Convergence Analysis of Noise Robust Modified AP(affine projection) Algorithm

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Abstract—**According to increasing projection order, the AP algorithm has noise amplification problem in large background noise. This phenomenon degrades the performances of the AP algorithm. In this paper, we analyze convergence characteristic of the AP algorithm and then suggest a noise robust modified AP algorithm for reducing this problem. The proposed algorithm normalizes the update equation to reduce noise amplification of AP algorithm, by adding the multiplication of error power and projection order to auto-covariance matrix of input signal. By computer simulation, we show the improved performance than conventional AP algorithm.**

Index Terms— **Projection Order, Noise Amplification Problem, Noise Robust, Modified AP Algorithm.**

I. INTRODUCTION

NLMS family algorithms are simple and numerically robust. But these algorithms have drawback of converging slowly, especially when input signal is colored. On the other hand, RLS algorithm exhibits fast convergence in colored or strongly correlated input signal, but it has heavy computational burdens. To solve these problems, affine projection(AP) algorithm was suggested[1].

The AP algorithm is a generalization of the NLMS algorithm. This algorithm lies somewhere between NLMS algorithm and RLS algorithm from a performance and computational complexity point of view. In many cases, it is possible to achieve an improved convergence rate with only a marginal increase in computation[2][3]. Nevertheless, when a projection is performed, noise amplification problem arises and this phenomenon degrades the performances of the AP algorithm.

II. CONVERGENCE ANALYSIS OF THE AP ALGORITHM

The convergence behavior is derived by using a simple model for the input signal vector with the usual independent assumption with theoretical analyses. The analysis is done based on the assumptions (A1~A4) [4-6]. The discrepancy between fundamental theoretical results and the true algorithm behavior was investigated in [7] and found to be relatively small.

< Assumption>

(A1) The signal vectors $\{\mathbf{x}_n\}$ have zero mean and are independent and identically distributed(i.i.d.) with covariance matrix

$$\mathbf{R} = E[\mathbf{x}_n \mathbf{x}_n^t] = \mathbf{V}\Lambda\mathbf{V}^t \qquad \text{(A-1)}$$

Where, $\Lambda = diag(\lambda_1, \lambda_2, \cdots, \lambda_L)$ and $\mathbf{V} = (\mathbf{v}_1\ \mathbf{v}_2 \cdots \mathbf{v}_L)$. Here, $\lambda_1, \lambda_2, \cdots, \lambda_L$ are the eigenvalues of $\mathbf{R}$ and $\mathbf{v}_1\ \mathbf{v}_2 \cdots \mathbf{v}_L$ are the corresponding orthonormal eigenvectors. ($\mathbf{V}^t\mathbf{V} = \mathbf{I}$). That is, $\mathbf{V}$ is a unitary matrix.

(A2) There exists a true adaptive filter weight $\mathbf{w}^o$ of dimension L such that the corresponding error signal

$$e_n = d_n - \mathbf{w}^{ot}\mathbf{x}_n \equiv \varepsilon_n \qquad \text{(A-2)}$$

Inherits the properties of the measurement noise ε_n, which is a zero mean white noise of variance ξ^0 that is independent of $\{\mathbf{x}_n\}$.

(A3) The random vector $\mathbf{x}_n$ is the product of three independent random variables which are i.i.d. That is,

$$\mathbf{x}_n = s_n r_n \mathbf{v}_n \qquad \text{(A-3)}$$

Where,

$$P\{s_n = \pm 1\} = \frac{1}{2}$$
$$r_n \sim \|\mathbf{x}_n\| \qquad \text{(A-4)}$$
$$P\{\mathbf{v}_n = \mathbf{v}_i\} = p_i = \frac{\lambda_i}{tr(\mathbf{R})}, \quad i = 1, 2, \cdots, L.$$

Where $r_n \sim \|\mathbf{x}_n\|$ means that r_n has the same distribution as the norm of the true input signal vectors.

