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Research on the Factors Influencing Middle School Teachers' Mathematics Pedagogical Content Knowledge[1]

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It is the development of a mathematics teachers' teaching knowledge that manifests a mathematics teacher's professional knowledge growth. It is becoming a direct and effective approach of conversion of mathematical knowledge into the knowledge of mathematics teaching. Through the investigation, the study revealed that the knowledge conversion process of mathematics teachers in middle school is restricted by three aspects including eight factors. From this point, the authors have structured the path and model on influencing factors of middle school Mathematics Teaching Knowledge Conversion (MPCK), and discuss the mechanism of the transformation process.

Keywords: knowledge of mathematics teaching, Mathematics Pedagogical Content Knowledge (MPCK), knowledge translating, influencing factors, teacher professional development
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1. INTRODUCTION

Mathematics Pedagogical Content Knowledge, short for MPCK, is an important manifestation of professional mathematics teacher. It is an important characteristic for

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distinguishing mathematics teachers from mathematicians, mathematics teachers and other subject teachers. A new round of curriculum reform put forward a number of new requirements on teaching knowledge. In the process of curriculum implementation, the development of teachers' mathematics teaching knowledge is far from satisfaction. It is because the lacks of MPCK that some teachers even are at loss on how to prepare lessons according to the new teaching materials. What's more, in pre and post-service teacher education in mathematics, the development of knowledge of mathematics teaching doesn't pay enough attention. And how to develop prospective teachers' mathematics teaching knowledge becomes a kind of "missing paradigm" (Shulman, 1986). After-service mathematics teacher training the majority of in-service mathematics teachers are based on years of teaching experience to develop mathematics teaching knowledge, and lack the support of theory. Therefore, based on the fact that our mathematic teachers' specialized and solid knowledge of the long-term learning of mathematics, it is necessary and essential for us to study on how to make mathematic teachers' Mathematics Knowledge converse into MPCK , especially on its influencing factors and their degree (Here in after referred to as "knowledge transformation").

2. RESEARCH DESIGN

On the hypothesis that expert teachers have more significant and higher level than the new teachers, and the analytical framework in light of subject knowledge transferring subject teaching knowledge theory proposed by American scholar Shulman (1986; 1987). The author took the middle school mathematics teachers as subject and conducted a survey by questionnaire to explore the influencing factors of teachers' knowledge conversion and their degree.

2.1. The theoretical framework of research design

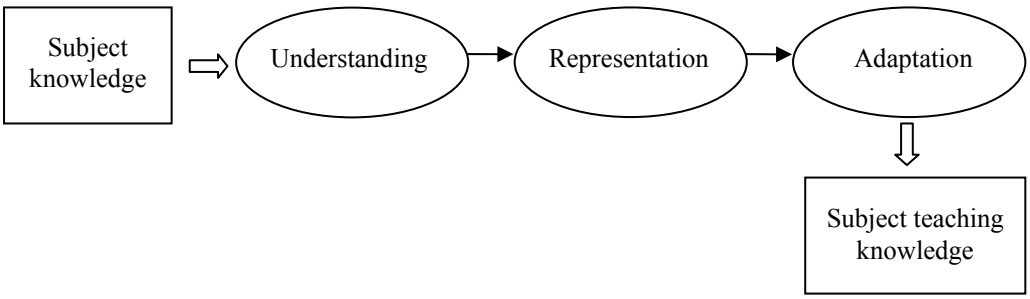

Figure 1. The process of knowledge transformation

Shulman (1987) divided the process that subject knowledge changes into subject teaching knowledge into three parts, *i.e*., understanding, representation, and adaptation as shown in Figure 1.

Of all them, understanding refers to teachers' cognitive for a mathematical knowledge which involves the depth, breadth and perforation of understanding. Representation mainly refers to teachers' outside show and expression to math knowledge during teaching; it is a kind of teachers' external performance of their inherent knowledge. Adaptation here means to whether the teachers' usage of the representation adjusts to students on the course of teachers' teaching; whether it is useful to reflect the nature of mathematical knowledge.

2.2. The Practical Basis of Study Design

In order to effectively study the impacted factors of mathematics teachers' knowledge transformation, the author selected six mathematics teachers (three new hand teachers, three expert teachers) as case study based on the above analysis framework to have a discussion on the influencing factors of middle school mathematics teachers' knowledge translating. Through classroom observation, video encoding and depth interviews to six teachers' teaching about "translation and rotation of graphics," "the nature explosion of quadrilateral" and "a function" three units, the author get an initial conclusion that teachers' knowledge, mathematical concept, teaching efficacy, mathematics teaching experience and other factors.

2.3. Investigation and analysis of influencing factors on middle mathematics teachers' knowledge transformation

Taking the above conclusion of cases study as a framework, the author designs questionnaire and uses the quantitative analysis to discuss about the influencing factors on junior mathematics teachers' knowledge transformation and the influencing degree of the factors.

