

# Temporal Price Reduction as Cooperative Price Discrimination

## 협력적 가격차별 수단으로서의 일시적 가격할인

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This paper considers a duopoly where switching costs exist. The analysis proves that temporal price reductions can be pure strategy equilibrium where firms earn more profit than in a regular price strategy. Greater profits result from price discrimination in temporal price reductions. The equilibrium is contrasted with previous studies, which explain temporal price reductions as a result of mixed strategy. In a given model with an assumption about forming switching cost, firms can control their range of loyal consumers by properly setting their regular and promotional prices. The model shows that temporal price reduction tends to raise the regular price and decrease the range of loyal consumers.

Key words: Price discrimination, Temporal promotions, Switching cost, Loyalty

### I. Introduction

Many researchers have raised the question of why manufacturers and retailers prefer offering substantial price reductions for short periods and then raising the price to its normal level rather than reducing the price permanently. This kind of periodic price

reduction is observable in many industries or distribution channels: as such, there have been countless studies on this phenomenon. Richards (2006) classified the rationales of periodic price reduction into five types. Of these, two are related with price discrimination, while the other three include coping with demand uncertainties,

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trigger strategies designed to support a collusive oligopoly implicitly, and introductory pricing.<sup>1)</sup>

This paper deals with periodic price reduction as a tool to discriminate the price offered to either loyal or non-loyal consumers. In many studies dealing with loyalty or information (e.g., Varian 1980, 1981; Narasiman 1988; Raju, Srinivasan, and Ral 1990; Baye and Morgan 2001), the main concern is competition focused on non-loyal or informed consumers, and they viewed periodic price reduction as a result of mixed strategy equilibrium.<sup>2)</sup> When rivals offer promotional price reduction, firms want to offer the regular price to secure a higher margin from loyal consumers. Conversely, when rivals do not offer promotions, firms want to offer promotions to secure a larger market share. However, the previous researches assert that rivals can exploit a predictable strategy and an unpredictable mixed strategy should be adopted.

The existence and proliferation of periodic price reduction can be observed from many empirical studies, Villas-Boas (1995), Baye, Morgan and Scholten (2004),

and Lach (2002) among others. However, the empirical analyses only show temporal price reduction itself and do not fully verify the unpredictability of pricing as we will see in section 2. Rao (1991) stated that the mixed strategy argument of promotion have no avowed economic purpose other than ensuring equilibrium. Temporal price reductions can be thought to occur under implicit approval of rivals. With this approval, they can avoid price wars. Furthermore, with alternating temporal price reduction, firms can extend their sales to non-loyal or price sensitive consumers.

With this line of thought, this paper tries to derive pure strategy equilibrium, which explains temporal price reduction, with the assumption of implicit cooperation. Lal (1990) showed a similar perspective. His study proved that two national brands could earn more profit against a private brand by offering temporal promotion in alternating way in repeated cooperative or non-cooperative game. He called this the alternating price strategy. However, Lal assumed a very restrictive situation. Furthermore, in Lal's study, the aim of

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1) A trigger strategy is a class of strategies employed in a repeated non-cooperative game. A player using a trigger strategy initially cooperates but punishes the opponent if a certain level of defection (i.e., the trigger) is observed.

2) In mixed strategy, firms select their prices probabilistically, contrastingly to pure strategy in which firms' actions are deterministically set.

promotion is to prevent the private brands from encroaching into the target market of national brands, which cannot be applied to situations where only similar firms compete. Therefore, we need a more generalized model to explain phenomenon found in empirical studies.<sup>3)</sup>

The model used in this paper considers a duopoly with a general demand function. Firms implicitly cooperate with respect to promotions. However, even if they compete in regular pricing, pure equilibrium with temporal price reductions can occur when they cooperate only with promotion. This should be noted as an aspect distinguishing the model in this paper from Lal (1990). Hence, a cooperative situation in this paper is not so far from competition. In addition, even when firms are fully non-cooperative, temporal price reduction can be equilibrium considering trigger strategy.

The situation where firms can control their range of loyal consumers is dealt with in this paper. In the studies of Varian (1980, 1981), Narasiman (1988), and Raju, Srinivasan, and Lal (1990), the number of loyal consumers and their reservation price are exogenously given and fixed. It is similar in Baye and Morgan (2001). Hence,

changes in the optimal price for loyal consumers arising from the introduction of temporal price reductions are not assessed. However, the range of loyal consumers generally varies according to the level of price. For example, even if a consumer prefers a specific firm, he/she should search for lower prices and become a shopper when the price of the firm is higher than his/her reservation price. When we assume that firms can control the range of loyal consumers with the level of the regular price, the introduction of temporal price reduction will affect the optimal price for loyal consumers. In this model, the existence of temporal price reduction tends to raise the regular price and decrease the range of loyal consumers.

In allowing firms to control their range of loyal consumers, it is assumed that consumers become loyal if they consecutively buy the same firm's product for several periods. If a consumer becomes loyal, switching cost occurs when he/she moves to another firm. These processes of forming loyalty and switching costs are generally observed in the real world. The existence of learning cost, uncertainty about quality, or the psychological aspect

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3) As Richards (2006) mentioned, some previous studies, such as Green and Porter (1984) and Rotemberg and Saloner (1986), viewed temporal price reduction as a trigger strategy designed to support a collusive oligopoly implicitly. However, in these studies, temporal price reduction is seen as a repetitive observation of collusion and competition.

of consumers can cause this kind of switching cost.<sup>4)</sup>

With this assumption, *alternating price strategy* (APS), defined in Lal (1990), is compared with the case where firms maintain a single *regular price strategy* (RPS). The model shows that APS can be more profitable than RPS and that it can be adopted in cooperative situations as well as in non-cooperative situations. The only conditions for APS is the lowest willingness-to-pay of loyal consumers should be more than a certain level. Therefore, APS can be adopted in general situation, which can be an explanation of temporal price reduction. The basic rationale for the fact that APS can be more profitable than RPS is the attribute of price discrimination in APS where loyal consumers alternately pay regular and promotional prices, and, on average, pay more than the non-loyal consumers do. On the other hand, by introducing the process of forming loyalty, we can see how loyal and non-loyal consumers are divided. Firms can control the range of loyal consumers by properly setting their regular and promotional prices. The analysis shows that regular prices are raised, and the range of loyal consumers decreases when

APS is adopted. Extending the range of loyal consumers is not always beneficial and this can be counter-intuitive.