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(A4) The target and reference signals are stationary, with known powers, so that the time varying power estimates can be replaced by their true values, that is,

$$\hat{\sigma}^2_{n,x} = \sigma^2_x, \quad \hat{\sigma}^2_{n,e} = \sigma^2_{n,e}, \quad E(\mathbf{x}_n^t \mathbf{x}_n) = L \cdot \sigma^2_x \quad \text{(A-5)}$$

Assumption (A3), first introduced by Slock [3], and assumption (A4) introduced by Greenberg [4], leads to a simple distribution for the vectors $\mathbf{x}_n$ consistent with the actual first- and second- order statistics of the input signal. Assumption (A3), as will be seen, makes the convergence analysis tractable. Under assumption (A3), the weight update equation of AP algorithm can be modified. Since $\mathbf{x}_n$ are either parallel or orthogonal to each other, the orthogonalization step to compute $\mathbf{x}_n^k$, for k = 1, 2 ,..., P-1 , becomes redundant. Thus the weight update equation is given by

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \mu_0 \mathbf{x}_n + \mu_1 \mathbf{x}_{n-1} + \cdots + \mu_{P-1}\mathbf{x}_{n-(P-1)} \quad (1)$$

$$\mu_k = \begin{cases} \dfrac{\mu e_n}{\mathbf{x}_n^t \mathbf{x}_n} & for\ k=0, \quad if\ \|\mathbf{x}_n\| \neq 0 \\ \dfrac{\mu e_n^k}{\mathbf{x}_{n-k}^t \mathbf{x}_{n-k}} & for\ k = 1,2,\cdots,P-1, \\ & if \quad \mathbf{x}_{n-k} \perp \mathbf{x}_{n-i} \quad \forall i<k \\ 0 & otherwise \end{cases} \quad (2)$$

$$\begin{cases} e_n = d_n - \mathbf{w}_n^t \mathbf{x}_n \\ e_n^k = d_{n-k} - \mathbf{w}_n^t \mathbf{x}_{n-k} & for\ k = 1,2,\cdots,P-1 \end{cases} \quad (3)$$

(Using (A3),

$$\mathbf{w}_n^{kt}\mathbf{x}_{n-k} = (\mathbf{w}_n + \mu_0\mathbf{x}_n + \mu_1\mathbf{x}_{n-1} + \cdots + \mu_{k-1}\mathbf{x}_{n-(k-1)})^t \mathbf{x}_{n-k} = \mathbf{w}_n^t\mathbf{x}_{n-k}$$

Since $\mathbf{x}_{n-k} \perp \mathbf{x}_{n-i} \quad \forall i<k$.)

To analyze the convergence behavior of (1), firstly, the weight adaptation is rewritten in terms of the weight error vector $\tilde{\mathbf{w}}_n$, where $\tilde{\mathbf{w}}_n = \mathbf{w}^o - \mathbf{w}_n$. Using this notation, we can rewrite e_n^k as $e_n^k = \tilde{\mathbf{w}}_n^t \mathbf{x}_{n-k} + \varepsilon_{n-k}$. Combining this result with (1) and (2), the adaptation equation in error form can be obtained as:

$$\begin{aligned} \tilde{\mathbf{w}}_{n+1} &= \mathbf{w}^o - \mathbf{w}_{n+1} \\ &= \mathbf{w}^o - (\mathbf{w}_n + \mu_0\mathbf{x}_n + \mu_1\mathbf{x}_{n-1} + \cdots + \mu_{P-1}\mathbf{x}_{n-(P-1)}) \\ &= \tilde{\mathbf{w}}_n - \left(\frac{\mu e_n}{\mathbf{x}_n^t\mathbf{x}_n}\mathbf{x}_n + \frac{\mu e_n^1}{\mathbf{x}_{n-1}^t\mathbf{x}_{n-1}}\mathbf{x}_{n-1} + \cdots + \frac{\mu e_n^{P-1}}{\mathbf{x}_{n-(P-1)}^t\mathbf{x}_{n-(P-1)}}\mathbf{x}_{n-(P-1)} \right) \end{aligned} \quad (4)$$