2.3.1. The investigation instrument

The investigation instrument we employed was self-made questionnaire, which made up by the test of knowledge changing level and teachers' MPCK influencing factors. As for the test questionnaire of knowledge changing level, it consists of four teaching scenarios tasks and its content is based on the established way of teaching contexts problem (Ma, 1999, pp. 120–150). Namely, alternating statements from the English to Chinese, in order to be conducive to the understanding of our teachers; alternating content from primary mathematics to junior mathematics, in order to adapt the needs of this study.

2.3.2. The process of investigation

After finishing the initial questionnaire, the final questionnaire was formed by consulting the relevant experts, amending the questionnaire, choosing some teachers to test, and then amending again. Then, we took 2 provincial key middle schools, 2 city key middle schools, 2 urban general middle schools and 2 rural general schools in Chong Qing, a city of China, as school sample. The test questionnaire should be finished about 30 minutes; the questionnaire of influencing factors should be finished about 40 minutes; the questionnaire should be immediately recovered when teachers completed them. In order to ensure the reliability, we use three-choice mark form and choose plural to the test questionnaire of teachers' knowledge translating level; Regarding to influencing factors questionnaire, we analysis its reliability and validity by statistical software SPSS12.0; 0.90 is the questionnaire's internal consistency coefficient α; its construct validity is 0.82, which proves that this questionnaire has good reliability and validity and it can be used for research work.

3. THE RESULTS OF THE STUDY

3.1. The factors influencing on Mathematics Middle School Teachers' Knowledge Transformation

The four questions of teaching contexts of middle school math teachers' knowledge transformation in test questionnaire on the basis of the model of 'explanation—representation—adaptation,' examine the subjects' transformation of knowledge. In order to know the teachers' level of knowledge conversion from the point view of quantity, we try to give marks to teachers' answer. The total scores of the four questions indicate all tested teachers' level of knowledge translating. The full score is 20. By statistical analysis, the investigated teachers' scores of knowledge translated level are as follow:

Minimum score 7 points; highest score 20 points; the average score 13 points.

By the correlation analysis of two variables, it can be concluded that there are eight factors influencing teachers' knowledge level: Scope, Profundity, Perforation, mathematics on instrument, mathematics on problem, feelings, feelings of teaching, and math teaching experiences. The concrete relevant degree is presented in Table 1. The data shows that all the factors have a positive relevance to teachers' knowledge transformation level and the correlation coefficients all reach to the level significance level ($p > 0.05$).

Table 1. Significance of the factors of teachers' knowledge transformation

Eight factors	Level of knowledge transformation	
	Correlation modulus (r)	Significant level (Sig.)
Teachers' understanding scope	0.706^{**}	0.000
Profundity	0.609^{**}	0.003
Perforation of teaching knowledge	0.837^{**}	0.000
Mathematics concept on instrumentalism orientation	0.472^{*}	0.04
Mathematics concept on problem solving orientation	0.674^{**}	0.005
General teaching efficacy	0.652^{**}	0.001
Personal teaching efficacy	0.777^{**}	0.000
Mathematics teaching experiences	0.820^{**}	0.000

Note: *: Means reaching the significant level $p > 0.05$.
**: Means reaching the significant level $p > 0.01$.

3.2. The analysis of the degree of middle school mathematic teachers' knowledge transformation

(1) The regression analysis of every factor on teachers' knowledge transform

To study the cause and effect connection of every factor to teachers' knowledge transform, we take the eight factors as independent variable, the level of teachers' knowledge transform as factor, and set up multi-regression equation by using the method of stepwise enter.

Table 2. Analyses of every factor on teachers' knowledge transformation

Predictor	Regression coefficient (R)	R^2	Adding R^2	Sig.
P	0.837	0.701	0.701	**
P/Q	0.668/0.319	0.774	0.073	**
P/Q/E	0.360/0.305/0.393	0.829	0.055	**

Note: P: Teachers' understanding perforation of mathematics knowledge
Q: Conception of mathematics on problem solving orientation
E: Mathematics teaching experiences.
**: Means reaching the significant level $p < 0.01$.

From Table 2, we can found that this research only take three variables into regression equation, they are teachers' understanding perforation of teaching knowledge, conception of mathematics on problem solving orientation and mathematics teaching experiences. This regression model indicates that the level of teachers' knowledge transform is directly

affected by teachers' understanding perforation of teaching knowledge, conception of mathematics on problem solving orientation and mathematics teaching experiences.