The remaining part of the paper is organized as follows. Some empirical studies are presented in Section 2 and an analytic framework in Section 3. Then, profit maximization in each of price strategies are analyzed in Section 4. Section 5 is the summary and further discussions.

## II. Empirical studies

Villas-Boas (1995), Lach (2002), and Baye, Morgan, and Scholten (2004) among others tried to verify empirically the mixed strategy equilibrium. Baye, Morgan, and Scholten (2004) assembled a dataset of 36 popular electronics products at shopper.com, a price comparison site, from November 1999 to May 2001. Over time, the firm that offered lowest price varied. In detail, over 19-month period, 11 different firms set the lowest prices in the case of product *3Com Homeconnect* and 7 of the 11 firms offered the lowest price just once during the period. This means that no firms consistently charge low prices.

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4) Klemperer (1995) classified the rationales of switching cost into six categories. The process of forming switching cost as detailed above can be rationally related to many of the existing categories.

Villas-Boas (1995) attempted to prove whether the real prices come from Varian's (1980) or Narasimahn's (1988) price density function. He used weekly price data on coffee and saltine crackers from six stores for 108-weeks (from mid-1985 to mid-1987). However, the null hypothesis (i.e., that observed prices come from estimated theoretical Varian's distribution) was rejected at the 5% significance level in 66% of the coffee brands and 50% of the saltine cracker brands. With Narasimahn's distribution the rejection rate was greater. In addition, the estimated parameters (e.g., number of competitors and reservation price), across seemingly competing brands were very different. With these results, appropriacy of explaining real situation with mixed strategy equilibrium becomes questionable.

Lach (2002) measured and analyzed the price dispersion of four homogeneous goods across stores in Israel for 48 months (1993-1996). The products included refrigerator, chicken, coffee, and flour. Cross-sectional data showed that the prices for each product at any given date were dispersed, and the distribution is stable over time. When the price level of a particular product was divided into four quartiles, the percentages of months spent by each store (in each of the four

quartiles) spread over the four quartiles, and the distribution looks different from Varian's distribution and different among firms, which can be compared with Villas-Boas (1995). The duration, which was defined as the number of consecutive months during which the store has a price in a given quartile, was generally one month or more. Apart from Lach's observation, it is generally observed that promotions continue for some time. Even if it cannot be predicted, rivals can react within a short period. Hence, we can believe that temporal price reductions occur under the implicit approval of rivals.

The existence and proliferation of periodic price reduction can be observed from these empirical studies. However, the unpredictability of pricing is not fully verified. As Lach (2002) mentioned, when many firms compete, consumers will find it hard to predict which firm will offer the lowest price. Baye, Morgan, and Scholten (2004) also showed that the identity of the low price firm is random over time. However, this does not mean that the pricing strategy of each firm is unpredictable or random. Furthermore, it is somewhat odd that rivals do not exploit a firm's price strategy due to the unpredictability. If every other firm is using random pricing, a firm which has observed

that no firm offers low price has incentive to lower its price. The assumption of mixed strategy is very strict. One possible explanation of temporal price reduction is cooperative situation.

### III. Model

Two firms,  $A$  and  $B$ , each producing a functionally homogeneous good is considered. The marginal costs of both firms are the same ( $c$ ), and a fixed cost is not considered. In this paper, two price strategies are compared. First, under RPS, firms offer consistently the same regular price ( $P^R$ ) to consumers. Second, under APS, one firm offers a regular price ( $P^R$ ), while the other offers a discounted promotional price ( $P^D$ ) in a particular period, and in the next period, they switch prices.<sup>5)</sup> Infinite periods are considered. In every period, there are identical consumers and they buy one product or nothing at all.<sup>6)</sup> Consumer  $j$ 's willingness-to-pay is denoted

as  $w_j$ . When price is  $P$ , the demand is  $f(P)$ . This means that the number of consumers whose willingness-to-pay are more than  $P$  is  $f(P)$ . We assume that  $f(P)$  satisfies the following:

If  $P > P^{Max}$ , then  $f(P) = 0$ ; otherwise  $f(P) > 0$ , and  $f'(P) < 0$ ,  $f''(P) \leq 0$ .

If consumers consecutively buy the same firm's product for more than  $n$  ( $\geq 2$ ) periods, such consumers become loyal to the firm. Once the loyalty is formed, it remains even though they do not buy in a single period. If a loyal customer does not buy consecutively more than two periods or buy the other firm's product, the loyalty disappears.<sup>7)</sup> Once consumers become loyal, switching cost ( $c_S > 0$ ) accrues to the price when they switch to another firm's product. The existence of learning cost, uncertainty about quality, or the psychological aspect of consumers can cause this kind of switching cost.

When prices of the firms are same and loyalty is not formed, it is assumed that each of the firms earn exactly half of the

5) As it will be shown in Proposition 2 and 3, the profit under APS is more than the profit under RPS, and the equilibrium  $P^R$  under APS ( $P^{R*}$ ) is more than equilibrium  $P^R$  under RPS ( $P_0^{R*} = P^M$ ). This means that if both firms offer  $P^{R*}$  in a specific period, industry profit is lower than monopolistic profit and is lower than profit from a firm's  $P^{R*}$  and the other firm's  $P^{D*}$ . That is, if no firm offers  $P^{D*}$ , industry profit is not maximized. APS is temporal and symmetric way in which a firm offers  $P^{D*}$  in every period.

6) This assumption implicitly considers perishable good or durable goods.