$$\begin{aligned} e_n &= d_n - \mathbf{w}_n^t\mathbf{x}_n = d_n - \mathbf{x}_n^t\mathbf{w}_n \\ &= \mathbf{x}_n^t\tilde{\mathbf{w}}_n + \varepsilon_n \end{aligned} \quad (5)$$

$$\begin{aligned} \mathbf{x}_n e_n &= \mathbf{x}_n(\mathbf{x}_n^t\tilde{\mathbf{w}}_n + \varepsilon_n) \\ &= \mathbf{x}_n\mathbf{x}_n^t\tilde{\mathbf{w}}_n + \varepsilon_n\mathbf{x}_n \end{aligned} \quad (6)$$

Inserting (6) in the equation (4), we can obtain final adaptation equation as (7).

$$\tilde{\mathbf{w}}_{n+1} = \left[\mathbf{I} - \sum_{j=0}^{P-1} \mu \frac{\mathbf{x}_{n-j}\mathbf{x}_{n-j}^t}{\mathbf{x}_{n-j}^t\mathbf{x}_{n-j}} \right] \tilde{\mathbf{w}}_n - \sum_{l=0}^{P-1} \mu \frac{\varepsilon_{n-l}\mathbf{x}_{n-l}}{\mathbf{x}_{n-l}^t\mathbf{x}_{n-l}} \quad (7)$$

Convergence in the mean square means that the steady-state value of the covariance cov($\tilde{\mathbf{w}}_n$)of the weight error vector is finite. If these two forms of convergence are satisfied, then the APA algorithm is said to be stable. We begin the convergence analysis with the computation of the weight error vector covariance. Using (7), the covariance of the weight error vector $\tilde{\mathbf{w}}_n$ is given by:

$$\begin{aligned} \text{cov}(\tilde{\mathbf{w}}_{n+1}) &= E\left(\left[\mathbf{I} - \sum_{j=0}^{P-1} \mu \frac{\mathbf{x}_{n-j}\mathbf{x}_{n-j}^t}{\mathbf{x}_{n-j}^t\mathbf{x}_{n-j}} \right] \tilde{\mathbf{w}}_n \tilde{\mathbf{w}}_n^t \left[\mathbf{I} - \sum_{l=0}^{P-1} \mu \frac{\mathbf{x}_{n-l}\mathbf{x}_{n-l}^t}{\mathbf{x}_{n-l}^t\mathbf{x}_{n-l}} \right] \right) \\ &+ E\left(\left[\sum_{j=0}^{P-1} \mu \frac{\varepsilon_{n-j}\mathbf{x}_{n-j}}{\mathbf{x}_{n-j}^t\mathbf{x}_{n-j}} \right] \left[\sum_{l=0}^{P-1} \mu \frac{\varepsilon_{n-l}\mathbf{x}_{n-l}^t}{\mathbf{x}_{n-l}^t\mathbf{x}_{n-l}} \right] \right) \\ &- E\left(\left[\mathbf{I} - \sum_{j=0}^{P-1} \mu \frac{\mathbf{x}_{n-j}\mathbf{x}_{n-j}^t}{\mathbf{x}_{n-j}^t\mathbf{x}_{n-j}} \right] \tilde{\mathbf{w}}_n \left[\sum_{l=0}^{P-1} \mu \frac{\varepsilon_{n-l}\mathbf{x}_{n-l}^t}{\mathbf{x}_{n-l}^t\mathbf{x}_{n-l}} \right] \right) \\ &- E\left(\left[\sum_{j=0}^{P-1} \mu \frac{\varepsilon_{n-j}\mathbf{x}_{n-j}}{\mathbf{x}_{n-j}^t\mathbf{x}_{n-j}} \right] \tilde{\mathbf{w}}_n^t \left[\mathbf{I} - \sum_{l=0}^{P-1} \mu \frac{\mathbf{x}_{n-l}\mathbf{x}_{n-l}^t}{\mathbf{x}_{n-l}^t\mathbf{x}_{n-l}} \right] \right) \end{aligned} \quad (8)$$

