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Changing Students' Conceptions of Mathematics through the Introduction of Variation[1]

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Some 400 Secondary One (*i.e.* seventh-grade) students from 10 schools were provided with non-routine mathematical problems in their normal mathematics classes as exercises for one academic year. Their attitudes toward mathematics, their conceptions of mathematics and their problem-solving performance were measured both in the beginning and at the end of the year. Hierarchical regression analyses revealed that the introduction of an appropriate dose of non-routine problems would generate some effects on the students' conceptions of mathematics. A medium dose of non-routine problems (as reported by the teachers) would result in a change of the students' conception of mathematics to perceiving mathematics as less of "a subject of calculables." On the other hand, a high dose would lead students to perceive mathematics as more useful and more as a discipline involving thinking. However, with a low dose of non-routine problems, students found mathematics more "friendly" (free from fear). It is therefore proposed that the use of non-routine mathematical problems to an appropriate extent can induce changes in students' "lived space" of mathematics learning and broaden their conceptions of mathematics and mathematics learning.

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INTRODUCTION

Understanding what affects students' learning has been a perennial question in education and has been revisited time and again from different paradigms and perspectives under different background assumptions and constraints. Numerous efforts have been made, resulting in the generation of innumerable publications even if they are confined to the subject of mathematics. More and more attention has been given in recent years to the relationship between modes of mathematics learning outcomes of mathematics learning, and beliefs about mathematics (Leder, Pehkonen & Törner, 2002). In fact, previous studies have revealed that the beliefs about mathematics as a discipline, about mathematics learning, about mathematics teaching and about the self situated in a social context in which mathematics is taught and learned are all closely related to a student's motivation to learn and his performance in the subject (Cobb, 1985; McLeod, 1992; Pehkonen & Törner, 1998).

Mathematics is often seen by students as a body of absolute truth (Fleener, 1996) and a set of rules for playing around with symbols (Clay & Kolb, 1983; Kloosterman, 1991; McLeod, 1992). In the United States, a study has revealed that 83% and 50% of the seventh-grade students agreed or strongly agreed respectively with the statements: "There is always a rule to follow in mathematics" and "Learning mathematics is mostly [memorizing]" (Dossey, Mullis, Lindquist & Chambers, 1988). Following an investigation of junior high school students, Frank (1988) came to the conclusion that, in students' eyes:

(a) Mathematics is computation;
(b) Mathematics problems should be solved in less than five steps (or else something is wrong with either the problem or the student);
(c) The goal of doing a mathematical problem is to obtain the correct answer;
(d) The role of the student is to receive mathematical knowledge and to demonstrate that it has been received; and
(e) In the teaching-learning process, the student is passive while the teacher is active.

Since there is always one correct way and/or rule to be followed to solve a mathematical problem, the task of the problem-solver is to search for such a way/rule to solve the problem (Carpenter, Lindquist, Silver & Matthews, 1983).

The similar general feature that students generally possess a narrow conception of mathematics was found in Hong Kong and the Chinese mainland. Students in these regions take mathematics as "a subject of calculables"; they believe that mathematics involves thinking (mathematics as a mental exercise) and is useful (in daily life and other disciplines) (Lam, Wong & Wong, 1999; Wong, 2002). Such a narrow conception of mathematics has been found to constrain students' mathematics learning and their approaches to solving mathematical problems. Students who see mathematics as a set of rules may assume that rote-memorization of facts, rules and procedures of stereotypical problems is an appropriate learning strategy, thus resulting in a lower level of understanding (Confrey, 1983; Peck, 1984; Underhill, 1988).

Naturally, such a classroom mathematics culture has been generated by a number of factors, among which how the teacher shapes the mathematics classroom is a prominent one. We call this the "lived space" of mathematics learning (Wong, Marton, Wong & Lam, 2002). In fact, teachers' conceptions of mathematics are frequently reflected in their teaching acts (Pehkonen & Törner, 1998; Shirk, 1972; Thompson, 1992). Those who see doing mathematics as following sets of procedures invented by others will provide their students with little opportunity for making sense out of mathematics (Battista, 1994). Some teachers think that students understand mathematics when they can successfully follow procedural instructions. The day-to-day experience of this "lived space", which is essentially shaped by the teacher (and the curriculum), influences students' conceptions of mathematics and how they approach mathematical problems. In fact, it was revealed that a great majority of the mathematical problems given to Hong Kong students by their teachers are close-ended and stereotyped, demanding only low-level cognitive skills (Wong, Lam & Chan, 2002). If most of the mathematical examples and problems that students meet in their daily mathematics classes exist in a restricted "lived space", they there is a good chance they will develop a narrow conception of mathematics. As such, students might be less successful on non-routine and Open- ended problems.