7) This assumption is the minimum required to obtain the equilibrium below. Even if loyalty remains when consumers do not buy consecutively several periods or buy the other product, the equilibrium does not change. Above condition means that loyalty is maintained for some times.

demand, and if the same prices are maintained, consumers choose the product same as their initial choice. Additionally, consumers only maximize the surplus of each period.<sup>8)</sup>

With these assumptions, the choice of consumers under each price strategy is considered. Under RPS, the offers of both firms are symmetric. Thus, consumers do not have to consider switching. Therefore, switching cost is meaningless and the model becomes the same as the one without switching costs.

Under APS, loyal consumers are confronted with four possible choices: i) loyally buy a product at every period

(*L-buy*), ii) loyally buy a product only when it is offered with a promotional price (*DL-buy*), iii) buy the product with the promotional price at every period (*D-buy*), and iv) buy nothing at every period (*Non-buy*). Given these possible choices, the condition for consumer  $j$ 's choices ( $Choice_j$ ) are as follows:

$$L-buy = \begin{cases} L-buy & \text{if } P^R - P^D \leq c_s \text{ and } w_j \geq P^R \\ DL-buy & \text{if } P^R - P^D \leq c_s \text{ and } P^R > w_j \geq P^D \\ & \text{if } P^R - P^D > c_s \text{ and } P^D + c_s > w_j \geq P^D \\ D-buy & \text{if } P^R - P^D > c_s \text{ and } w_j \geq P^D + c_s \\ Non-buy & \text{if else.} \end{cases} \quad (1)$$

[Table 1] Assumptions of the model

Categories	Description
Pricing	<ul style="list-style-type: none"> <li>Under APS, a firm offers promotional price (<math>P^D</math>) and the other offers regular price (<math>P^R</math>), and they switch their prices in the next period.</li> <li>Under RPS, Both firms offer regular price.</li> </ul>
Consumer	<ul style="list-style-type: none"> <li>Buy one or nothing in each period.</li> <li>The number of consumers whose willingness-to-pay are more than <math>P</math> is <math>f(P)</math>.</li> <li>Consumers who buy the same firm's product consecutively more than <math>n</math> periods becomes loyal, and the loyalty remains more than a period.</li> <li>Switching cost (<math>c_s</math>) occurs to loyal consumers when they switch to the other firm's product.</li> </ul>
Others	<ul style="list-style-type: none"> <li>Marginal cost (<math>c</math>) and no fixed cost.</li> <li>Infinite periods</li> </ul>

8) When consumers maximize their multi-period surplus, some can consider the case where consumers choose whether or not they should become loyal to a product. If we assume that consumers cannot choose between being loyal or disloyal, then, maximizing multi-period surplus does not change consumers' choice defined in Equation (1).

There are two possible choices for consumers with no loyalty: *D-buy* or *Non-buy*. Hence, when  $P^R - P^D > c_S$ , no consumer will buy at the price of  $P^R$ . This means that APS become the same as RPS. Therefore, we consider APS only with  $P^R - P^D \leq c_S$ .

#### IV. Profit-maximizing prices

In this section, we will analyze profit-maximizing prices. First, we consider cases where firms cooperatively determine their profit-maximizing symmetric prices. This cooperative situation reflects “bandwagon strategy” wherein a firm follows a leader’s price system. It may also reflect an implicit collusion or a monopoly operating two brands. Subsequently, we will show that APS can be adopted even in non-cooperative situations.

##### 4.1 Cooperative regular price strategy

When both firms offer symmetric regular prices, consumers will not consider switching. Hence, firms only set the price to maximize the profit at each period. Under RPS, the profit of each firm at each period is:

$$\Pi_i^R = (P^R - c)f(P^R)/2,$$

where  $i = A, B$ . We can then obtain the condition for the profit-maximizing price ( $P_0^{R*}$ ) by partially differentiating the profit function with  $P^R$ :

$$\frac{\partial \Pi_i^R}{\partial P^R} = f(P_0^{R*}) + (P_0^{R*} - c)f'(P_0^{R*}) = 0.$$

The condition is the same in a general monopolistic case ( $P_0^{R*} = P^M$ ).

**Proposition 1.** Under cooperative RPS, firms choose a monopolistic price at each period ( $P_0^{R*} = P^M$ ).

##### 4.2 Cooperative alternating promotion strategy

As earlier mentioned, when firms choose symmetric APS, we only consider cases where  $P^R - P^D \leq c_S$  because no consumers will choose a regular price when  $P^R - P^D > c_S$ . Consumers become loyal only if they consecutively buy the same firm’s product for more than  $n$  periods. Hence, periods can be classified into two: one with no loyal consumers (initial  $n$  periods) and another with loyal consumers. Moreover, because no consumers will choose a higher price at periods with no loyal consumers, we assume that firms offer only regular prices



in initial  $n$  periods. After these periods, firms alternately offer  $P^R$  and  $P^D$  ( $\leq P^R$ ).

Defining  $w^L$  as  $Max(P_1^R, P_2^R, \dots, P_n^R)$  where  $P_i^R$  is the regular price in initial  $n$  periods, only consumers with  $w_j > w^L$  are considered loyal to a firm at the end of period  $n$ . Then,  $w^L$  is a parameter representing the minimum level of willingness-to-pay of loyal consumers. We also define  $\Pi_R^{w^L}$  with the given  $w^L$  as the present value of the future industry profit from consumers who adopt  $P^R$  after period  $n+1$ . Similarly,  $\Pi_D^{w^L}$  comes from consumers who adopt  $P^D$ ; and  $\Pi_{R+D}^{w^L}$  is the sum of  $\Pi_R^{w^L}$  and  $\Pi_D^{w^L}$ . Specifically,  $\Pi_k^{w^L}(x, y)$  ( $k = R, D, R+D$ ) is the present value of profit with  $P^R = x$ ,  $P^D = y$ . If  $P^D$  is lower than  $w^L$ , the function of profits, either with  $P^R > w^L$  or not, are as follows:

$$\Pi_{R+D}^{w^L}(P^R \geq w^L, P^D) = \sum_{\tau=t}^{\infty} \delta^{\tau-t} \left[ (P^R - c)f(P^R)/2 + (P^D - c)(f(P^D) - f(w^L)/2) \right], \quad (2)$$

$$\Pi_{R+D}^{w^L}(P^R \leq w^L, P^D) = \sum_{\tau=t}^{\infty} \delta^{\tau-t} \left[ (P^R - c)f(w^L)/2 + (P^D - c)(f(P^D) - f(w^L)/2) \right]. \quad (3)$$