If the dependency of $\tilde{\mathbf{w}}_n$ on past measurement noise is neglected, using that ε_n is of zero mean, the last two terms of the above expression vanish. Furthermore, if we neglect the dependency of $\tilde{\mathbf{w}}_n$ on the past input vectors that appear in the first term of the above expression and use (A2) to simplify the second term, we can rewrite (8) as

$$\mathrm{cov}(\tilde{\mathbf{w}}_{n+1}) = E\left(\left[\mathbf{I} - \sum_{j=0}^{P-1}\mu\frac{\mathbf{x}_{n-j}\mathbf{x}_{n-j}^t}{\mathbf{x}_{n-j}^t\mathbf{x}_{n-j}}\right]\mathrm{cov}(\tilde{\mathbf{w}}_n)\left[\mathbf{I} - \sum_{l=0}^{P-1}\mu\frac{\mathbf{x}_{n-l}\mathbf{x}_{n-l}^t}{\mathbf{x}_{n-l}^t\mathbf{x}_{n-l}}\right]\right) + E\left(\mu^2\sum_{j=0}^{P-1}\left|\varepsilon_{n-j}\right|^2\frac{\mathbf{x}_{n-j}\mathbf{x}_{n-j}^t}{\left\|\mathbf{x}_{n-j}\right\|^2\mathbf{x}_{n-j}^t\mathbf{x}_{n-j}}\right) \tag{9}$$

Using (A3), we can rewrite the outer- to inner-product ratios as follows:

$$\frac{\mathbf{x}_{n-j}\mathbf{x}_{n-j}^t}{\mathbf{x}_{n-j}^t\mathbf{x}_{n-j}} = \frac{s_{n-j}r_{n-j}\mathbf{v}_{n-j}\mathbf{v}_{n-j}^t r_{n-j}s_{n-j}}{s_{n-j}^2 r_{n-j}^2\left\|\mathbf{v}_{n-j}\right\|^2} = \mathbf{v}_{n-j}\mathbf{v}_{n-j}^t \tag{10}$$

where $\mathbf{v}_{n-j}$ is one of $\{\mathbf{v}_1\ \mathbf{v}_2 \cdots \mathbf{v}_L\}$. Note that the above result is independent of the norm of $\mathbf{x}_{n-j}$. Now, substituting (10) into (9), we get,

$$\mathrm{cov}(\tilde{\mathbf{w}}_{n+1}) = E\left(\left[\mathbf{I} - \sum_{j=0}^{P-1}\mu\mathbf{v}_{n-j}\mathbf{v}_{n-j}^t\right]\mathrm{cov}(\tilde{\mathbf{w}}_n)\left[\mathbf{I} - \sum_{l=0}^{P-1}\mu\mathbf{v}_{n-l}\mathbf{v}_{n-l}^t\right]\right) + E\left(\mu^2\sum_{j=0}^{P-1}\left|\varepsilon_{n-j}\right|^2\frac{1}{r_{n-j}^2}\mathbf{v}_{n-j}\mathbf{v}_{n-j}^t\right) \tag{11}$$

Since ε_n is independent of $\mathbf{x}_n$ and r is independent of $\mathbf{v}_n$, from (A2) and (A3) respectively, we can rewrite (11) as

$$\mathrm{cov}(\tilde{\mathbf{w}}_{n+1}) = E\left(\left[\mathbf{I} - \sum_{k\in K_n}\mu\mathbf{v}_k\mathbf{v}_k^t\right]\mathrm{cov}(\tilde{\mathbf{w}}_n)\left[\mathbf{I} - \sum_{l\in K_n}\mu\mathbf{v}_l\mathbf{v}_l^t\right]\right) + \mu^2\xi^0 E\left(\frac{1}{r^2}\right)E\left(\sum_{k\in K_n}\mathbf{v}_k\mathbf{v}_k^t\right) \tag{12}$$

Where

$$K_n = \left\{k : \exists j \ni \frac{\mathbf{x}_{n-j}\mathbf{x}_{n-j}^t}{\mathbf{x}_{n-j}^t\mathbf{x}_{n-j}} = \mathbf{v}_k\mathbf{v}_k^t\right\} \subseteq \{1,2,\ldots,L\} \tag{13}$$

Let us define the diagonal elements of transformed covariance matrix $\mathbf{V}^t\mathrm{cov}(\tilde{\mathbf{w}}_n)\mathbf{V}$ as $\tilde{\lambda}_{n,i}$ for $i = 1,2,\ldots,L$.

