $\beta$, when the quality level is unacceptably low. On the other hand, the producer desires that the lot rejection probability should be smaller than the specified producer's risk when the quality level is acceptably high. Therefore, we want to find the design parameters of the proposed plan such that the following two inequalities are satisfied at the same time.

$$L\left(p \,\middle|\, \frac{\mu}{\mu_0} = r_1\right) \le \beta, \tag{3.1}$$

$$L\left(p \,\middle|\, \frac{\mu}{\mu_0} = r_2\right) \ge 1 - \alpha, \tag{3.2}$$

where $r_1$ is the life ratio at the consumer's risk and $r_2$ is the life ratio at the producer's risk. In this study, the ratio $r_1$ was set to 1, in which case the LTRL is $\mu_0$.

Let $p_1$ be the probability of failure corresponding to the consumer's risk and $p_2$ be the failure probability corresponding to the producer's risk. Then, the above Equations (3.1) and (3.2) reduce to

$$L(p_1) \leq \beta, \tag{3.3}$$

$$L(p_2) \geq 1 - \alpha. \tag{3.4}$$

It should be noted that the plan parameters can be determined independently of the life distributions if $p_1$ and $p_2$ are directly, not through the life ratio, specified as LTRL and ARL, respectively. However, in this paper the ARL and the LTRL will be specified in terms of life ratios, which may be more practical.

There may exist a multiple solutions for the design parameters satisfying (3.3) and (3.4). In order to resolve this, we use the strategy to choose these parameters which lead to the smallest average sample number(ASN). The ASN for the proposed plan is a function of p and is given by

$$\mathrm{ASN}(p) = rg_1 + rg_2\left(1 - P_a^{(1)} - P_r^{(1)}\right). \tag{3.5}$$

Therefore, the design parameters can be found from the solution of the following optimization problem:

$$\text{Minimize } \mathrm{ASN}(p_1) = rg_1 + rg_2\left(1 - P_a^{(1)} - P_r^{(1)}\right). \tag{3.6a}$$

Subject to

$$L(p_1) \leq \beta \tag{3.6b}$$

$$L(p_2) \geq 1 - \alpha \tag{3.6c}$$

$$1 \leq g_2 \leq g_1 \tag{3.6d}$$

$$0 \leq c_1 < c_2 \tag{3.6e}$$

$$g_1, g_2, c_1, c_2 : \text{ integers.} \tag{3.6f}$$

Note that the ASN at $p_1$ is larger than the ASN at $p_2$, so it is reasonable to minimize the larger ASN as in (3.6a). The constraint (3.6d) is added because it may not be desirable if the number of groups in the second stage is larger than that in the first stage. This optimization problem can be simply solved by a search method that can be implemented on Excel sheets.

Tables 1–6 present the design parameters of the proposed sampling plan for different values of shape parameters($m = 1, 2, 3$). Two values($r = 3$ and $r = 5$) were chosen as the tester (group) size and two values of termination time ratios($a = 0.5$ and $a = 1.0$) were used. Four levels of consumer's risk($\beta = 0.25, 0.10, 0.05, 0.01$) were specified and several life ratios were considered as the ARLs with the producer's risk of 5 percent. The cell with upward arrow(↑) indicates that the same value applies as in the upper cell. Tables for other combinations of values can be similarly constructed.

It is seen from these tables that the numbers of groups required($g_1$ and $g_2$) decrease as the group size increases from $r = 3$ to $r = 5$ when other conditions remain the same. The sample size($rg_1$ or $rg_2$) also decreases as the group size increases, which means that the use of a larger group size may be more economical. The change in the number of groups according to different shape parameters is not monotonic. When the shape parameter increases from $m = 1$ (exponential case) to $m = 2$, the number of groups decreases. When the shape parameter increases from $m = 2$ to $m = 3$, the number of groups increases for $a = 0.5$ but decreases for $a = 1.0$. It is also observed that the number of groups increases