In the previous equation with discount

rate  $\delta$ , the term on the left enclosed in square brackets is  $\Pi_R^{w^L}$ , while the term on the right is  $\Pi_D^{w^L}$ . The term  $\Pi_{R+D}^{w^L}$  shows that when  $P^R > w^L$ , some of the loyal consumers ( $P^R > w_j \geq w^L$ ) will not buy at the price of  $P^R$ . Moreover, when  $P^R \leq w^L$ , all loyal consumers will buy. However, the equation of  $\Pi_D^{w^L}$  is always the same when  $P^R - P^D \leq c_S$  is satisfied.

When we differentiate the profit function with  $P^D$  ( $< w^L$ ), then

$$\frac{\partial \Pi_{R+D}^{w^L}}{\partial P^D} = \sum_{\tau=t}^{\infty} \delta^{\tau-t} \left[ (P^D - c)f'(P^D) + f(P^D) - f(w^L)/2 \right] \quad (4)$$

$$\text{and } \frac{\partial^2 \Pi_{R+D}^{w^L}}{\partial P^D \partial P^D} = \sum_{\tau=t}^{\infty} \delta^{\tau-t} \left[ (P^D - c)f''(P^D) + 2f'(P^D) \right] < 0. \quad (5)$$

Considering Equation (5) and the fact that  $(P^D - c)f'(P^D) + f(P^D) = 0$  with  $P^D = P^M$ , the Equation (4) can be zero only when  $c < P^D < P^M$ . Given  $w^L$ , we now define  $P_{w^L}^D$  ( $c < P_{w^L}^D < P^M$ ) as the  $P^D$  that makes Equation (4) zero. Given  $w^L$  and  $P^R$ , we can then define  $P_{P^R}^D$ , the profit-maximizing  $P^D$ , as follows:

$$P_{P^R}^D = \begin{cases} P^R - c_S & \text{if } P^R - c_S > P_{w^L}^D \\ P_{w^L}^D & \text{if } P^R - c_S \leq P_{w^L}^D \end{cases} \quad (6)$$

Equation (2), (3), and (4) are induced by assuming  $P^D < w^L$ . If we consider  $P^D \geq w^L$  ( $w^L \leq P^M$ ), we can intuitively conclude that  $P^D$ , which maximizes  $\Pi_D^{w^L}$ , is  $P^M$ . Therefore, if we do not consider the restriction of  $P^D < w^L$ , the profit-maximizing  $P^D$  should be  $P_{P^R}^D$  or  $P^M$ . On the other hand, if  $w^L > P^M$ , then the profit-maximizing  $P^D$  should be  $P_{P^R}^D$ . In addition, the case where profit-maximizing  $P^D$  is  $P^M$  can only occur when  $w^L < P^M$ . In this case profit-maximizing  $P^R$  should be  $P^M$ . Thus, there are some cases where APS does not occur, and  $P^R = P^D = P^M$  can be adopted. Therefore, we should induce the condition under which APS does not occur.

Particularly, we deal with the case where  $P^M > w^L$  and  $w^L - P_{w^L}^D \leq c_S$ . Considering Equations (4) and (5), and the fact that  $f(w^L)/2$  is a decreasing function of  $w^L$ , then  $P_{w^L}^D$  is considered an increasing function of  $w^L$ . Therefore, if we define  $w^{\bar{L}}$  as  $w^L$  which makes  $P_{w^L}^D$  the same as  $w^L$ ,  $w^{\bar{L}}$  will always exist, thereby satisfying  $c < w^{\bar{L}} < P^M$ . This is because when

$w^L = c$ , then  $P_{w^L}^D > w^L$ , and when  $w^L = P^M$ , then  $P_{w^L}^D < w^L$ . The partial derivative of  $\Pi_D^{w^L}(x, P_{w^L}^D)$  with respect to  $w^L$  is as follows:

$$\begin{aligned} \frac{\partial \Pi_D^{w^L}(x, P_{w^L}^D)}{\partial w^L} &= \\ &= \sum_{\tau=t}^{\infty} \delta^{\tau-t} \left[ \frac{\partial P_{w^L}^D}{\partial w^L} (f(P_{w^L}^D) - f(w^L)/2) + (P_{w^L}^D - c) \left( f'(P_{w^L}^D) \frac{\partial P_{w^L}^D}{\partial w^L} - f'(w^L)/2 \right) \right] \\ &= \\ &= \sum_{\tau=t}^{\infty} \delta^{\tau-t} \left[ \frac{\partial P_{w^L}^D}{\partial w^L} ((P_{w^L}^D - c) f'(P_{w^L}^D) + f(P_{w^L}^D) - f(w^L)/2) - (P_{w^L}^D - c) f'(w^L)/2 \right] \\ &> 0. \end{aligned} \quad (7)$$

Considering Equation (4), the term on the right enclosed in square brackets in the lower part of Equation (7) is zero. Thus, Equation (7) is positive. We can say that  $\Pi_D^{w^L = P^M}(P^M, P_{w^L}^D)$  is higher than  $\Pi_i^R(P^M)$  defined in §4.1. If we define  $\bar{w}^L$  as  $w^L$  which makes  $\Pi_D^{w^L}(\text{optimal } P^R, P_{w^L}^D)$  the same as  $\Pi_i^R(P^M)$ , then  $\bar{w}^L$  will satisfy  $\bar{w}^L < P^M$ . On the other hand,  $\Pi_D^{w^{\bar{L}}}(w^{\bar{L}}, w^{\bar{L}})$  is as follows:

$$\begin{aligned} \Pi_D^{w^{\bar{L}}}(w^{\bar{L}}, P_{w^{\bar{L}}}^D) &= (w^{\bar{L}} - c)(f(w^{\bar{L}}) - f(w^{\bar{L}})/2) \\ &= (w^{\bar{L}} - c)f(w^{\bar{L}})/2 = \Pi_i^R(w^{\bar{L}}). \end{aligned}$$