That is ,

$$\left[\mathbf{V}^t\mathrm{cov}(\tilde{\mathbf{w}}_n)\mathbf{V}\right]_{ii} = \mathbf{v}_i^t\mathrm{cov}(\tilde{\mathbf{w}}_n)\mathbf{v}_i = \tilde{\lambda}_{n,i} \tag{14}$$

With the above notation, the pre- and post-multiplication of (12) by $\mathbf{v}_i^t$ and $\mathbf{v}_i$ respectively results in

$$\tilde{\lambda}_{n+1,i} = E\left(\mathbf{v}_i^t\left[\mathbf{I} - \sum_{k\in K_n}\mu\mathbf{v}_k\mathbf{v}_k^t\right]\mathrm{cov}(\tilde{\mathbf{w}}_n)\left[\mathbf{I} - \sum_{l\in K_n}\mu\mathbf{v}_l\mathbf{v}_l^t\right]\mathbf{v}_i\right) + \mu^2\xi^0 E\left(\frac{1}{r^2}\right)E\left(\mathbf{v}_i^t\left[\sum_{k\in K_n}\mathbf{v}_k\mathbf{v}_k^t\right]\mathbf{v}_i\right) \tag{15}$$

From the orthonormality of the $\mathbf{v}_k$,

$$\mathbf{v}_i^t\sum_{k\in K_n}\mathbf{v}_k\mathbf{v}_k^t = \begin{cases}\mathbf{v}_i^t, & \text{if } i\in K_n\\ 0, & \text{if } i\notin K_n.\end{cases} \tag{16}$$

Using the above result, (16) can be rewritten as

$$\begin{aligned}\tilde{\lambda}_{n+1,i} &= \mathbf{v}_i^t\mathrm{cov}(\tilde{\mathbf{w}}_n)\mathbf{v}_i + E\left(\mathbf{v}_i^t\left[\sum_{k\in K_n}\mu\mathbf{v}_k\mathbf{v}_k^t\right]\mathrm{cov}(\tilde{\mathbf{w}}_n)\left[\sum_{l\in K_n}\mu\mathbf{v}_l\mathbf{v}_l^t\right]\mathbf{v}_i\right)\\ &\quad - E\left(\mathbf{v}_i^t\mathrm{cov}(\tilde{\mathbf{w}}_n)\left[\sum_{l\in K_n}\mu\mathbf{v}_l\mathbf{v}_l^t\right]\mathbf{v}_i\right) - E\left(\mathbf{v}_i^t\left[\sum_{k\in K_n}\mu\mathbf{v}_k\mathbf{v}_k^t\right]\mathrm{cov}(\tilde{\mathbf{w}}_n)\mathbf{v}_i\right)\\ &\quad + \mu^2\xi^0 E\left(\frac{1}{r^2}\right)E\left(\mathbf{v}_i^t\left[\sum_{k\in K_n}\mathbf{v}_k\mathbf{v}_k^t\right]\mathbf{v}_i\right)\\ &= \tilde{\lambda}_{n,i}[1 - \mu(2-\mu)\rho(i\in K_n)] + \mu^2\xi^0 E\left(\frac{1}{r^2}\right)\rho(i\in K_n)\end{aligned} \tag{17}$$

The probability $\rho(i\in K_n)$. is the same as the probability of drawing (with replacement) the ball marked i, at least once in P trials, from a collection of L balls marked $1,2,\ldots,L$, where the probability of drawing the ball marked j is p_j. Hence

$$\rho(i\in K_n) = 1-(1-p_i)^P \tag{18}$$

By substituting (18) into (17), we get

$$\tilde{\lambda}_{n+1,i} = (1-\alpha\beta_i)\tilde{\lambda}_{n,i} + \mu^2\xi^0 E\left(\frac{1}{r^2}\right)\beta_i \tag{19}$$

Where $\alpha = \mu(2-\mu)$ and $\beta_i = 1-(1-p_i)^P$.