The results show: teachers' understanding perforation of mathematics knowledge accounts 70% for information transformation. Regression gets significant meaning, and partial regression coefficient is 0.837; Adding as variable into regression equation has two effects. For one thing, it turns partial regression coefficient of understanding perforation of mathematics knowledge from 0.837 to 0.668. For another, it makes partial regression coefficient of conception of mathematics on problem solving orientation towards 0.319. The two variables' explanation of the level of knowledge transformation arrive at 77.4%; Finally, adding mathematics teaching experiences into regression equation leads to the partial regression coefficient of understanding perforation of mathematics knowledge 0.360, the coefficient of conception of mathematics on problem solving orientation 0.305 and the coefficient of mathematics teaching experiences 0.393 meanwhile three variables' R2 of the level of teachers' knowledge transform becomes 82.9%.

Therefore, by taking teachers' understanding perforation of teaching knowledge, conception of mathematics on problem solving orientation, and mathematics teaching experiences as variables, their standard partial regression coefficient turns out to be 0.360, 0.305, and 0.393 respectively. So we can find the standardized regression equation:

$$Y = 0.360Z_1 + 0.305Z_2 + 0.393Z_3$$

Y stands for the level of teachers' knowledge transform, Z_1 for teachers' understanding perforation of teaching knowledge, Z_2 for teachers' conception of mathematics on problem solving orientation, and Z_3 for teachers' mathematics teaching experiences.

(2) The route analyses of the every factor to the level of transformation

To acquaint ourselves more with the influencing degree and route between every affecting factor and teachers' knowledge transform, we analysis these eight affecting factors and the level of teachers' knowledge transform respectively, altogether nine elements, to gain standard partial regression coefficient and find the cause and effect route model by applying the method of analyzing route (Figure 2).

According to the route model of middle school teachers' knowledge transformation, the evident factors of the level of teachers' knowledge transform can be found as follows: teachers' mathematics teaching experiences (0.393), teachers' understanding perforation of teaching knowledge (0.360) teachers' Conception of mathematics on problem solving orientation (0.305); The effect degree of these direct factors to teachers' mathematics teaching experiences are teacher' personal feelings of teaching efficiency (0.529), and common feelings of teaching efficiency (0.454); Although -teachers' understanding scope, profundity of teaching knowledge, teacher' personal feelings of teaching efficiency and

common feelings of teaching efficiency do not effect the level of teachers' knowledge transform directly, there are some domino effects.