Table 1: Two-stage group sampling plan with $r = 3$ for Weibull having $m = 1$

| $\beta$ | $\mu/\mu_0$ | $a = 0.5$ | | | | | $a = 1.0$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $= r_2$ | $g_1$ | $g_2$ | $c_1$ | $c_2$ | $L(p_2)$ | $g_1$ | $g_2$ | $c_1$ | $c_2$ | $L(p_2)$ |
| 0.25 | 2 | - | - | - | - | - | - | - | - | - | - |
| | 4 | 5 | 4 | 1 | 2 | 0.9676 | 2 | 1 | 1 | 2 | 0.9518 |
| | 6 | ↑ | ↑ | ↑ | ↑ | 0.9914 | ↑ | ↑ | ↑ | ↑ | 0.9854 |
| | 8 | ↑ | ↑ | ↑ | ↑ | 0.9968 | ↑ | ↑ | ↑ | ↑ | 0.9940 |
| | 10 | ↑ | ↑ | ↑ | ↑ | 0.9985 | ↑ | ↑ | ↑ | ↑ | 0.9970 |
| 0.10 | 2 | - | - | - | - | - | - | - | - | - | - |
| | 4 | 48 | 47 | 2 | 3 | 0.9945 | 11 | 10 | 2 | 3 | 0.9884 |
| | 6 | 7 | 7 | 1 | 2 | 0.9824 | 3 | 2 | 1 | 2 | 0.9686 |
| | 8 | ↑ | ↑ | ↑ | ↑ | 0.9934 | ↑ | ↑ | ↑ | ↑ | 0.9873 |
| | 10 | ↑ | ↑ | ↑ | ↑ | 0.9970 | ↑ | ↑ | ↑ | ↑ | 0.9938 |
| 0.05 | 2 | - | - | - | - | - | - | - | - | - | - |
| | 4 | 59 | 58 | 2 | 3 | 0.9918 | 13 | 13 | 2 | 3 | 0.9826 |
| | 6 | 9 | 8 | 1 | 2 | 0.9753 | 3 | 3 | 1 | 2 | 0.9592 |
| | 8 | ↑ | ↑ | ↑ | ↑ | 0.9074 | ↑ | ↑ | ↑ | ↑ | 0.9835 |
| | 10 | ↑ | ↑ | ↑ | ↑ | 0.9958 | ↑ | ↑ | ↑ | ↑ | 0.9921 |
| 0.01 | 2 | - | - | - | - | - | - | - | - | - | - |
| | 4 | 85 | 84 | 2 | 3 | 0.9836 | 19 | 18 | 2 | 3 | 0.9668 |
| | 6 | 12 | 12 | 1 | 2 | 0.9561 | ↑ | ↑ | ↑ | ↑ | 0.9958 |
| | 8 | ↑ | ↑ | ↑ | ↑ | 0.9833 | 5 | 3 | 1 | 2 | 0.9733 |
| | 10 | ↑ | ↑ | ↑ | ↑ | 0.9924 | ↑ | ↑ | ↑ | ↑ | 0.9870 |

Note: The cells with hyphens(-) indicate that $g$ and $c$ cannot be found to satisfy the conditions. The cell with upward arrow(↑) indicates that the same value applies as in the upper cell.