Further, considering  $w^{\bar{L}} < P^M$ , then  $\Pi_D^{w^{\bar{L}}}(w^{\bar{L}}, w^{\bar{L}}) < \Pi_i^R(P^M)$ . Thus,  $P^M > \bar{w}^L > w^{\bar{L}} > c$ . Using the same logic,  $P^M > \bar{w}^L > w^{\bar{L}} > c$  is satisfied even when

$w^L - P_{w^L}^D > c_S$ . If  $w^L$  is lower than  $\bar{w}^L$ , APS cannot occur. With this fact, we can define the profit-maximizing APS,  $P^{R^*}$  and  $P^{D^*}$ .

**Proposition 2.** The profit-maximizing APS can be defined as follows:

- a) If  $w^L \geq P^M + c_S$ ,  

$$P^{R^*} = P^M + c_S \text{ and } P^{D^*} = P^M.$$
- b) If  $P^M + c_S \geq w^L \geq P^M$ ,  $P^{R^*} = w^L$ ,  

$$P^{D^*} = \begin{cases} w^L - c_S & \text{if } w^L - P_{w^L}^D > c_S \\ P_{w^L}^D & \text{if } w^L - P_{w^L}^D \leq c_S \end{cases}.$$
- c) If  $P^M > w^L > \bar{w}^L$  and  

$$P^M - P_{w^L}^D \leq c_S, \quad P^{R^*} = P^M, \quad \text{and}$$

$$P^{D^*} = P_{w^L}^D.$$
- d) If  $P^M > w^L > \bar{w}^L$  and  $P^M - P_{w^L}^D > c_S$ ,  

$$P^M > P^{R^*} \geq w^L \quad \text{and} \quad P^{D^*} = P^{R^*} - c_S$$

$$(\geq P_{w^L}^D) \text{ or } P^{R^*} = P^{D^*} = P^M.$$
- e) If  $w^L \leq \bar{w}^L$ ,  $P^{R^*} = P^{D^*} = P^M$ .

**Proof:** See Appendix A. Q.E.D.

Based on Proposition 2, APS is preferred to RPS: i) when  $w^L > \bar{w}^L$  in case where  $c_S$  is sufficiently large and, ii) when  $w^L$  is larger than a certain value in  $(P^M, \bar{w}^L)$  in case where  $c_S$  is not so large. In comparison with RPS (monopolistic profit), it is clear that firms can earn more profit if APS is adopted, because firms adopt

APS even if they can adopt RPS. When APS is adopted, the attribute of alternating price plays three roles. First, price differentiation can occur by making loyal consumers alternately buy at  $P^R$  or  $P^D$  ( $< P^R$ ). On average, loyal consumers pay more compared with the non-loyal consumers who always buy at price ( $P^D$ ). Price differentiation is the factor that makes firms earn more than monopolistic profit. Second, alternating price prevents non-loyal consumers from becoming loyal by making them switch at every period. If the range of loyal consumers grows or if  $w^L$  decreases, the profit from APS will be reduced as seen in Equation (7) or APS will not be adopted. This means that a larger number of loyal consumers can harm industry profit by deleting the chance of price differentiation. Third, it distributes increased industry profit to firms by making firms' prices symmetric. This effect was also mentioned by Lal (1990): "Alternating promotions are then simply a way to equalize their equilibrium-payoffs. Such promotions can be motivated by equity considerations." If this attribute is not satisfied, then cooperative pricing will not easily occur or a firm will have the incentive to deviate from cooperation and occupy a superior status.

The above analysis is performed under

the assumption that  $w^L$  is given after period  $n$ . We can consider the case where firms can control the range of loyal consumers to earn more profit through APS. This consideration also means examining the condition under which APS can be conducted. Thus, we first examine which  $w^L$  can give the most profit to firms.

**Lemma 1.** Profit-maximizing  $w^{L*}$  satisfies  $P^M + c_S > w^L > P^M$ , except in some extreme cases with  $c_S \leq P^M - P_{w^L}^D$  where  $w^{L*}$  is  $P^M$ .

**Proof:** See Appendix B. Q.E.D.

Considering Lemma 1, firms should set their prices in initial  $n$  periods to control  $w^L$  and to maximize profit after period  $n$ . However,  $w^L$  is  $Max(P_1^R, P_2^R, \dots, P_n^R)$ , and  $w^L$  should be higher than  $P^M$  in most cases considering Lemma 1. Firms should thus sacrifice profit before  $n$  periods because, without loyal consumers, the price that maximizes the profit at each period is  $P^M$ . Therefore, firms should balance the profit from before and after the  $n$  period. As such, firms should first set their price as  $P^M$  in initial  $n-1$  periods, because  $\delta \leq 1$  and  $w^L$  can be controlled at period  $n$ . Second, firms should set their price at period  $n$  as  $P^M + c_S > P^R \geq P^M$ . This is

considered as straightforward because  $P^R$  at period  $n$  should be between  $w^{L*}$  and  $P^M$ . Under Lemma 1,  $P^R = P^M$  occurs in some extreme cases. If  $\delta$  becomes close to 1,  $P^R$  becomes close to  $w^{L*}$  as firms increasingly consider future profit, and if  $\delta$  becomes close to zero, it also becomes close to  $P^M$ .

**Proposition 3.** If firms try to maximize their discounted total profit, they should set  $w^L$  (their price at period  $n$ ) as  $P^M + c_S > w^L \geq P^M$ . If  $\delta$  becomes close to 1, then  $w^L$  becomes close to  $w^{L*}$ , and if  $\delta$  becomes close to 0, it becomes close to  $P^M$ .