Despite the extensive literature on students' conceptions of mathematics and their relations with mathematics performance, few studies have examined how they can be changed. This is precisely the aim of the present study.

Revisiting the earlier findings of Marton & Booth (1997) and Bowden & Marton (1998) leads us to the conclusion that a way of experiencing a phenomenon can be characterized in terms of those aspects of the phenomenon that are discerned and kept in focal awareness by the learner (see also Runesson, 1999). Thus discernment is an essential element to learning and variation is crucial to bringing about discernment. The lack of variation in the "lived space" of mathematics learning experienced by students would inevitably lead to a relatively narrow conception of mathematics. Furthermore, they would tend to hold a narrow conception of mathematics learning and would possess limited strategies when they are confronted with mathematical problems. In a nutshell, less variation is associated with narrower ways of experiencing a phenomenon and more variation is associated with wider ways of experiencing that phenomenon. Such a restricted conception may be the outcome of a “lived space” shaped by teachers who have not introduced enough variation in mathematics teaching. The systematic introduction of non-routine problems should help in widening the “lived space” of mathematics learning; students could acquire broader conceptions of mathematics and become more capable mathematical problem-solvers (Wong, Marton, Wong & Lam, 2002).

Thus, it is logical to hypothesize that students’ conceptions of mathematics would be broadened if their “lived space” of mathematics learning were widened by systematically introducing variation. They will become more capable problem-solvers too. The aim of the present research is, therefore, to explore the effects of systematically introducing more non-routine questions to students.

The aims of the present study are:

1. Enhance secondary school students’ capabilities of dealing with mathematical problems, word problems and open problems;
2. Cultivate among the students more effective approaches to solving mathematical problems; and
3. Examine if students’ conceptions of mathematics can thereby be broadened;

by letting students, in the mathematics class:

(a) engage in more varied and more open tasks;
(b) a simpler word would be explain to the teacher and to their fellow students their ways of solving mathematical problems; and
(c) compare and reflect on their own and others’ methods of going about mathematical learning.

The results of the students’ problem-solving performances were reported in Wong, Chiu, Wong & Lam (2005). In general, it was found that the exposure to non-routine problems helped improve the students’ performance in solving open-ended problems but the appropriate type of problems depends on the academic standards of the students. In brief, students from schools with higher academic standards benefit from high and medium doses of non-routine tasks whereas those from schools with lower academic standard only benefit from medium and low doses. In this paper, we will be reporting on the effects the introduction of non- routine problems will have on the students’ conceptions of mathematics.

METHODOLOGY

Participants

The new Secondary One (*i.e.* seventh-grade) mathematics curriculum is the target curriculum content for our project. On promotion to the secondary level, students can more easily accept new ways of doing mathematics. Furthermore, the introduction of a variety of student activities is in line with the new 2001 mathematics curriculum implemented in Hong Kong. A Secondary One class each from 10 schools were chosen to take part in this study, among which 3 were "high academic standard schools" (mainly admitting students who scored top one-third in the aptitude test at the end of their primary schooling), 4 were "medium standard" ones and the remaining 3 were "low standard" ones.

Apart from the experimental group, there was a "reference group" of eight other classes for the purpose of comparison. There were 410 students in the experimental group and 275 students in the reference group. By and large, the same amount of time was used in all the classrooms for teaching mathematics. Furthermore, the textbooks used and the topics for consideration were similar. Background information regarding academic standards in general, those of mathematics in particular, attitudes toward mathematics and conceptions of mathematics were collected. The teachers in the reference group did not overlap with teachers in the experimental group. The former conducted their lessons as what they used to do without the introduction of non-routine problems.