Using (A4), we can rewrite equation (19) to more definite forms.

$$\tilde{\lambda}_{n+1,i} = (1-\alpha\beta_i)\tilde{\lambda}_{n,i} + \mu^2\xi^0\left(\frac{1}{L\cdot\sigma_x^2}\right)\beta_i \tag{20}$$

In the steady-state region, the eigenvalues are not changed($\tilde{\lambda}_{n+1,i} = \tilde{\lambda}_{n,i}$). With this, we rewrite (20) as below,

$$\begin{aligned}\lim_{n\to\infty}\tilde{\lambda}_{n+1,i} = \tilde{\lambda}_{\infty,i} &= \frac{\mu^2\xi^0}{\alpha}\left(\frac{1}{L\cdot\sigma_x^2}\right) \quad \Leftarrow \alpha = \mu(2-\mu)\\ &= \frac{\mu}{2-\mu}\xi^0\left(\frac{1}{L\cdot\sigma_x^2}\right)\end{aligned} \tag{21}$$

The steady-state mean-squared error, $J_n = E(e_n^2)$ in the output estimate can be written as

$$\begin{aligned} J_{\infty,AP} &= \xi^0 + \sum_{i=1}^{L} \lambda_i \tilde{\lambda}_{\infty,i} \\ &= \xi^0 + tr(\mathbf{R})\tilde{\lambda}_{\infty,i} \\ &= \xi^0\left[1 + \frac{\mu}{2-\mu}\left(\frac{P\cdot L\cdot\sigma_x^2}{L\cdot\sigma_x^2}\right)\right] \\ &= \xi^0\left(1 + \frac{\mu P}{2-\mu}\right) \end{aligned} \tag{22}$$

From the equation (22), we find steady-state mean-squared error increase according to increasing projection order in AP algorithm.

The steady-state echo gain, which is defined as the ratio of excess mean-squared error to far-end signal power(Echo Gain), equals

$$\begin{aligned} \mathrm{EG}_{AP} &= \frac{j_{\infty,AP} - \xi^0}{\sigma_x^2} = \frac{j_{\infty,AP} - \sigma_t^2}{\sigma_x^2} \\ &= \frac{\sigma_t^2}{\sigma_x^2}\left(\frac{P\cdot\mu}{2-\mu}\right) \quad \Leftarrow \mu = 1 \quad \textit{for simplicity} \\ &= \frac{P\cdot\sigma_t^2}{\sigma_x^2} \end{aligned} \tag{23}$$

Where, $\xi^0 = \sigma_t^2$, steady-state error power would be measurement noise power($\lim_{n\to\infty}\sigma_{e,n}^2 = \sigma_t^2$). From the equation (23), we find steady-state echo gain linearly increase with projection order of AP algorithm and steady-state error power.

III. PROPOSED MODIFIED AP ALGORITHM

A new modified AP algorithm is proposed to reduce noise amplification problem of AP algorithm. The proposed modified AP algorithm normalizes the update equation to reduce noise amplification of AP algorithm, by adding the multiplication of error power and projection order to auto-covariance matrix of input signal.

In the proposed algorithm, we use (24) instead of (2). To analyze the convergence behavior of the proposed algorithm, we use same manner as in chapter **II**.

By using (24), the steady-state mean-squared error is given by (25).

