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SOME RESULTS ON FUZZY IDEAL EXTENSIONS OF BCK-ALGEBRAS

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Abstract. In this paper, we prove that the extension ideal of a fuzzy characteristic ideal of a positive implicative BCK-algebra is a fuzzy characteristic ideal. We introduce the notion of the extension of intuitionistic fuzzy ideal of BCK-algebras and some properties of fuzzy intuitionistic ideal extensions of BCK-algebra are investigated.

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [14]. Since then these ideas have been applied to other algebraic structures such as semigroups, groups and rings, etc. In 1991, Xi [12] applied the concept of fuzzy sets to BCK-algebras which are introduced by Y. Imai and K. Iséki in 1966.

Recently, Xie [13] introduced the concept of the extension of a fuzzy ideal of semigroups and investigated some properties of a fuzzy ideal of semigroups. Modifying his idea, in [4], we applied the idea to BCK-algebras. We introduced the notion of the extension of a fuzzy ideal of BCK-algebras and investigated its properties. In this paper, we show that the extension ideal of a fuzzy characteristic ideal of a positive implicative BCK-algebra is a fuzzy characteristic ideal. We introduce the notion of the extension of intuitionistic fuzzy ideal of BCK-algebras and some properties of fuzzy intuitionistic ideal extensions of BCK-algebra are investigated.

2. Preliminaries

We begin with the following well-known definitions and results which are necessary for completeness.

Definition 2.1. An algebra $(X; *, 0)$ of type (2,0) is called a *BCK-algebra* if for all $x, y, z \in X$ the following conditions hold:

(a) $((x * y) * (x * z)) * (z * y) = 0$,
(b) $(x * (x * y)) * y = 0$,
(c) $x * x = 0$,

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(d) $0 * x = 0$,
(e) $x * y = 0$ and $y * x = 0$ imply $x = y$.

For any BCK-algebra X, the relation $\leq$ defined by $x \leq y$ if and only if $x * y = 0$ is a partial order on X.

A BCK-algebra X has the following properties for all $x, y, z \in X$:

(1) $x * 0 = x$,
(2) $(x * y) * z = (x * z) * y$,
(3) $x * y \leq x$,
(4) $x \leq y$ implies $x * z \leq y * z$ and $z * y \leq z * x$.

A mapping $f : X \to Y$ of BCK-algebras is called a *homomorphism* if $f(x * y) = f(x) * f(y)$ for all $x, y \in X$. A bijective homomorphism on X is called an *automorphism* on X. Let $\mathrm{Aut}(X)$ denote the set of all automorphisms of a BCK-algebra X.

In what follows, X would mean a BCK-algebra unless otherwise specified.

Definition 2.2. ([3]) X is said to be *positive implicative* if it satisfies for all x, y and z in X,

$$(x * z) * (y * z) = (x * y) * z.$$

Definition 2.3. ([2]) A nonempty subset I of X is called an *ideal* of X if

(I1) $0 \in I$,
(I2) $x * y \in I$ and $y \in I$ imply $x \in I$, for all $x, y \in X$.

We now review some fuzzy logic concepts. A *fuzzy subset* μ in a set S is a function from S into $[0, 1]$. For any fuzzy subsets μ and ν of S, we define

$$\mu \subseteq \nu \Leftrightarrow \mu(x) \leq \nu(x) \text{ for all } x \in S.$$

A fuzzy subset μ in X is called a *fuzzy subalgebra* of X if

$$\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$$

for all $x, y \in X$.

Definition 2.4. ([12]) A fuzzy subset μ in X is called a *fuzzy ideal* of X if

(FI1) $\mu(0) \geq \mu(x)$ for all $x \in X$,
(FI2) $\mu(x) \geq \min\{\mu(x * y), \mu(y)\}$ for all $x, y \in X$.

Lemma 2.5. ([5]) *If μ is a fuzzy ideal of X and if $x \leq y$, then $\mu(x) \geq \mu(y)$.*

3. Fuzzy ideal extensions

Definition 3.1. ([4]) Let μ be a fuzzy subset of X and $a \in X$. The fuzzy subset $\langle \mu, a \rangle : X \longrightarrow [0, 1]$ defined by

$$\langle \mu, a \rangle(x) := \mu(x * a)$$

is called the *extension* of μ by a.

Proposition 3.2. ([4]) *Let μ be a fuzzy ideal of X. If $a \leq b$, then $\langle \mu, a\rangle \subseteq \langle \mu, b\rangle$.*

Definition 3.3. ([6]) If μ is a fuzzy ideal of X and θ is a map from X into itself, we define a map $\mu^{\theta} : X \to [0,1]$ by $\mu^{\theta}(x) = \mu(\theta(x))$ for every $x \in X$.