Procedure

After having established shared views about the pedagogy of variation in the group of participating teachers through joint study activities and discussions, we designed mathematical tasks for the 10 classrooms in the experimental group by considering the different ways in which students dealt with mathematics, the ways they discussed their views and strategies with one another and their ways of reflecting on and learning from such discussions.

First, exemplars of non-routine mathematical problems were given to the teachers and they designed more of such problems afterwards. Non-routine mathematical problems are not frequently encountered by students or found in the standard textbooks. Such non-routine tasks may include mathematical problems that:

(a) Appear in unfamiliar formats;
(b) Have more than one answer;
(c) Allow openness in the solving process;

(d) Contain missing or irrelevant data;
(e) can be solved by a variety of means (for instance, by algebraic means, geometric means, graphical methods, concrete objects, calculators and micro-computers);
(f) involve problem-posing.

The problems provided include open problems (problems that possess openness either in the "given," "process," or "goal"; see Becker, 1998), problems found in overseas textbooks, problems found on the Internet and even problems found in local textbooks that appear very different from those used in individual schools. Problem-posing here refers to both the formulation of new problems and the conversion of existing problems into new ones. These problems were naturally incorporated into the day-to-day teaching so that there was no need to alter the curriculum which the teachers were required to follow. Examples of non-routine problems used can be found in the Appendix.

Around half of all the tasks used in each classroom were taken from a common pool of tasks constructed by a number of teachers. These tasks were more varied and included a greater number of open problems than usual.

This proves that variation in the students' learning experience was produced through the openness of each task and the variation between tasks. This was achieved by:

(a) the designing of more open tasks by individual teachers;
(b) shared tasks developed by different teachers;
(c) reflection during sharing sessions within the teacher groups; or
(d) sharing and discussion among the students of different solutions offered by others in the classroom.

In this way, the students encountered not only different tasks authored by other teachers but also different solutions offered by other students to the same problem. As a result, the "lived space" of variation of the students' mathematical experience was expected to be broadened, as was that of the teachers.

At the end of the project, the teachers reported the extent of use of such non-routine problems. Low use of the non-routine problems occurred in 3 classes, medium use in 4 and high use in 3 classes. In classes of both groups, were accessed both at the beginning and toward the end of the academic year based on the cognitive learning outcomes of the students in computational problems, word problems and open problems.

Instruments

Conceptions of Mathematics.

The Conception of Mathematics Scale, a 5-point Likert scale, consists of 9 questions on "mathematics is a subject of calculables", 9 questions on "mathematics involves thinking" and 9 questions on "mathematics is useful". The scale was developed locally by grounded research and prior application of the scale has yielded excellent reliability indices (Wong, Lam & Wong, 1998). The reliability indices of the scale obtained in the present project would be reported in the "Results" section.

Attitudes toward Mathematics. Aiken's (1974) Mathematics Attitude Scale comprises of the four subscales of "enjoyment", "motivation", "importance of mathematics" and "free from fear." Each of these four subscales consists of 6 items. An additional subscale of "mathematics self-concept," which consists of 8 items, from Self Description Questionnaire - I (Marsh, 1992), was adapted for this purpose of measuring attitudes. All the items were on a 5-point scale (from 1 to 5: strongly disagree, disagree, fairly agree, agree, strongly agree). Sample items of the above four subscales of Mathematics Attitude Scale include respectively, "I have usually enjoyed studying mathematics in school", "I want to develop my mathematical skills and study this subject more", "Mathematics is a very worthwhile and necessary subject" and "I don't get upset while trying to solve mathematics problems."

Minato's (1983) Mathematics Semantic Differential was used in the questionnaire to tap students' attitudes toward mathematics. It consists of 14 bipolar statements on a 6-point response scale. Sample items include "School mathematics is (simple → complicated)" and "School mathematics is (beautiful → ugly)."