Table 2: Two-stage group sampling plan with $r = 5$ for Weibull having $m = 1$

| $\beta$ | $\mu/\mu_0$ | $a = 0.5$ | | | | | $a = 1.0$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $= r_2$ | $g_1$ | $g_2$ | $c_1$ | $c_2$ | $L(p_2)$ | $g_1$ | $g_2$ | $c_1$ | $c_2$ | $L(p_2)$ |
| 0.25 | 2 | 23 | 21 | 3 | 4 | 0.9519 | 19 | 19 | 4 | 5 | 0.9729 |
| | 4 | 2 | 1 | 1 | 2 | 0.9539 | 2 | 1 | 1 | 2 | 0.9709 |
| | 6 | ↑ | ↑ | ↑ | ↑ | 0.9857 | 1 | 1 | 1 | 3 | 0.9686 |
| | 8 | ↑ | ↑ | ↑ | ↑ | 0.9940 | 1 | 1 | 1 | 2 | 0.9762 |
| | 10 | 1 | 1 | 0 | 2 | 0.9502 | ↑ | ↑ | ↑ | ↑ | 0.9876 |
| 0.10 | 2 | 314 | 313 | 4 | 5 | 0.9766 | - | - | - | - | - |
| | 4 | 8 | 7 | 2 | 3 | 0.9844 | 2 | 2 | 2 | 3 | 0.9621 |
| | 6 | 3 | 2 | 1 | 2 | 0.9716 | 1 | 1 | 1 | 3 | 0.9686 |
| | 8 | ↑ | ↑ | ↑ | ↑ | 0.9883 | 1 | 1 | 1 | 2 | 0.9762 |
| | 10 | ↑ | ↑ | ↑ | ↑ | 0.9942 | ↑ | ↑ | ↑ | ↑ | 0.9876 |
| 0.05 | 2 | 388 | 388 | 4 | 5 | 0.9655 | - | - | - | - | - |
| | 4 | 10 | 8 | 2 | 3 | 0.9792 | 3 | 3 | 2 | 4 | 0.9547 |
| | 6 | 4 | 2 | 1 | 2 | 0.9628 | 3 | 2 | 2 | 3 | 0.9885 |
| | 8 | ↑ | ↑ | ↑ | ↑ | 0.9845 | 2 | 1 | 1 | 2 | 0.9539 |
| | 10 | ↑ | ↑ | ↑ | ↑ | 0.9923 | ↑ | ↑ | ↑ | ↑ | 0.9756 |
| 0.01 | 2 | - | - | - | - | - | - | - | - | - | - |
| | 4 | 14 | 12 | 2 | 3 | 0.9637 | 10 | 10 | 3 | 4 | 0.9863 |
| | 6 | ↑ | ↑ | ↑ | ↑ | 0.9942 | 4 | 3 | 2 | 3 | 0.9821 |
| | 8 | 5 | 4 | 1 | 2 | 0.9723 | 2 | 1 | 1 | 2 | 0.9539 |
| | 10 | ↑ | ↑ | ↑ | ↑ | 0.9865 | ↑ | ↑ | ↑ | ↑ | 0.9756 |

Note: The cells with hyphens(-) indicate that $g$ and $c$ cannot be found to satisfy the conditions. The cell with upward arrow(↑) indicates that the same value applies as in the upper cell.

rapidly as the consumer's risk becomes smaller when the test duration is short but the increase rate is quite slow when the test duration is long.

***Example 1.*** Suppose that a manufacturer adopts the two-stage group sampling plan with $r = 5$ to

Table 3: Two-stage group sampling plan with $r = 3$ for Weibull having $m = 2$

| $\beta$ | $\mu/\mu_0$ | $a = 0.5$ | | | | | $a = 1.0$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $= r_2$ | $g_1$ | $g_2$ | $c_1$ | $c_2$ | $L(p_2)$ | $g_1$ | $g_2$ | $c_1$ | $c_2$ | $L(p_2)$ |
| 0.25 | 2 | 23 | 22 | 1 | 2 | 0.9783 | 12 | 11 | 2 | 3 | 0.9960 |
| | 4 | 4 | 3 | 0 | 1 | 0.9841 | 1 | 1 | 0 | 1 | 0.9755 |
| | 6 | ↑ | ↑ | ↑ | ↑ | 0.9966 | ↑ | ↑ | ↑ | ↑ | 0.9947 |
| | 8 | ↑ | ↑ | ↑ | ↑ | 0.9989 | ↑ | ↑ | ↑ | ↑ | 0.9983 |
| 0.10 | 2 | 33 | 33 | 1 | 2 | 0.9579 | 17 | 17 | 2 | 3 | 0.9915 |
| | 4 | 5 | 5 | 0 | 1 | 0.9699 | 2 | 1 | 0 | 1 | 0.9536 |
| | 6 | ↑ | ↑ | ↑ | ↑ | 0.9934 | ↑ | ↑ | ↑ | ↑ | 0.9896 |
| | 8 | ↑ | ↑ | ↑ | ↑ | 0.9978 | ↑ | ↑ | ↑ | ↑ | 0.9966 |
| 0.05 | 2 | 647 | 647 | 2 | 3 | 0.9953 | 21 | 21 | 2 | 3 | 0.9873 |
| | 4 | 6 | 6 | 0 | 1 | 0.9586 | 2 | 1 | 0 | 1 | 0.9536 |
| | 6 | ↑ | ↑ | ↑ | ↑ | 0.9908 | ↑ | ↑ | ↑ | ↑ | 0.9896 |
| | 8 | ↑ | ↑ | ↑ | ↑ | 0.9970 | ↑ | ↑ | ↑ | ↑ | 0.9966 |
| 0.01 | 2 | 932 | 932 | 2 | 3 | 0.9905 | 31 | 30 | 2 | 3 | 0.9747 |
| | 4 | 59 | 58 | 1 | 2 | 0.9992 | 6 | 6 | 1 | 2 | 0.9978 |
| | 6 | 9 | 8 | 0 | 1 | 0.9825 | 3 | 1 | 0 | 1 | 0.9848 |
| | 8 | ↑ | ↑ | ↑ | ↑ | 0.9941 | ↑ | ↑ | ↑ | ↑ | 0.9949 |