Proposition 3 means that if firms can control  $w^L$ , it becomes higher than  $P^M$  in most cases. It also means that firms do not want to extend the range of loyal consumers, considering price differentiation through APS. This is an important aspect of APS. In other words, APS or temporal price reduction can raise the regular prices of firms and decrease the range of loyal consumers.

In Lemma 1 and in Proposition 3, we deal with cases where firms control  $w^L$ . However, we can consider the situation where firms consider APS at a certain time

before which they competed with regular prices without considering APS. According to Proposition 2, firms can earn more profit by adopting APS if  $w^L$  (or the price maintained previously) is higher than a certain value ( $\bar{w}^L$ ) which is lower than  $P^M$ . This means that firms can introduce APS in a market where competition is not so intense. Further, when switching cost occurs, firms will have monopolistic power over their loyal consumers, resulting in lessened competition.<sup>9)</sup> Thus, APS can be adopted in most markets with switching costs, even if firms do not cooperate with regard to regular pricing.

### 4.3 Non-cooperative alternating promotion strategy

In this subsection, we consider a non-cooperative situation. As such, we only consider the case where firms begin to adopt APS from a point in time after period  $n$ , assuming that the conditions for APS in Proposition 2 are satisfied. With a sufficiently large  $\delta$ , profit-maximizing APS can be adopted even in non-cooperative situations if we define a trigger strategy.

**Proposition 4.** If  $\delta$  is sufficiently large and  $n > 3$ , following a punishment strategy will makes profit-maximizing APS a perfect Nash equilibrium path. If the other firm deviates from the profit-maximizing APS at a previous period, price should be set to  $c$ , and optimal APS should again be started with  $P^{D*}$  in the next period.

**Proof:** If a firm deviates from the profit-maximizing APS, the price should be  $P^{R*} - c_S - \epsilon$  or  $P^{D*} - c_S - \epsilon$  ( $\epsilon \rightarrow 0+$ ). Let  $\Pi_i^D$  be the profit from the deviation to satisfy the following:

$$\Pi_i^D < \Pi_{R+D}^{w^{L*}}(P^{R*}, P^{D*}).$$

This is because the price in deviation is lower than  $P^M$ . In addition, the deviating firm will earn zero profit as punishment in the next period. Therefore, the sum of the discounted profits from the deviating and punishment periods is lower than that of the discounted profit in adopting the profit-maximizing APS, if and only if  $\delta$  is sufficiently large. Thus, no firm can have the incentive to deviate from the profit-maximizing APS. The condition of  $n > 3$  is needed to prevent the range of

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9) Klemperer (1987) showed that non-cooperative equilibrium in an oligopoly with switching costs may be the same as collusive outcome in an otherwise identical market without switching costs. Furthermore, Taylor (2003) mentioned that switching costs give individuals incentives to remain with the same supplier over time, granting firms a degree of monopoly power over their base of subscribers.

loyal consumers from changing during the deviation and punishment periods. Q.E.D.

Considering Proposition 4, ASP can be adopted in non-cooperative situations. Specifically, the punishment period is operative for only a limited time, allowing firms to get on with the collusive prices again. This argument is similar with that presented by Lal (1990).

## V. Summary and further discussions

From the analysis, it is clear that APS can be more profitable than RPS, and that APS can be adopted in cooperative situations reflecting the “bandwagon strategy” where a firm follows a leader’s price system, or implicit collusions, or situations where a monopoly operates two brands, as well as in non-cooperative situations. The conditions for APS are the existence of switching cost (high or low) and the range of loyal consumers higher than a certain value ( $< P^M$ ). If firms considering APS control the range of loyal consumers, then the condition on the range of loyal consumers will be accomplished. Even when firms begin to consider APS at a certain point in the operation, the condition remains unrestrictive. This is because the value

representing the range is lower than the monopolistic price, and competition with RPS can yield high equilibrium prices when there is switching cost. Thus, even when firms do not cooperate with respect to regular prices, APS can be an equilibrium.

This result is very important because the dominant view claims that temporal price reductions occur as a result of mixed strategy. The view emphasizes competitive pressure. However, the important aspect of temporal price reduction is that firms want to maintain a higher price to their loyal consumers, and they do not react immediately to the promotions of rivals even if they can do so. This aspect is reflected in this paper as the alternating promotion strategy. If the range of loyal consumers is given, one firm can generate more profit through promotions without harming the competitors. However, price reduction, which is not temporal, will change the range of loyal consumers and will take away the chances of other firms to attract consumers, leading firms into a price war. Therefore, promotions should be temporal. For an equitable conduct of such a strategy, the symmetric alternating method is the best option to adopt.

However, situations in real world do not have to be the exact alternating way. Cooperation can be maintained if two

conditions are satisfied; not to offer promotion when rivals do promotion and not to offer promotion consecutively. The symmetric alternating method is the way to maximize the profit of each firm under these conditions. However, if there are other factors such as variation of demand or lack of inventory which are not considered in the model, there can be periods where both firm offer promotion or where no firm offers promotion. Specifically, if there are many firms, temporal price reduction cannot become exactly alternating, and can be considered a significant variation in the identity of the low-price firm as in the previous empirical studies.

The important thing is that temporal price reduction does not necessarily result from the incentives of competition. Even when firms do compete on regular pricing, firms can cooperate with respect to promotions, which do not harm the profit of rivals from their own loyal consumers. Hence, in this paper, a cooperative situation is not far from competition.

By introducing the process of forming loyalties, we can observe how loyal and non-loyal consumers are divided. Some consumers switch only because they want to avail of the promotional price. However, they can be locked-in if firms do not give

them the incentive to change their choice. In this case, firms can control the range of loyal consumers by setting their regular and promotional prices accordingly. The analysis shows the counter-intuitive implication that extending the range of loyal consumers is not always beneficial. When promotions exist, firms have incentives to increase regular price and decrease the range of loyal consumers. Maintaining too many loyal consumers lowers the regular price and diminishes the demand from promotions. In this situation, temporal price reduction is a tool for price discrimination between loyal and non-loyal consumers. It is not for obtaining more loyal consumers or market share.