Problem-solving Items

Problem-solving items with openness in either (a) the given information, (b) the goals to be achieved or (c) the solving process (Becker, 1998; Silver & Mamona, 1989) were used in assessing students' performances in problem-solving. While doing research for this report, the following types of open problems were used:

(a) problems with irrelevant information (Low & Over, 1989);
(b) "problematic word problems" (Verschaffel, Greer & DeCorte, 2000);
(c) problem which allow more than one solution;
(d) problem which allow multiple methods;
(e) problems with different possible interpretations of the question;
(f) problems asking for communication;
(g) problems involving judgment;

(h) problems involving decision-making.

The scoring of open problems was developed according to the rubrics established in Cai, Lane & Jakabcsin (1996).

Statistical Analysis

The major concern of this study is whether students' conceptions of mathematics can be affected through the introduction of non-routine problems in classroom teaching. First, the psychometric properties of the affective measures of mathematics learning were examined by a reliability test (*Cronbach alpha*). Then the intervention effects of the experimental design were examined using the hierarchial regression analysis. Once the hierarchical has been set, ordinary least squares regressions were used to model predictors of these measures (conceptions of mathematics and mathematics self-concept) at the end of the year (Cohen & Cohen, 1983). Entering predictors in hierarchical sets allows the estimation of the portion of variance among student scores explained by each set of predictors. The sets of predictor variables were entered mostly in chronological order into a hierarchical regression. Predicting the outcome-variable post-intervention measures with the pre-intervention measures can give an effective estimation of the effects of later predictors on students' conceptions of mathematics and mathematics self-concept. Using the difference between the two scores assumes inevitably that the pre-intervention score has a unitary effect on post-intervention score on the post – intervention score. Student gender is entered next as it is not affected by any of the later variables. Students' average academic standards of schools exist prior to the type of intervention, so it is entered before the intervention type.

RESULTS

The reliability indices of the measures were summarized in Table 1. The results show that the Cronbach alpha fell between 0.66 and 0.94. Indicating that the instruments had satisfactory internal consistencies. Also, the instruments were generally stable, except for those on conceptions of mathematics, particularly when they were administered to the reference group.

Hierarchical regression analysis was conducted on every dimension of the affective measures including Conception of Mathematics Scale (mathematics is a subject of calculables; mathematics involves thinking; and mathematics is useful), Mathematics Attitude Scale (enjoyment, motivation, importance and free from fear), Mathematics Semantic Differential and mathematics self-concept. Among the 9 dimensions, 4 of them yield significant differences between the experimental group and the reference group.

Table 1. The Cronbach Alpha Reliability Indices of the Instruments Used in the Pre-test and Post-test

Subscales	Experimental group		Reference group	
	Pre-test	Post-test	Pre-test	Post-test
Enjoyment	0.88	0.88	0.80	0.82
Motivation	0.83	0.86	0.77	0.80
Importance	0.78	0.78	0.76	0.78
Free from fear	0.89	0.89	0.81	0.83
Self-concept	0.92	0.91	0.89	0.88
Calculable	0.66	0.70	0.70	0.65
Thinking	0.69	0.82	0.78	0.79
Useful	0.89	0.87	0.85	0.88
Attitude	0.94	0.94	0.92	0.94

Table 2. Regression Models Predicting Students' Ratings on the Conception "Mathematics Is a Subject of Calcuables"

Predictor	Students' ratings on the conception "mathematics is a subject of calculables"							
	Model 1		Model 2		Model 3		Model 4	
Intercept	1.836	***	1.845	***	1.843	***	1.960	***
	(0.121)		(0.123)		(0.125)		(0.131)	
Pre-intervention measure	0.398	***	0.394	***	0.394	***	0.382	***
	(0.036)		(0.036)		(0.036)		(0.037)	
Girl			0.005		0.004		0.020	
			(0.043)		(0.043)		(0.044)	
Hi_standard					0.022		0.001	
					(0.057)		(0.064)	
Med_standard					–0.004		–0.063	
					(0.049)		(0.063)	
Hi_use							–0.101	
							(0.065)	
Med_use							–0.167	**
							(0.060)	
Low_use							0.008	
							(0.065)	
R-squared	0.158		0.156		0.156		0.169	

Note: Standard errors are in parentheses. * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$.

They included "mathematics is a subject of calculables", "mathematics involves thinking", "mathematics is useful" and "free from fear." In addition, a gender difference in the growth of mathematics self-concept was also found.