Note: The cell with upward arrow(↑) indicates that the same value applies as in the upper cell.

Table 4: Two-stage group sampling plan with $r = 5$ for Weibull having $m = 2$

| $\beta$ | $\mu/\mu_0$ | $a = 0.5$ | | | | | $a = 1.0$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $= r_2$ | $g_1$ | $g_2$ | $c_1$ | $c_2$ | $L(p_2)$ | $g_1$ | $g_2$ | $c_1$ | $c_2$ | $L(p_2)$ |
| | 2 | 8 | 7 | 1 | 2 | 0.9717 | 2 | 2 | 2 | 3 | 0.9851 |
| 0.25 | 4 | 2 | 2 | 0 | 1 | 0.9841 | 1 | 1 | 0 | 2 | 0.9518 |
| | 6 | ↑ | ↑ | ↑ | ↑ | 0.9966 | 1 | 1 | 0 | 1 | 0.9853 |
| | 8 | ↑ | ↑ | ↑ | ↑ | 0.9989 | ↑ | ↑ | ↑ | ↑ | 0.9951 |
| | 2 | 12 | 12 | 1 | 3 | 0.9500 | 3 | 3 | 2 | 3 | 0.9737 |
| 0.10 | 4 | 3 | 3 | 0 | 1 | 0.9681 | 1 | 1 | 0 | 2 | 0.9518 |
| | 6 | ↑ | ↑ | ↑ | ↑ | 0.9930 | 1 | 1 | 0 | 1 | 0.9853 |
| | 8 | ↑ | ↑ | ↑ | ↑ | 0.9977 | ↑ | ↑ | ↑ | ↑ | 0.9951 |
| | 2 | 81 | 81 | 2 | 3 | 0.9918 | 4 | 4 | 2 | 3 | 0.9600 |
| 0.05 | 4 | 4 | 3 | 0 | 1 | 0.9586 | 1 | 1 | 0 | 2 | 0.9518 |
| | 6 | ↑ | ↑ | ↑ | ↑ | 0.9908 | 1 | 1 | 0 | 1 | 0.9853 |
| | 8 | ↑ | ↑ | ↑ | ↑ | 0.9970 | ↑ | ↑ | ↑ | ↑ | 0.9951 |
| | 2 | 117 | 116 | 2 | 3 | 0.9848 | 18 | 17 | 3 | 4 | 0.9916 |
| 0.01 | 4 | 21 | 19 | 1 | 2 | 0.9988 | 3 | 2 | 1 | 2 | 0.9945 |
| | 6 | 5 | 5 | 0 | 1 | 0.9825 | 2 | 1 | 0 | 1 | 0.9718 |
| | 8 | ↑ | ↑ | ↑ | ↑ | 0.9941 | ↑ | ↑ | ↑ | ↑ | 0.9904 |

Note: The cell with upward arrow(↑) indicates that the same value applies as in the upper cell.