This paper abstracted the real world and explains the temporal price reduction as a cooperative pure strategy. However, some of the assumptions are strict and diminishes the reality of the model. Specifically, the process of forming loyalty should be more realistic. Moreover, many important aspects of real world such as variation of demand or lack of inventory are not considered. The existence of these aspects induces various phenomena and should be incorporated to the model.

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## Appendix A : Proof of Proposition 2

Proof a: With the given  $P^R$ ,  $P^D$  should be defined as in Equation (6). If  $P^R \geq w^L$ ,  $P^R - c_S \leq P_{w^L}^D$  cannot occur because  $w^L \geq P^M + c_S$  and  $P_{w^L}^D < P^M$ . The present value of the industry profit is then given as Equation (2) with  $P^D = P^R - c_S$ , and the partial derivative with respect to  $P^R$  is presented by:

$$\begin{aligned} & \frac{\partial \Pi_{R+D}^{w^L}(P^R > w^L, P^R - c_S)}{\partial P^R} = \\ & \sum_{\tau=t}^{\infty} \delta^{\tau-t} [f(P^R) + (P^R - c)f'(P^R)]/2 + \\ & \sum_{\tau=t}^{\infty} \delta^{\tau-t} [f(P^R - c_S) + (P^R - c_S - c)f'(P^R - c_S) - f(w^L)/2]. \end{aligned} \quad (A1)$$

In Equation (A1), the first term is the marginal profit with the demand function  $f(P)$ ; if  $P^R$  is above  $P^M$ , then it is negative. Furthermore, the second term is the same as in Equation (4) with  $P^D = P^R - c_S$ ; given that  $P^R - c_S > P_{w^L}^D$ , then it is considered negative. Therefore, Equation (A1) is also negative.

When  $P^R < w^L$ , we first consider the case of  $P^R - c_S \leq P_{w^L}^D$  wherein  $P^D$  should then be  $P_{w^L}^D$ . In this case, because  $P^R < w^L$ , when  $P^R$  increases,  $\Pi_D^{w^L}$  is maintained but  $\Pi_R^{w^L}$  increases. Therefore,

when  $P^R < w^L$  and  $P^R - c_S \leq P_{w^L}^D$ , profit is maximized with the condition of  $P^R - c_S = P_{w^L}^D$ . On the other hand, in the case of  $P^R - c_S \geq P_{w^L}^D$ , the industry profit is given as in Equation (3) with  $P^D = P^R - c_S$ . The partial derivative with respect to  $P^R$  is given by:

$$\begin{aligned} & \frac{\partial \Pi_{R+D}^{w^L}(P^R < w^L, P^R - c_S)}{\partial P^R} = \\ & \sum_{\tau=t}^{\infty} \delta^{\tau-t} [f(P^R - c_S) + (P^R - c_S - c)f'(P^R - c_S)]. \end{aligned} \quad (A2)$$

This is the marginal profit with the demand function  $f(P)$  as well. Therefore, Equation (A2) is negative if  $P^R - c_S > P^M$  and positive if  $P^R - c_S < P^M$ . Summarizing the above results, we can conclude that if  $w^L \geq P^M + c_S$ , then  $P^{R*} = P^M + c_S$  and  $P^{D*} = P^M$ .

Proof b: We first consider the case  $P^R \geq w^L$ . If  $P^R - c_S \leq P_{w^L}^D$ ,  $P^D = P_{w^L}^D$ . If  $P^R$  increases, then  $\Pi_D^{w^L}$  is maintained but  $\Pi_R^{w^L}$  decreases. If  $P^R - c_S \geq P_{w^L}^D$ ,  $P^D = P^R - c_S$ , and the partial derivative of the industry profit with respect to  $P^R$  is the same as in (A1). Thus, using the same logic as in the proof of Proposition 2-a, (A1) is negative. Hence, with the condition

of  $P^R \geq w^L$ ,  $P^R = w^L$  maximizes the profit.

Next, we consider the case of  $P^R \leq w^L$ . If  $P^R - c_S \leq P_{w^L}^D$ ,  $P^D = P_{w^L}^D$ . If  $P^R$  increases, then  $\Pi_D^{w^L}$  is maintained but  $\Pi_R^{w^L}$  increases. If  $P^R - c_S \geq P_{w^L}^D$ ,  $P^D = P^R - c_S$  and the partial derivative of the industry profit with respect to  $P^R$  is the same as in (A2). As  $P^R - c_S \leq P^M$ , it can be positive or zero. Summarizing these,  $P^{R*} = w^L$  and  $P^{D*}$  is the same as in Equation (6).

Proof c: The case of  $P^R - c_S > P_{w^L}^D$  means  $P^R > P^M$  and  $P^D = P_{w^L}^D$ . The profit in this case becomes lower than that with  $P^R = P^M$  and  $P^D = P_{w^L}^D$ . On the other hand, when  $P^R - c_S \leq P_{w^L}^D$ ,  $P^D = P_{w^L}^D$ . When  $P^R$  increases in the range of  $P^R < P^M$ , then  $\Pi_R^{w^L}$  increases, and when  $P^R$  increases in the range of  $P^R \geq P^M$ ,  $\Pi_R^{w^L}$  decreases. It does not change in case of  $\Pi_D^{w^L}$ . Therefore,  $P^{R*} = P^M$  and  $P^{D*} = P_{w^L}^D$ .

Proof d: If  $P^R - c_S < P_{w^L}^D$ ,  $P^D = P_{w^L}^D$ . However,  $P^R - c_S < P_{w^L}^D$  means that  $P^R < P^M$ , and  $\Pi_R^{w^L}$  can be increased with the increase of  $P^R$ . If  $P^R - c_S \geq P_{w^L}^D$ ,

$P^D = P^R - c_S$ . When  $P^R \leq w^L$ , the partial derivative of the industry profit with respect to  $P^R$  is the same as in (A2). As  $P^R - c_S < P^M$ , (A2) is positive and  $P^R \leq w^L$  cannot maximize the profit. When  $P^R \geq w^L$ , the partial derivative is the same as in (A1). Thus, using the same logic as in the proof of Proposition 2-a, if  $P^R \geq P^M$ , (A1) is negative, and profit cannot be maximized. If  $P^R < P^M$ , it can be negative or positive. Summarizing these, if APS is adopted,  $P^M > P^{R*} \geq w^L$  and  $P^{D*} = P^{R*} - c_S$ . However, if  $P^M \geq P^R$  and  $P^D \geq P_{w^L}^D$ , cases where the profit is lower than  $\Pi_{A+B}^R(P^M)$  emerge. Therefore, Proposition 2-d is formed.