Definition 3.4. ([6]) A fuzzy ideal μ of X is called a *fuzzy characteristic ideal* of X if $\mu(\theta(x)) = \mu(x)$ for all $x \in X$ and all $\theta \in \mathrm{Aut}(X)$.

Definition 3.5. Let $\theta : X \to X$ be a homomorphism. The set

$$\mathrm{Fix}_{\theta}(X) = \{x \in X \mid \theta(x) = x\}$$

is called the *fixed point set* of X with respect to θ. The set

$$\mathrm{Fix}(X) = \cap_{\theta \in \mathrm{Aut}(X)} \mathrm{Fix}_{\theta}(X)$$

is called the *fixed point set* of X.

For every homomorphism θ, the fixed point set $\mathrm{Fix}_{\theta}(X)$ of X with respect to θ is nonempty.

Example 3.6. Let $X = \{0, a, b, c\}$ in which $*$ is defined by :

$*$	0	a	b	c
0	0	0	0	0
a	a	0	0	0
b	b	a	0	a
c	c	a	a	0

Then $(X; *, 0)$ is a BCK-algebra ([9]). Let 1_X be an identity homomorphism on X, and let f be the function defined by

$$f(0) = 0, f(a) = a, f(b) = c \text{ and } f(c) = b.$$

Then we can find that $\mathrm{Aut}(X) = \{1_X, f\}$. Thus, $\mathrm{Fix}_{1_X}(X) = X$ and $\mathrm{Fix}_f(X) = \{0, a\}$. Hence, the fixed point set of X is $\mathrm{Fix}(X) = \{0, a\}$.

Proposition 3.7. *Let μ be a fuzzy subset of X and let $\theta : X \to X$ be a homomorphism. Then $\langle \mu^{\theta}, a\rangle = \langle \mu, \theta(a)\rangle^{\theta}$, for all $a \in X$.*

Proof. Let $\theta : X \to X$ be a homomorphism and let $a \in X$. For all $x \in X$, we have

$$\begin{aligned}
\langle \mu^{\theta}, a\rangle(x) &= \mu^{\theta}(x * a) \\
&= \mu(\theta(x * a)) \\
&= \mu(\theta(x) * \theta(a)) \\
&= \langle \mu, \theta(a)\rangle(\theta(x)) \\
&= \langle \mu, \theta(a)\rangle^{\theta}(x).
\end{aligned}$$

Hence $\langle \mu^{\theta}, a\rangle = \langle \mu, \theta(a)\rangle^{\theta}$, completing the proof. $\square$

Lemma 3.8. *Let μ be a fuzzy ideal of a positive implicative BCK-algebra X and let $\theta : X \to X$ be an onto homomorphism and $a \in X$. Then $\langle \mu, a \rangle^{\theta}$ is a fuzzy ideal of X.*

Proof. It follows from [6] and [4] that $\langle \mu, a \rangle^{\theta}$ is a fuzzy ideal of X. □

Theorem 3.9. *Let μ be a fuzzy ideal of a positive implicative BCK-algebra X. If μ is a fuzzy characteristic ideal of X and $a \in Fix(X)$, then the extension $\langle \mu, a \rangle$ of μ by a is a fuzzy characteristic ideal of X.*

Proof. Let μ be a fuzzy characteristic ideal of X and let $a \in \mathrm{Fix}(X)$. Let θ be any automorphism of X. Since μ is a fuzzy ideal of X, it follows from Lemma 3.8 that $\langle \mu, a \rangle^{\theta}$ is a fuzzy ideal of X. Since μ is a fuzzy characteristic ideal of X, we have $\mu^{\theta} = \mu$, that is, $\mu^{\theta}(x) = \mu(\theta(x)) = \mu(x)$, for all $x \in X$. It follows that

$$\begin{aligned}
\langle \mu, a \rangle^{\theta}(x) &= \langle \mu, a \rangle(\theta(x)) \\
&= \mu(\theta(x) * a) \\
&= \mu(\theta(x * a)) \\
&= \mu^{\theta}(x * a) \\
&= \mu(x * a) \\
&= \langle \mu, a \rangle(x),
\end{aligned}$$

for all $x \in X$. Hence, $\langle \mu, a \rangle^{\theta} = \langle \mu, a \rangle$. Thus, $\langle \mu, a \rangle$ is a fuzzy characteristic ideal of X. This completes the proof. □

Example 3.10. Let $X = \{0, a, b, c\}$ in which $*$ is defined by :