decide about the acceptance or rejection of the lot under inspection. The specified life of the product is $\mu_0 = 1000$ hours and the test duration is 500 hours. The producer's risk is 0.05 when the true mean is 4 times $\mu_0$ and consumer's risk is 0.10 when the true mean is $\mu_0$. The lifetime of this product is known to follow the Weibull distribution. In order to estimate its shape parameter, failure data were collected from 10 products of the previous lots as follows: 507, 720, 892, 949, 1031, 1175, 1206, 1428, 1538, 2083. Then, the maximum likelihood estimate of the shape parameter is obtained by $\hat{m} = 2.883$. So, let us assume that $m = 3$ now. As $r = 5$ and $m = 3$, the design parameters can be found from Table 6 and they are chosen by $(g_1, g_2, c_1, c_2) = (7, 6, 0, 1)$ since $a = 0.5$ and $r_2 = 4$. This sampling plan will be implemented as follows: the first sample of size $n_1 = 35$ should be drawn and allocated to 7 testers. If there are no failures from each tester, then accept the lot. Terminate the test and reject the lot as soon as 2 failures are recorded from any tester. If one failure is observed from any test, then

Table 5: Two-stage group sampling plan with $r = 3$ for Weibull having $m = 3$

| $\beta$ | $\mu/\mu_0$ | $a = 0.5$ | | | | | $a = 1.0$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $= r_2$ | $g_1$ | $g_2$ | $c_1$ | $c_2$ | $L(p_2)$ | $g_1$ | $g_2$ | $c_1$ | $c_2$ | $L(p_2)$ |
| 0.25 | 2 | 96 | 95 | 1 | 2 | 0.9987 | 3 | 2 | 1 | 2 | 0.9958 |
| | 4 | 8 | 7 | 0 | 1 | 0.9990 | 1 | 1 | 0 | 1 | 0.9986 |
| | 6 | ↑ | ↑ | ↑ | ↑ | 0.9999 | ↑ | ↑ | ↑ | ↑ | 0.9999 |
| 0.10 | 2 | 142 | 140 | 1 | 2 | 0.9973 | 4 | 4 | 1 | 2 | 0.9914 |
| | 4 | 11 | 11 | 0 | 1 | 0.9979 | 2 | 1 | 0 | 1 | 0.9972 |
| | 6 | ↑ | ↑ | ↑ | ↑ | 0.9998 | ↑ | ↑ | ↑ | ↑ | 0.9997 |
| 0.05 | 2 | 175 | 175 | 1 | 2 | 0.9960 | 5 | 5 | 1 | 2 | 0.9875 |
| | 4 | 14 | 13 | 0 | 1 | 0.9969 | 2 | 1 | 0 | 1 | 0.9972 |
| | 6 | ↑ | ↑ | ↑ | ↑ | 0.9997 | ↑ | ↑ | ↑ | ↑ | 0.9999 |
| 0.01 | 2 | 252 | 251 | 1 | 2 | 0.9920 | 7 | 7 | 1 | 2 | 0.9780 |
| | 4 | 20 | 19 | 0 | 1 | 0.9938 | 3 | 2 | 0 | 1 | 0.9928 |
| | 6 | ↑ | ↑ | ↑ | ↑ | 0.9994 | ↑ | ↑ | ↑ | ↑ | 0.9993 |

Note: The cell with upward arrow(↑) indicates that the same value applies as in the upper cell.