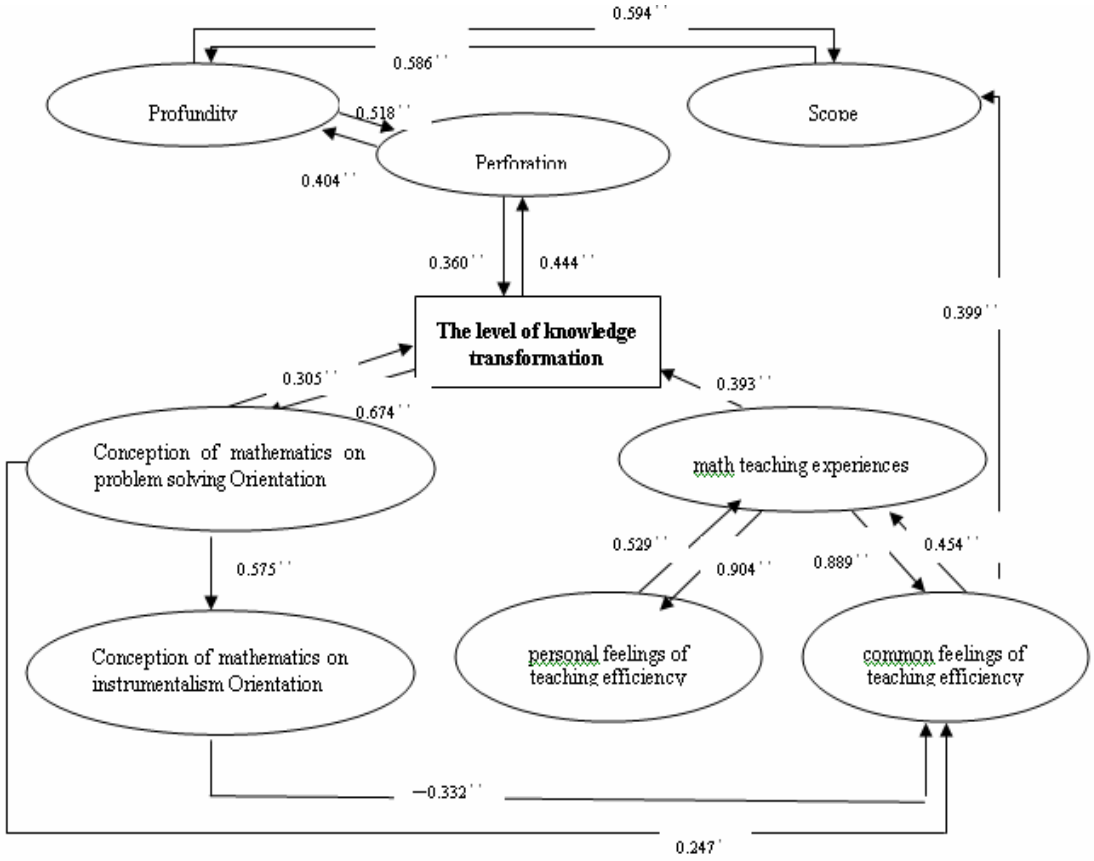

Figure 2. Route model of middle school teachers' knowledge transformation

Teachers' understanding profundity of mathematics knowledge indirectly affect the level of teachers' knowledge transformation through teachers' understanding perforation of teaching knowledge; teachers' understanding scope of teaching knowledge through the route – teachers' understanding profundity of teaching knowledge → teachers' understanding perforation of teaching knowledge; personal feelings of teaching efficiency through teachers' mathematics teaching experiences; teachers' common feelings of teaching efficiency through teachers' mathematics teaching experiences and understanding scope of teaching knowledge → understanding profundity of teaching knowledge → understanding perforation of teaching knowledge.

The sum of the direct effect and the indirect one are called the general effect of variable on factor. So we can find the general effect of every factor in route model on the level of teachers' knowledge transform (see Table 3).

Table 3. The general effect analysis of every factor to the level of teachers' knowledge transformation

Eight factors	Total effect
Teachers' understanding scope	0.2761
Profundity	0.3078
Perforation of teaching knowledge	0.360
Mathematics concept on instrumentalism orientation	0.061
Mathematics concept on problem solving orientation	0.643
General teaching efficiency	0.3502
Personal teaching efficiency	0.3553
Mathematics teaching experiences	0.705

Shown in Table 3, the influencing data of the general effect of every different factor in route model on the level of teachers' knowledge transform are: teachers' mathematics teaching experiences (0.705), teachers' conception of mathematics on problem solving orientation (0.643), teachers' understanding perforation of teaching knowledge teachers' personal feelings of teaching efficiency (0.3553), teachers' common feelings of teaching efficiency (0.3502), teachers' understanding profundity of teaching knowledge (0.3078), their understanding scope of teaching knowledge (0.2761), teachers' conception of mathematics on instrumentalism orientation (0.061).

4. CONCLUSION AND DISCUSSION

4.1. The pedagogical schema—the representation of the mathematics teachers' essential knowledge transform

The research results show that there are great differences between new teachers and specialized ones as far the level of knowledge transform. The essential difference between new and specialized teachers in the progress of knowledge transformation lies in the different teaching schema of mathematics knowledge explored by them. (Qian Xu-sh eng & Tong Li, 2009)However, the schema organizes related information and experiences and composes certain frame and construction in their mind according to some teaching of mathematics knowledge. The schema is created to solve various complex or ill-constructed problems in the lengthy process of teachers' engaging in teaching about this topic. And it is organized with the center of a certain teaching topic. Also, it varies along with the changing class, then be continually modified and improved, and

reproduces new schema (Wang, 2005, pp. 47–48).

4.2. Three system model—knowledge of mechanisms transformation of middle school mathematics teachers

Learning from the research result, the eight factors of middle school mathematics teachers' knowledge transform (teachers' understanding scope, profundity, perforation of teaching knowledge, conception of mathematics on instrumentalism orientation, conception of mathematics on problem solving orientation, common feelings of teaching efficiency, personal feelings of teaching efficiency, and mathematics teaching experiences) can be divided into three interactive systems: basic system, motive system, and efficiency system. Among them, basic system refers to teachers' understanding on mathematics knowledge.

In the process of teachers' knowledge transformation, teachers' knowledge construction possesses foundational action and developmental value. Teachers' knowledge construction are based on a certain mathematics knowledge conformed to other knowledge such as pedagogic, psychology, course and educational environment and so on. As an integrative and effective system, it mainly uses some knowledge in other field to understand mathematics knowledge deeply and act as a transform foundation; motive system refers to teachers' belief. Belief is a conjunction of sense and sensibility.

The teachers' belief mentioned here mainly refers to teachers' conception of mathematics, conception of mathematics teaching and learning, and the understanding and faith about their own teaching ability. Namely on the answers on the questions like these: "what is mathematics?", "how to carry through the teaching and learning of mathematics?", "whether my mathematics teaching is effective?" and so forth. Correct teaching belief advance the transformation from mathematics knowledge to mathematics teaching knowledge effectively, while some false belief and altitude restrict it severely. Efficiency system refers to teachers' teaching experiences. It mainly included teaching cases and students' mathematics study experience and other else (Tong, 2009).

All in all, the process of teachers' knowledge transformation is a united process which takes teachers' teaching experiences as an activator and based on the understanding on mathematics knowledge and promoted by teachers' belief.

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