$*$	0	a	b	c
0	0	0	0	0
a	a	0	0	0
b	b	b	0	b
c	c	c	c	0

Then $(X; *, 0)$ is a positive implicative BCK-algebra ([9]). Let θ be the function defined by

$$\theta(0) = 0, \theta(a) = a, \theta(b) = c \text{ and } \theta(c) = b.$$

Then we can easily show that $\theta \in \mathrm{Aut}(X)$. Moreover, we find that $\mathrm{Aut}(X) = \{1_X, \theta\}$, where 1_X is the identity automorphism on X. Since $\mathrm{Fix}(X) = \{0, a\}$, we have $b \notin \mathrm{Fix}(X)$. Let μ be the fuzzy subset in X defined by

$$\mu(0) = 0.9, \mu(a) = 0.7, \mu(b) = 0.3 \text{ and } \mu(c) = 0.5.$$

Then it is a fuzzy ideal of X. It follows from [4] that $\langle \mu, b \rangle$ is also a fuzzy ideal of X. But, $\langle \mu, b \rangle$ is not a fuzzy characteristic ideal of X, since $\langle \mu, b \rangle^{\theta}(c) = 0.9 \neq 0.5 = \langle \mu, b \rangle(c)$.

Definition 3.11. ([8]) Let X and Y be two sets, let $f : X \to Y$ be a map. A fuzzy subset μ in X is said to be *f-invariant* if $f(x) = f(y)$ implies $\mu(x) = \mu(y)$ for all $x, y \in X$.

Theorem 3.12. *Let μ be a fuzzy subset of a BCK-algebra X and let $f : X \to X$ be a homomorphism. If μ is f-invariant, then $\langle \mu, a \rangle$ is f-invariant, for all $a \in X$.*

Proof. Suppose that μ is an f-invariant fuzzy subset of X. Let $f : X \to X$ be a homomorphism, and let $x \in X$. If $f(x) = f(y)$, then $f(x * a) = f(x) * f(a) = f(y) * f(a) = f(y * a)$. Since μ is f-invariant, we have $\mu(x * a) = \mu(y * a)$. It follows that $\langle \mu, a \rangle(x) = \langle \mu, a \rangle(y)$. Thus, $\langle \mu, a \rangle$ is f-invariant, for all $a \in X$. □

Corollary 3.13. *Let $f : X \to X$ be a homomorphism on a positive implicative BCK-algebra X and let μ be an f-invariant fuzzy ideal of X and $a \in X$. Then the extension $\langle \mu, a \rangle$ of μ by a is an f-invariant fuzzy ideal of X.*

Proof. It follows from [4] that $\langle \mu, a \rangle$ of μ by a is a fuzzy ideal of X. By the above theorem, $\langle \mu, a \rangle$ is f-invariant, and the result follows. □

Definition 3.14. ([10]) A nonempty subset Q of X is called a *quasi-left (resp. quasi-right) ideal* of X if

(QI1) $0 \in Q$,

(QI2) if $x \in Q$ and $y \in X$, then $y \wedge x \in Q$ (resp. $x \wedge y \in Q$).

Moreover, Q is called a *quasi-ideal* of X if it satisfies the left and right conditions.

Note that if X is a BCK-algebra, then $\{0\}$ and X are quasi-ideals of X.

Definition 3.15. ([10]) A fuzzy subset μ of X is called a *fuzzy quasi-left (resp. quasi-right) ideal* in X if

(FQI1) $\mu(0) \geq \mu(x)$, for all $x \in X$,

(FQI2) $\mu(x \wedge y) \geq \mu(y)$ (resp. $\mu(x \wedge y) \geq \mu(x)$), for all $x, y \in X$.

Moreover, μ is called a *fuzzy quasi-ideal* in X if it satisfies the fuzzy quasi-left and quasi-right conditions.

Note that if X is a commutative BCK-algebra, then the fuzzy quasi-left ideal and the fuzzy quasi-right ideal both coincide to fuzzy quasi-ideal.

Definition 3.16. ([11]) A nonconstant fuzzy quasi-left (resp. quasi-right) ideal of X is called a *fuzzy prime left (resp. right) quasi-ideal* of X if

$$\mu(x \wedge y) = \max\{\mu(x), \mu(y)\} \text{ (resp. } \mu(y \wedge x) = \max\{\mu(x), \mu(y)\}),$$

for all $x, y \in X$.