Table 6: Two-stage group sampling plan with $r = 5$ for Weibull having $m = 3$

| $\beta$ | $\mu/\mu_0$ | $a = 0.5$ | | | | | $a = 1.0$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $= r_2$ | $g_1$ | $g_2$ | $c_1$ | $c_2$ | $L(p_2)$ | $g_1$ | $g_2$ | $c_1$ | $c_2$ | $L(p_2)$ |
| 0.25 | 2 | 31 | 30 | 1 | 2 | 0.9983 | 1 | 1 | 1 | 2 | 0.9912 |
| | 4 | 5 | 4 | 0 | 1 | 0.9990 | 1 | 1 | 0 | 1 | 0.9959 |
| | 6 | ↑ | ↑ | ↑ | ↑ | 0.9999 | ↑ | ↑ | ↑ | ↑ | 0.9996 |
| 0.10 | 2 | 46 | 45 | 1 | 2 | 0.9966 | 2 | 1 | 1 | 2 | 0.9827 |
| | 4 | 7 | 6 | 0 | 1 | 0.9979 | 1 | 1 | 0 | 1 | 0.9959 |
| | 6 | ↑ | ↑ | ↑ | ↑ | 0.9998 | ↑ | ↑ | ↑ | ↑ | 0.9959 |
| 0.05 | 2 | 57 | 55 | 1 | 2 | 0.9951 | 2 | 2 | 1 | 2 | 0.9765 |
| | 4 | 8 | 8 | 0 | 1 | 0.9969 | 1 | 1 | 0 | 1 | 0.9959 |
| | 6 | ↑ | ↑ | ↑ | ↑ | 0.9997 | ↑ | ↑ | ↑ | ↑ | 0.9996 |
| 0.01 | 2 | 82 | 80 | 1 | 2 | 0.9905 | 3 | 2 | 1 | 2 | 0.9654 |
| | 4 | 12 | 11 | 0 | 1 | 0.9939 | 2 | 1 | 0 | 1 | 0.9920 |
| | 6 | ↑ | ↑ | ↑ | ↑ | 0.9994 | ↑ | ↑ | ↑ | ↑ | 0.9993 |

Note: The cell with upward arrow(↑) indicates that the same value applies as in the upper cell.

go to the second stage, where the second sample of size $n_2 = 30$ is drawn and allocated to 6 testers. Accept the lot if there are no failures from the second sample, but terminate the test and reject the lot as soon as the first failure is observed from any tester.

## 4. Comparisons with Single-Stage Group Sampling Plans

In this section we will compare the proposed plan with the ordinary (single-stage) group sampling plan in terms of the ASN and the lot acceptance probability at ARL. A sampling plan can be said to be more efficient if it requires a smaller ASN (or sample size) when the lot acceptance probability remains the same or if the lot acceptance probability becomes higher when the ASN keeps the same. The results of the single-stage group sampling plan can be obtained from Aslam and Jun (2009b). The ASN for the single-stage group sampling plan is just the sample size required, which will be $g$ times $r$.

Table 7 compares the results from two plans. This table shows the ASN and the lot acceptance probability of each plan for the consumer's risk of 0.25, the producer's risk of 0.05, $r = 5$ and $a = 0.5$ when the shape parameter of the Weibull is $m = 2$ or $m = 3$.

The two plans may not be directly compared because the ASN or the lot acceptance probability of each plan is not exactly same. But, the ASN for the two-stage plan is smaller than that for the

Table 7: Comparisons of two-stage group sampling plans with single-stage group sampling plans

| $m$ | $\mu/\mu_0 = r_2$ | Two-stage group sampling plan | | | Single-stage group sampling plan | | |
|---|---|---|---|---|---|---|---|
| | | Plan parameters | $L(p_2)$ | ASN | Plan parameters | $L(p_2)$ | ASN |
| 2 | 2 | ( 8, 7, 1, 2) | 0.9717 | 59.9 | (32, 2) | 0.9878 | 160 |
| | 4 | ( 2, 2, 0, 1) | 0.9841 | 14.7 | ( 6, 1) | 0.9913 | 30 |
| | 6 | ( 2, 2, 0, 1) | 0.9966 | 14.7 | ( 6, 1) | 0.9982 | 30 |
| 3 | 2 | (31, 30, 1, 2) | 0.9983 | 179.6 | (23, 1) | 0.9728 | 115 |
| | 4 | ( 5, 4, 0, 1) | 0.9990 | 37.4 | ( 4, 0) | 0.9726 | 20 |
| | 6 | ( 5, 4, 0, 1) | 0.9999 | 37.4 | ( 4, 0) | 0.9918 | 20 |