First of all, we focus on the subscale "mathematics is a subject of calculables" (Table 2). Model 1 shows the effects of pre-intervention measure (0.398) on the post-intervention measure of the students' ratings on the conception "mathematics is a subject of calculables." Out of all the effects of the pre – intervention measure, gender and academic standard had no net effects on the post-intervention score (Models 2 and 3 respectively). The number of teachers self-reported use of open problems showed a small non-linear effect (see Model 4). Students tended to see mathematics less as a subject of calculables if the teacher reported medium use of open problems in their classes.

Table 3. Regression Models Predicting Students' Ratings on the Conception "Mathematics Involves Thinking"

Predictor	Students' ratings on the conception "mathematics involves thinking"							
	Model 1		Model 2		Model 3		Model 4	
Intercept	2.096	***	2.049	***	2.034	***	2.107	***
	(0.148)		(0.149)		(0.150)		(0.153)	
Pre-intervention measure	0.425	***	0.426	***	0.416	***	0.407	***
	(0.039)		(0.039)		(0.039)		(0.040)	
Girl			0.082		0.067		0.045	
			(0.048)		(0.048)		(0.049)	
Hi_standard					0.118		0.037	
					(0.064)		(0.072)	
Med_standard					0.087		–0.021	
					(0.055)		(0.071)	
Hi_use							0.166	*
							(0.074)	
Med_use							–0.061	
							(0.067)	
Low_use							0.091	
							(0.071)	
R-squared	0.157		0.166		0.171		0.180	

Note: Standard errors are in parentheses.
* $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$.

We have similar findings for the subscale "mathematics involves thinking" (Table 3). Model 1 shows the effects of the pre-intervention measure (0.425) on the post-intervention measure of the students' ratings on the conception "mathematics involves thinking." As can be seen from the results, gender and academic standard had no net effects on the post-intervention score (Models 2 and 3 respectively). The number of teachers self-reported use of open problems showed a small non-linear effect (Model 4). Students tended to see mathematics as a subject that involves thinking if the teacher reported a high use of open problems in their classes.

The findings for the subscale "mathematics is useful" are similar too (Table 4). Model 1 shows the effects of the pre-intervention measure (0.297) on the post-intervention measure of the students' ratings on the conception "mathematics is useful." Based on the research, gender had insignificant effect on the post-intervention scores (Model 2). However, when comparing equally able students with similar pre-treatment score (controlling for both the effects of academic standard and pre-intervention score), girls reported a slightly lower ratings on the feelings that "mathematics is useful" in the post-intervention measure (Model 3). Model 4 shows that students tended to see mathematics as more useful if the teacher reported a high use of open problems in their classes.

Table 4. Regression Models Predicting Students' Ratings on the Conception "Mathematics Is Useful"

Predictor	Students' ratings on the conception "mathematics is useful"							
	Model 1		Model 2		Model 3		Model 4	
Intercept	2.231	***	2.267	***	2.258	***	2.306	***
	(0.125)		(0.129)		(0.130)		(0.132)	
Pre-intervention measure	0.297	***	0.297	***	0.292	***	0.284	***
	(0.038)		(0.039)		(0.039)		(0.039)	
Girl			–0.065		–0.073	*	–0.091	*
			(0.035)		(0.036)		(0.037)	
Hi_standard					0.068		0.015	
					(0.047)		(0.053)	
Med_standard					0.034		–0.033	
					(0.040)		(0.052)	
Hi_use							0.124	*
							(0.054)	
Med_use							–0.028	
							(0.049)	
Low_use							0.056	
							(0.053)	
R-squared	0.084		0.090		0.093		0.102	

Note: Standard errors are in parentheses. * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$.

As for the students' rating on "free from fear" (Table 5), Model 1 shows the effects of the pre-intervention measure (0.227) on the post-intervention measure of the students' ratings on "free from fear." Based on the findings/results, gender and academic standard had no net effects on the post-intervention score (Models 2 and 3 respectively). Students tended to see mathematics as something less scaring (+0.130) if the teacher reported a low use of open problems in their classes. However, after studying the effects of the pre-intervention score, gender and amount of open problems in class, more medium-ability students tended to report slightly negative feelings (–0.115) compared to the low-ability students in their ratings on "free from fear."