$$\overline{\mu}_k = \begin{cases} \dfrac{\mu e_n}{\mathbf{x}_n^t\mathbf{x}_n + P\cdot L\cdot\sigma_{e,n}^2} & \textit{for } k=0, \quad \textit{if } \|\mathbf{x}_n\| \neq 0 \\ \dfrac{\mu e_n^k}{\mathbf{x}_{n-k}^t\mathbf{x}_{n-k} + P\cdot L\cdot\sigma_{e,n}^2} & \textit{for } k = 1,2,\cdots,P-1, \\ & \textit{if } \|\mathbf{x}_{n-k}\| \neq 0 \\ 0 & \textit{otherwise} \end{cases} \tag{24}$$

$$\begin{aligned} J_{\infty,proposed} &= \xi^0 + \sum_{i=1}^{L}\lambda_i\tilde{\lambda}_{\infty,i} \\ &= \xi^0 + tr(\mathbf{R})\tilde{\lambda}_{\infty,i} \\ &= \xi^0\left[1 + \frac{\mu}{2(L\cdot\sigma_x^2 + P\cdot L\cdot\sigma_{e,n}^2) - \mu L\cdot\sigma_x^2}P\cdot L\cdot\sigma_x^2\right]. \end{aligned} \tag{25}$$

From the equation (25), we can find steady-state mean-squared error on the proposed algorithm cancels the influence from projection order in AP algorithm.

Using (25), the steady-state echo gain equals

$$\begin{aligned} \mathrm{EG}_{proposed} &= \frac{j_{\infty,proposed} - \xi^0}{\sigma_x^2} = \frac{j_{\infty,proposed} - \sigma_t^2}{\sigma_x^2} \\ &= \frac{\sigma_t^2}{\sigma_x^2}\left(\frac{\mu\cdot P\cdot L\cdot\sigma_x^2}{2(L\cdot\sigma_x^2 + P\cdot L\cdot\sigma_t^2) - \mu\cdot L\cdot\sigma_x^2}\right) \\ &= \frac{\sigma_t^2}{\sigma_x^2}\left(\frac{\mu\cdot P}{2 + 2P\cdot(\sigma_t^2/\sigma_x^2) - \mu}\right) \Leftarrow \mu = 1 \quad \textit{for simplicity} \\ &= \frac{\sigma_t^2}{\sigma_x^2}\left(\frac{P}{1 + 2P\cdot\dfrac{\sigma_t^2}{\sigma_x^2}}\right) \end{aligned} \tag{26}$$

Where, σ_t^2/σ_x^2 is defined as Near- to Far-end signal power ratio(NFR). From the equation (26), we can also show that steady-state echo gain for the proposed algorithm cancels the influence from projection order and steady-state error power.

IV. COMPUTER SIMULATION AND RESULTS

A. Steady-state echo gain

First, we calculate steady-state echo gain with the proposed algorithm and AP by computer simulation. In the simulation, step-size, $\mu = 0.2$, NFR are ranged from 10^{-3} to 10^{3}, and projection order are 2, 6, 10 in AP and proposed algorithm, respectively.

From the fig 1, the echo gain of the AP algorithm increases linearly with target power and projection order. This is a serious shortcoming of the AP algorithm. In comparison, the proposed algorithm provides substantially improved steady-state performance at high NFR.

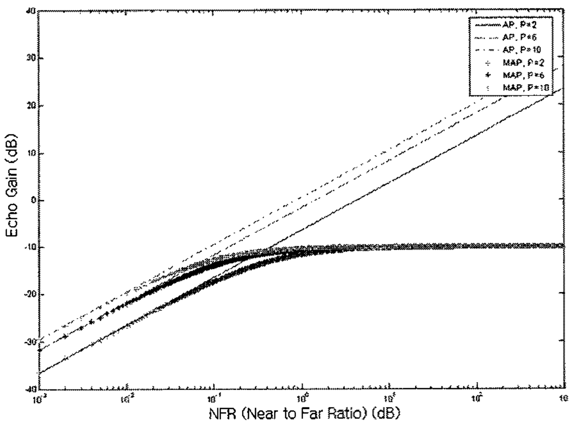

Fig. 1. Steady-state echo gain as a function of NFR.