Theorem 3.17. *Let μ be a fuzzy quasi-left (resp. quasi-right) ideal of a positive implicative BCK-algebra X. Then $\langle \mu, a\rangle$ is also a fuzzy quasi-left (resp. quasi-right) ideal of X, for all $a \in X$.*

Proof. Let μ be a fuzzy quasi-left ideal of X, and let $a \in X$. If $x \in X$, then

$$\begin{aligned}\langle \mu, a\rangle(0) &= \mu(0 * a)\\ &= \mu(0)\\ &\geq \mu(x * a)\\ &= \langle \mu, a\rangle(x).\end{aligned}$$

Next, let $x, y \in X$. Since X is positive implicative, we have

$$\begin{aligned}\langle \mu, a\rangle(x \wedge y) &= \mu((x \wedge y) * a)\\ &= \mu((x * a) \wedge (y * a))\\ &\geq \mu(y * a)\\ &= \langle \mu, a\rangle(y).\end{aligned}$$

Thus, $\langle \mu, a\rangle$ is a fuzzy quasi-left ideal of X. Similarly, if μ is a fuzzy quasi-right ideal of X, then so is $\langle \mu, a\rangle$, for all $a \in X$. ☐

The converse of Theorem 3.17 need not be true, as shown by the following example.

Example 3.18. Let $(X; *, 0)$ be as in Example 3.10. Let μ be the fuzzy subset in X defined by

$$\mu(0) = 0.9, \mu(a) = 1, \mu(b) = 0.8 \text{ and } \mu(c) = 1.$$

Then it is a routine to verify that $\langle \mu, c\rangle$ is a fuzzy quasi-left ideal of X. But, μ is not a fuzzy quasi-left ideal of X, since $\mu(0) \ngeq \mu(a)$.

Theorem 3.19. *Let μ be a fuzzy prime quasi-left (resp. quasi-right) ideal of a positive implicative BCK-algebra X and let $a \in X$. Then $\langle \mu, a\rangle$ is also a fuzzy prime quasi-left (resp. quasi-right) ideal of X, if $\langle \mu, a\rangle$ is nonconstant.*

Proof. Let μ be a fuzzy quasi-left ideal of X and let $a \in X$. Assume that the fuzzy subset $\langle \mu, a\rangle$ is nonconstant. Theorem 3.17 says that $\langle \mu, a\rangle$ is a fuzzy quasi-left ideal of X. Let $x, y \in X$. Since μ is a fuzzy prime quasi-left ideal of X, we have

$$\begin{aligned}\langle \mu, a\rangle(x \wedge y) &= \mu((x \wedge y) * a)\\ &= \mu((x * a) \wedge (y * a)) \quad \text{since } X \text{ is positive implicative}\\ &= \max\{\mu(x \wedge a), \mu(y \wedge a)\}\\ &= \max\{\langle \mu, a\rangle(x), \langle \mu, a\rangle(y)\}.\end{aligned}$$

Thus, $\langle \mu, a\rangle$ is a fuzzy prime quasi-left ideal of X. ☐

Example 3.20. Let $(X; *, 0)$ be as in Example 3.10. Let μ be the fuzzy subset in X defined as follows :

$$\mu(0) = \mu(a) = \mu(b) = 1 \text{ and } \mu(c) = 0.5.$$

Then it is a fuzzy prime quasi-left ideal of X. It is easily check that

$$\langle \mu, c\rangle(x \wedge y) = \max\{\langle \mu, c\rangle(x), \langle \mu, c\rangle(y)\},$$

for all $x, y \in X$. But the fuzzy subset $\langle \mu, c\rangle$ is constant.

4. Intuitionistic fuzzy ideals extensions

Definition 4.1. ([1]) An *intuitionistic fuzzy set* (briefly, IFS) in a nonempty set X is a pair (μ, γ) such that the functions $\mu : X \to [0, 1]$ and $\gamma : X \to [0, 1]$ satisfy

$$0 \leq \mu(x) + \gamma(x) \leq 1, \text{ for all } x \in X.$$

Definition 4.2. ([7]) An IFS (μ, γ) in X is called an *intuitionistic fuzzy subalgebra* of X if it satisfies:

(IS1) $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$,
(IS2) $\gamma(x * y) \leq \max\{\gamma(x), \gamma(y)\}$,

for all $x, y \in X$.

Definition 4.3. ([7]) An IFS (μ, γ) in X is called an *intuitionistic fuzzy ideal* of X if it satisfies

(IF1) $\mu(0) \geq \mu(x)$ and $\gamma(0) \leq \gamma(x)$,
(IF2) $\mu(x) \geq \min\{\mu(x * y), \mu(y)\}$,
(IF3) $\gamma(x) \leq \max\{\gamma(x * y), \gamma(y)\}$,

for all $x, y \in X$.