Proof e: If  $w^L \leq \bar{w}^L$ , there is no  $P^R$  and  $P^D (< P^R)$  making  $\Pi_{R+D}^{w^L}$  more than  $\Pi_{A+B}^R(P^M)$ . Therefore,  $P^{R*} = P^{D*} = P^M$ . Q.E.D.

## Appendix B : Proof of Lemma 1

i) If  $w^L \geq P^M + c_S$ , when  $w^L$  decreases,  $\Pi_D^{w^L}$  is maintained but  $\Pi_R^{w^L}$  increases. If  $w^L = P^M + c_S$ , then  $P^{R*} = w^L$  and  $P^{D*} = w^L - c_S$  because  $w^L - P_{w^L}^D > c_S$ . Thus, the partial derivative of the

industry profit with respect to  $w^L$  is given by:

$$\frac{\partial \Pi_{R+D}^{w^L}(w^L, w^L - c_S)}{\partial w^L} = \sum_{\tau=t}^{\infty} \delta^{\tau-t} [f(w^L - c_S) + (w^L - c_S - c)f'(w^L - c_S) + c_S f'(w^L)/2] \quad (A3)$$

As such, when  $w^L = P^M + c_S$ , the sum of the first two terms is zero, and the last term is negative. Therefore,  $w^L$  should be lower than  $P^M + c_S$ .

ii) Consider the case of  $P^M \geq w^L > \bar{w}^L$ . If  $P^{R^*} = P^M$  and  $P^{D^*} < P^M$ , as  $w^L$  increases,  $\Pi_R^{w^L}$  is maintained but  $\Pi_D^{w^L}$  increases, which can be seen in Equation (7). If  $P^{R^*} = P^{D^*} = P^M$ , then the industry profit is lower than when  $w^L = P^M$ . If  $P^{R^*} < P^M$ , then the industry profit is also lower than when  $w^L = P^M$ . Therefore,  $w^L$  should be equal to or more than  $P^M$ .

iii) If  $w^L = P^M$ ,  $P^{R^*} = P^M$ , and  $P^{D^*} = w^L - c_S$  or  $P^{D^*} = P_{w^L}^D$ . When  $P^{D^*} = P_{w^L}^D$  ( $P^M - P_{w^L}^D < c_S$ ), the partial derivative of the industry profit with respect to  $w^L$  is given by:

$$\frac{\partial \Pi_{R+D}^{w^L}(w^L, P_{w^L}^D)}{\partial w^L} =$$

$$\sum_{\tau=t}^{\infty} \delta^{\tau-t} [f(w^L) + (w^L - P_{w^L}^D)f'(w^L)]/2 + \sum_{\tau=t}^{\infty} \delta^{\tau-t} [f(P_{w^L}^D) + (P_{w^L}^D - c)f'(P_{w^L}^D) - f(w^L)/2] \frac{\partial P_{w^L}^D}{\partial w^L}.$$

The lower term is zero considering Equation (4). The upper term is positive as  $w^L = P^M$  and  $P_{w^L}^{D^*} > c$ . Therefore, in this case,  $w^L$  should be higher than  $P^M$ .

When  $P^{D^*} = w^L - c_S$  ( $P^M - P_{w^L}^D \geq c_S$ ), the partial derivative with respect to  $w^L$  is Equation (A3) and the partial derivative of Equation (A3) with respect to  $c_S$  is given by:

$$\frac{\partial^2 \Pi_{R+D}^{w^L}(w^L, w^L - c_S)}{\partial w^L \partial c_S} = \sum_{\tau=t}^{\infty} \delta^{\tau-t} [-2f'(w^L - c_S) - (w^L - c_S - c)f''(w^L - c_S) + f'(w^L)/2] \quad (A4)$$

Considering  $w^L = P^M$ , if  $c_S = 0$ , Equation (A3) becomes zero, and Equation (A4) becomes positive in most cases especially where  $c_S$  is small. Therefore, Equation (A3) is generally positive with  $w^L = P^M$ , except in some extreme cases where Equation (A4) is negative in the large part of  $0 < c_S \leq P^M - P_{w^L}^D$ .

By summarizing i) ~ iii), Lemma 1 is formed.

# 협력적 가격차별 수단으로서의 일시적 가격할인

송 재 도\*

## Abstracts

이 연구는 교체비용이 존재하는 상황에서 복점기업간 경쟁을 다룬다. 분석에서는 일시적 가격할인 현상이 순수전략균형(Pure Strategy Equilibrium)의 결과일 수 있음을 보인다. 이는 기존 대다수 연구들에서 일시적 가격할인 현상을 혼합전략균형(Mixed Strategy Equilibrium)의 결과로 해석해왔던 것과 구별된다. 본 연구에서 구해진 일시적 가격할인의 순수전략균형 하에서 기업들은 정규가격만을 제공하는 경우보다 높은 이윤을 얻게 되며, 이러한 현상은 일시적 가격할인이 충성가입자와 비충성 가입자간 가격차별의 역할을 수행하는 것에 기인한다. 한편 본 연구에서는 기업들이 정규가격과 할인가격을 적절히 조정함으로써 충성가입자들의 수를 통제할 수 있음을 가정하고 있다. 이를 통해 기업들은 정규가격 수준을 일정 수준 이상으로 유지함으로써 충성가입자들의 수를 지나치게 늘리지 않으려는 유인이 있음을 보인다.

핵심개념: 가격차별, 가격할인, 교체비용, 충성도

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