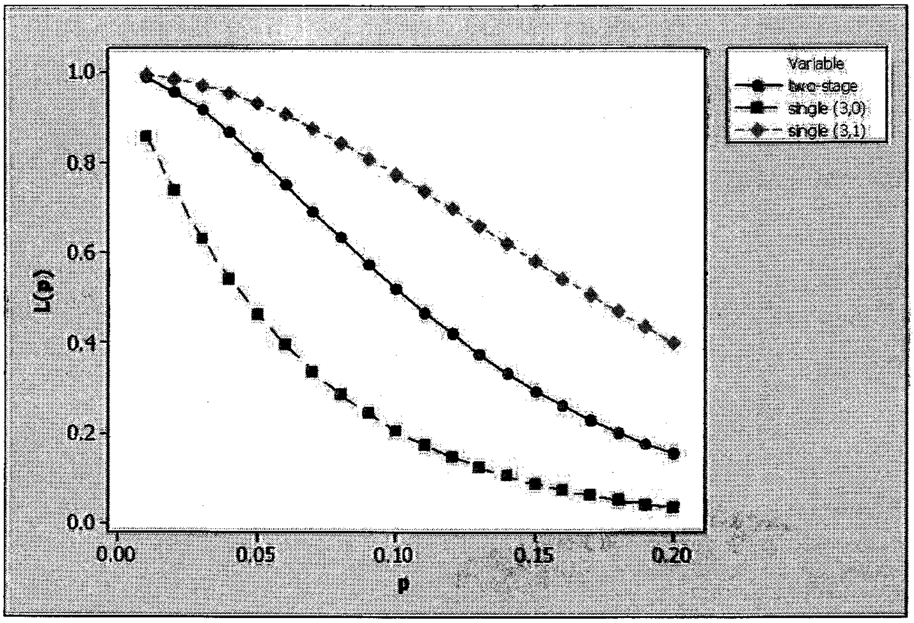


Figure 1: *OC curves of two-stage and single-stage group sampling plans*

single-stage plan when $m = 2$ although the lot acceptance probability for the single-stage plan is a little higher. When $m = 3$, the ASN for the two-stage plan is larger than that for the single-stage plan but the lot acceptance probability for the former is quite larger than the latter.

In order to compare the lot acceptance probabilities or operating characteristics(OC) of two plans having the equal sample size, we chose a case of $m = 2$, $r = 5$, $\beta = 0.25$, $r_2 = 4$ and $a = 0.5$ from Table 7. Under these conditions, the ARL is 0.012 with the producer's risk of 5% and the LTRL is 0.178 with the consumer's risk of 25%. As shown in Table 7, the two-stage group sampling plan with (2, 2, 0, 1) requires the ASN of 14.7 ($\approx$ 15), which is equivalent to 3 groups in the single-stage group sampling plan. So, we may compare the two-stage plan having (2, 2, 0, 1) and the single-stage plan having (3, 0) or (3, 1). We may choose some other cases as long as the ASN of a two-stage plan is same as the sample size of a single-stage plan. Figure 1 shows the OC curves of these three plans.

As seen in Figure 1 the OC value for the two-stage plan is larger than 0.95 at the ARL and smaller than 0.25 at the LTRL. However, the OC curve for the single-stage plan with (3, 0) is placed lower than that for the two-stage plan, whereas the OC curve for the single-stage plan with (3, 1) is placed higher. So, the single-stage plans do not meet the producer's risk or the consumer's risk.

## 5. Conclusion

A two-stage group sampling plan was proposed when the lifetime of a product is distributed as Weibull with known shape parameter. The number of groups and the acceptance number in each stage were determined so as to minimize the ASN subject to satisfying the consumer's and the producer's risks at the same time under a variety of conditions. Particularly, the ARL was expressed by the ratio of the true mean life to the specified life, so it depends on the shape parameter. Tables for the plan parameters were constructed under various combinations of the specified conditions such as shape parameters, group sizes, levels of consumer's risk and so on. It was seen from these tables that the numbers of groups required decrease as the group size increases when other conditions remain the same. The sample size also decreases as the group size increases, which means that the use of a larger group size may be more economical. It was also observed from the comparison with the single-stage sampling plans that the proposed two-stage group sampling plan performs better in terms of the average sample number and the operating characteristics.

It was assumed in this study that the shape parameter of the Weibull distribution is known or its estimate is available if it is unknown. Some future works may be needed to handle unknown shape parameter by estimating with failure data when determining the plan parameters. There may be further studies needed in order to develop a more efficient version of two-stage group sampling plans.

## Acknowledgement

The authors would like to thank the reviewers and Editor for their valuable comments which led to improve the presentation of this paper

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*Received August 2009; Accepted October 2009*