B. AIC(acoustic interference cancellation) in acoustic echo canceller

Next, we apply to acoustic echo canceller with the proposed algorithm in hands-free environments. Far-end signal recorded with 8 kHz sampling rate, 16 bits quantization-level, and 10 second-long man and woman alternate pronounced English sentences. Far-end signal to background noise ratio set to 60 dB and 20dB by assuming low level and high level white-Gaussian background noise, respectively. And acoustic echo path impulse response measured in small size office room with 512^{th} order lengths. Adaptive filter has the same length, step-size, $\mu = 0.125$ and projection order $P=2$. For performance evaluation with proposed algorithm, AIC(acoustic interference cancellation) were used[8].

$$AIC(k) = 10\log_{10}\frac{E\{y^2(k)\}}{E\{\hat{z}^2(k)\}} = 10\log_{10}\frac{E\{y^2(k)\}}{E\{y^2(k)-\hat{i}^2(k)\}} \quad [dB] \tag{27}$$

Where, $\hat{i}(k)=\hat{d}(k)+\hat{n}(k)$ means estimated interference signal including estimated echo and background noise. Equation (27) means power ratio of microphone input signal $y(k)$ (including acoustic echo and background noise) and transmitted signal $\hat{z}(k)$ (or residual error signal). Therefore as acoustical echo is eliminated more by acoustic echo canceller, AIC has larger value.

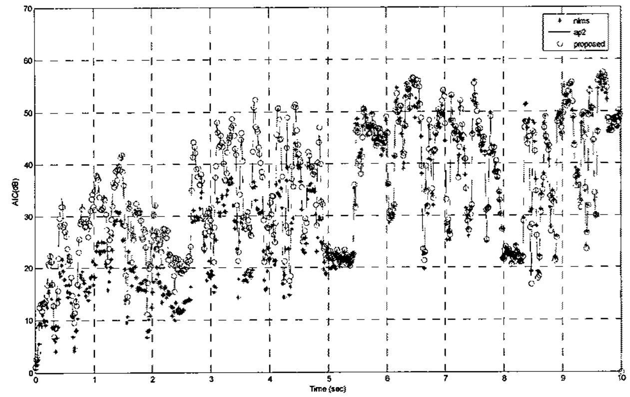

Fig. 2. AIC comparison with 60dB white Gaussian noise.

From the fig. 2, it is impossible to distinguish performance of the AP algorithm from the proposed algorithm in relatively low background noise power.

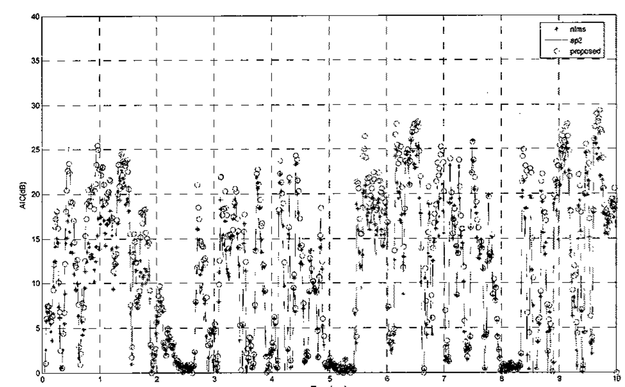

Fig. 3. AIC comparison with 20dB white Gaussian noise.

From the fig. 3, it shows that the proposed algorithm has about 3dB better performance than NLMS and AP algorithms in relatively high background noise power.

C. Comparison of Algorithm Complexity

Table. I shows the complexity of each algorithms. It compares algorithm complexity with number of multiplication in unit iteration. The number of multiplication of Modified AP is almost same to AP when the filter length is long.

TABLE I
Comparison of Algorithm Complexity

Algorithm	Multiplication	Complexity (P=2, L=1024)
NLMS	$2L+4$	2,052
AP	$2P\cdot L+7P^2$	16,412
Modified AP	$2P\cdot L+7P^2+P+3$	16,417

IV. CONCLUSIONS

We derive the convergence behavior of the AP algorithm using a simple model for the input signal vector with the usual independent assumption with theoretical analyses. From the analysis of convergence behavior, we were able to suggest a modified AP algorithm. By computer simulation, the proposed algorithm substantially improves the problem of the AP algorithm.

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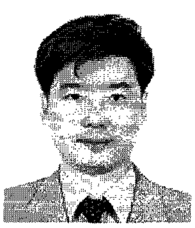

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