Table 5. Regression Models Predicting Students' Ratings on "Free from Fear"

Predictor	Students' ratings on "free from fear"							
	Model 1		Model 2		Model 3		Model 4	
Intercept	2.179	***	2.160	***	2.178	***	2.206	***
	(0.110)		(0.111)		(0.112)		(0.114)	
Pre-intervention measure	0.227	***	0.231	***	0.231	***	0.225	***
	(0.038)		(0.038)		(0.038)		(0.038)	
Girl			0.006		0.011		0.014	
			(0.031)		(0.031)		(0.031)	
Hi_standard					0.007		–0.063	
					(0.041)		(0.046)	
Med_standard					0.004		–0.115	*
					(0.036)		(0.046)	
Hi_use							0.054	
							(0.048)	
Med_use							–0.026	
							(0.043)	
Low_use							0.130	**
							(0.047)	
R-squared	0.052		0.054		0.057		0.070	

Note: Standard errors are in parentheses. * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$.

Finally, when we come to mathematics self-concept, the results showed that there were no significant differences in mathematics self-concept between students in the reference group and the experimental group (Table 6). However, the results revealed a net gender effect (negative for female students) on students' mathematics self-concept even after controlling the effects of the pre-intervention self-concept. In other words, Secondary One female students tended to have an additional decline in mathematics self-concept, over and above their already lower self-concept, compared to that of the male students in their first year of secondary schooling.

Table 6. Regression Models Predicting Students' Ratings on Mathematics Self-Concept

Predictor	Students' ratings on mathematics self-concept							
	Model 1		Model 2		Model 3		Model 4	
Intercept	0.944	***	1.154	***	1.157	***	1.214	***
	(0.087)		(0.097)		(0.101)		(0.107)	
Pre-intervention measure	0.618	***	0.593	***	0.595	***	0.598	***
	(0.030)		(0.030)		(0.030)		(0.031)	
Girl			–0.258	***	–0.255	***	–0.259	***
			(0.053)		(0.053)		(0.055)	
Hi_standard					–0.030		–0.064	
					(0.069)		(0.078)	
Med_standard					–0.002		–0.066	
					(0.060)		(0.066)	
Hi_use							0.004	
							(0.080)	
Med_use							–0.125	
							(0.073)	
Low_use							–0.001	
							(0.079)	
R-squared	0.399		0.423		0.424		0.426	

Note: Standard errors are in parentheses. * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$.

DISCUSSIONS

The major focus of this study is to examine the potential effects of introducing non-routine problems in classroom teaching on students' attitude towards mathematics. A number of affective measures of mathematics learning are adopted including Conception of Mathematics Scale (mathematics is a subject of calculables; mathematics involves thinking; and mathematics is useful), Mathematics Attitude Scale (enjoyment, motivation, importance and free from fear), Mathematics Semantic Differential and mathematics self-concept.

The results revealed that the introduction of an appropriate dose of non-routine problems would achieve those educational goals we expected in the study. A medium dose (as reported by the teachers) of non-routine problems would result in the change of students' conception of mathematics to perceiving mathematics less as "a subject of calculables." On the other hand, a high dose would lead the students to perceive mathematics as more of a discipline that involves thinking and be more useful. However, with the low use of non-routine problems, students found mathematics more “friendly” (free from fear). Thus, the results support the findings for the students’ problem- solving abilities experiment done in the same study, as reported in Wong, Chiu, Wong, and Lam (2004). In general, the broadening of a student’s mathematical “lived space” by the introduction of variation (through the use of non-routine problems) generates expected educational outcomes but, one needs to moderate the extent at which variation is introduced. In the case of problem-solving abilities, we found that teaching students to do open problems generally improved their abilities to solve open problems but the appropriate amount depends on their school standard. Students in high-standard schools benefit from high and medium doses of open problem-solving while a low dose may not be sufficient. In contrast, students in low-standard schools benefit from medium and low doses rather than high doses. Undoubtedly, it is not sensible to think that the greater the extent of implementation of a certain initiative, the greater the effect will be the effect.