Theorem 4.4. ([7]) *Every intuitionistic fuzzy ideal of X is an intuitionistic fuzzy subalgebra of X.*

Lemma 4.5. ([7]) *Let (μ, γ) be an intuitionistic fuzzy ideal of X. Then μ is order-reversing and γ is order-preserving.*

Definition 4.6. Let (μ, γ) be an IFS in X and $a, b \in X$. The IFS $\langle(\mu, \gamma), (a, b)\rangle$ defined by

$$\langle(\mu, \gamma), (a, b)\rangle = (\langle \mu, a\rangle, \langle \gamma, b\rangle)$$

is called the *extension* of (μ, γ) by (a, b). If $a = b$, then we denote it by $\langle(\mu, \gamma), a\rangle$.

Lemma 4.7. *Let (μ, γ) be an IFS in X and $a, b \in X$. Then the extension $\langle(\mu, \gamma), (a, b)\rangle$ of (μ, γ) by (a, b) is an IFS in X.*

Proof. It is straightforward. □

Theorem 4.8. *Let (μ, γ) be an intuitionistic fuzzy subalgebra of a positive implicative BCK-algebra X and $a, b \in X$. Then the extension $\langle(\mu, \gamma), (a, b)\rangle$ of (μ, γ) by (a, b) is also an intuitionistic fuzzy subalgebra of X.*

Proof. Let (μ, γ) be an intuitionistic fuzzy subalgebra of a positive implicative BCK-algebra X and $a, b \in X$. Let $x, y \in X$. Then we have

$$\begin{aligned}\langle\mu, a\rangle(x * y) &= \mu((x * y) * a) \\ &= \mu((x * a) * (y * a)) \\ &\geq \min\{\mu(x * a), \mu(y * a)\} \\ &= \min\{\langle\mu, a\rangle(x), \langle\mu, a\rangle(y)\}\end{aligned}$$

and

$$\begin{aligned}\langle\gamma, b\rangle(x * y) &= \gamma((x * y) * b) \\ &= \gamma((x * b) * (y * b)) \\ &\leq \max\{\gamma(x * b), \gamma(y * b)\} \\ &= \max\{\langle\gamma, b\rangle(x), \langle\gamma, b\rangle(y)\}.\end{aligned}$$

Thus, $\langle(\mu, \gamma), (a, b)\rangle$ is an intuitionistic fuzzy subalgebra of X. □

Theorem 4.9. *Let (μ, γ) be an intuitionistic fuzzy ideal of a positive implicative BCK-algebra X and $a, b \in X$. Then the extension $\langle(\mu, \gamma), (a, b)\rangle$ of (μ, γ) by (a, b) is an intuitionistic fuzzy ideal of X.*

Proof. Since (μ, γ) is an intuitionistic fuzzy ideal of X, we have $\langle\mu, a\rangle(0) = \mu(0 * a) = \mu(0) \geq \mu(x * a) = \langle\mu, a\rangle(x)$ and $\langle\gamma, b\rangle(0) = \gamma(0 * b) = \gamma(0) \leq \gamma(x * b) = \langle\gamma, b\rangle(x)$ for all $x \in X$. Thus, $\langle(\mu, \gamma), (a, b)\rangle$ satisfies condition (IF1) of definition 4.2. Next, let $x, y \in X$. Then

$$\begin{aligned}\langle\mu, a\rangle(x) &= \mu(x * a) \\ &= \min\{\mu((x * a) * (y * a)), \mu(y * a)\} \\ &\geq \min\{\mu((x * y) * a), \mu(y * a)\} \\ &= \min\{\langle\mu, a\rangle(x * y), \langle\mu, a\rangle(y)\}\end{aligned}$$

and

$$\begin{aligned}\langle\gamma, b\rangle(x) &= \gamma(x * b) \\ &\leq \max\{\gamma((x * b) * (y * b)), \gamma(y * b)\} \\ &= \max\{\gamma((x * y) * b), \gamma(y * b)\} \\ &= \max\{\langle\gamma, b\rangle(x * y), \langle\gamma, b\rangle(y)\}.\end{aligned}$$

Therefore, $\langle(\mu, \gamma), (a, b)\rangle$ is an intuitionistic fuzzy ideal of X. This completes the proof. □

Corollary 4.10. *Let (μ, γ) be an intuitionistic fuzzy subalgebra (resp. ideal) of a positive implicative BCK-algebra X and $a \in X$. Then the extension $\langle(\mu, \gamma), a\rangle$ of (μ, γ) by a is an intuitionistic fuzzy subalgebra (resp. ideal) of X.*

Proof. It is straightforward. □

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