From the results obtained above, we observe that though medium to high doses of non-routine problems can change students’ conceptions of mathematics to a desirable direction, there is a possibility of creating discomfort among them (if we look at the results on the subscale “free from fear”). This is further reflected in interviews conducted with students participating in this study most students who were not good at mathematics disliked open problems and their interest in learning mathematics even declined after the experimental phase. The interview showed that the non-routine problems were very challenging to the less able students, and even created frustration among them. This negative feeling, if prolonged, could become a demotivating force (Lam, Wong & Wong, 2004). It is reasonable to speculate that there could be an optimal level of dosage and it is possible that the level could have an individual difference.

In contrast to what we have found for problem-solving abilities (where girls tend to learn more about solving non-routine problems), there were no significant gender differences found for these affective variables except the subscale “mathematics is useful.” With many studies on the relationship between gender and mathematics being done in the last decade, it is found that there has been a narrowing down in gender differences in mathematics learning in recent years (Leder, 1992). The results which show girls experienced a lesser impact on the subscale “mathematics is useful” is in line with results in previous studies that proves females hold less functional beliefs about themselves as learners than do males (Leder, Forgasz & Solar, 1996). Thus usefulness of mathematics may not be a major concern among girls.

In conclusion, the present study has demonstrated that by familiarizing students with a more diverse variety of mathematical problems in their day-to-day classroom learning, their conceptions of mathematics were changed. They found mathematics as less of “a subject of calculables," as more of a discipline that involves thinking, more useful and more friendly. How one could optimize such effect could be something that is worth further exploration.

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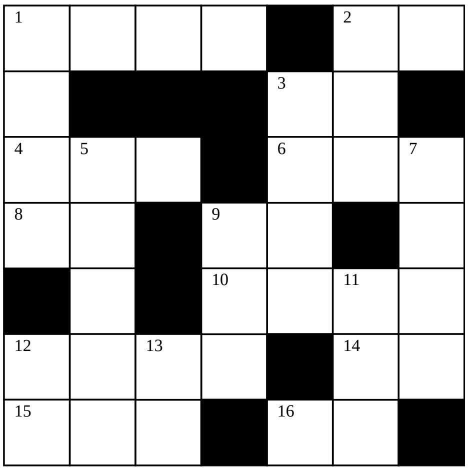
APPENDIX: EXAMPLES OF NON-ROUTINE PROBLEMS

Crossnumber



Across:

1. 1786 rounded to the nearest thousand
2. 53 rounded to the nearest ten
3. Weight in grams of a slice of bread
4. Approximate answer of 81 × 7.9
6. 800.86 rounded to the nearest whole number
8. Weight in kilograms of an average man
9. 37.34 rounded to the nearest whole number
10. 2536.875 rounded to the nearest ten
12. 6082.999 rounded to the nearest ten
14. Weight of a slice of bread to the nearest 10g
15. Number of bricks in a wall with 12 rows of bricks and 50 bricks in each row.
16. Weight in grams 3 slices of bread

Down:

1. 2467.32 rounded to the nearest whole number
2. 511 rounded to the nearest ten
3. 25 × 155
5. Approximate answer to 89.7 × 503
7. Total weight in kg of 16 average men
9 .Approximate answer to 4.35 × 79.72
12. 3 × 22
13.Approximate answer to 3.835 × 23

(Modified from an overseas textbook which was then translated into Chinese)

Jenny responded to this advertisement and was offered a retainer of $640 per month, $140 per month car allowance, plus 1.5% commission on any houses she sells.

WANTED

REAL ESTATE

SALES CONSULTANTS

2 positions, must be licenced, currently employed/or looking for a start, must have completed full schooling.

Further details, George or Milli

(0096) 15426666

If during her first month of work for her new company, she sold four houses valued at $121 000, $162 000, $87 000, and $196 000 respectively, determine her total income for the month.

(Taken from an overseas textbook which was then translated into Chinese)

[Each of the nine animals below worth different amounts, can you find out which is the highest and which is the lowest? The way of finding the answer is simple. The value on the top of each column is the sum of the values of all the animals in that column and the value at the left is the total of the values of all the animals in that row.]