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A STUDY ON THE RELATION BETWEEN MATHEMATICS AND FOREIGN LANGUAGE

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Abstract. We observed the symptoms that occur to students who dislike mathematics when they study mathematics and the data that mathematics is related to foreign language. This study investigated the relation between mathematics and foreign language. Continuous immersion aids not only in acquiring language but also in learning mathematics. For continuous immersion, it is essential to organize small class. We organized small class and compared large class with small class about how the relation between mathematics and language appears in achievement, rate of presence, rate of submission of report, and attitude and enthusiasm. Based on the result, we try to find out the way to increase understanding mathematics and level up the achievements.

1. Introduction

To solve mathematical problems to students who have difficulties in mathematics learning makes fears, stresses, and anxieties. These feelings are called mathematical disabilities and these are defined as frustration and languor on ability to solve mathematical problems. These feelings are a factor of challenge of all the students who have learned mathematics.

We know that to increase fears and stresses about mathematics is consistent with the diminish of pride and the tendency that confidence and interest fall. Students with prejudice against mathematics suffer from lack of understanding mathematics. On the other hand, students

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with mathematical talents admit mathematical problems analytically and synthetically. They have the mathematical feelings of pleasures instead of anxieties and fears. They have good understandings and achievements.

To students who cause psychological disabilities because they think that mathematics is uninteresting and worthless subject, we should get rid of the fears about the mathematics, make them study mathematics hard, and increase understanding mathematics.

Bloom classified learning objective as six regions but mathematics was often classified as three regions[6]. One of the regions is the region of understanding. Language includes sorts of logic(classification, relation, proposition operation), and numeral. This says that mathematics is related with language.

Since mathematics uses symbols much and condenses thought process and represents it economically, most students feel difficult in mathematics. This study observed the relation between mathematics and language and compared large class with small class about how the relation appears in achievement, rate of presence, rate of submission of report, and attitude and enthusiasm. Based on the result, language activities between teacher and students are done much. We hope that fears against mathematics are got rid of and students are immersed in mathematics. The purpose of this study is to increase understanding mathematics and level up the achievements. In this study, the relation between mathematics and foreign language was centered on the learning.

2. Theoretical background

The symptoms that occur to students who dislike mathematics when they study mathematics are fears, headache, nervousness, stomachache, and etc. The cause of these symptoms is that students feel difficult in mathematics and they can't solve the problems. There is the case they can't understand mathematics problems among the reasons that they can't solve the problems. This represents that mathematics is related to language.

Egyptian and Mesopotamian achievement played a major role in the development of mathematics but up to B.C. 600 the development of mathematics became slow and other methods except trial and error or mathematical method to observe result of chance were not presented[6].

From Greek era mathematics started to be developed with different method from form which was developed till then. Based on contents of mathematics to be developed by that time, Greeks added the contents of mathematics much. What was more noticeable was that they completely changed the method or character to study their mathematics. Many historians presented different opinions each other about why Greeks used these techniques and innovated in the method to study mathematics but common views were as follows. Because there was social wealth or slavery, social atmosphere to seek knowledge and create the culture as not a means of living innovated in the method to study mathematics. Thus Greeks seemed to try to improve the fault of mathematics system that Egyptian and Mesopotamians introduced. Anyway, Greeks were familiar with logic and critical thinking and studied mathematics vitally by introducing this techniques[6].

As a result, they played a major role in the field to study phenomenon to occur in the world of nature and started to concentrate to build the hard logical foundation rather than to simply apply certain mathematical knowledge to natural phenomena. Greece was the center of mathematics in ancient countries. Greece was conquered by Rome, but Rome did not conquer mathematical language of Greece. Greece thought mathematics as language.

Theoretical positivism started by Vienna Circle promoted this interpretation by classifying two kinds of truth, i.e. experimental and synthetic truth based on perception and analytic truth based to topological combination started from definition. Carnap to try to restore analytic truth to pure syntax noticed necessity to introduce semantics. This analysis says that mathematics is related with language since they look at logical and mathematical truth as meaningfully completed general topological syntax[4].

Adams displayed common points between mathematics and language as the following[1].
-Abstraction is used to communicate.
-Symbols and rules are uniform and consistent.
-Expressions are linear, continuous, and consecutive.
-To improve understanding needs practices.
-Success needs memorization of symbols and rules.
-Interpretation is necessary to beginners.
-Order of symbols affects the meaning of words.

level	English	Mathematics
1	Stage which neither understands nor answers	Stage which doesn't understand teacher's explanation
2	Stage which understands but speaks only yes or no	Stage which understands teacher's explanation and solves basic problems
3	Stage which speaks with sentences	Stage which solves basic problems but doesn't apply to application problems
4	Stage which speaks somewhat fluently	Stage which applies to application problems

TABLE 1. Comparison of level of English and mathematics

-Communication needs decoding and encoding.
-Intuition, insight, and talking with thinking accompany fluency.

According to Adams, mathematics is the language that people use to communicate, to solve the problems, to relax, and to make artistic job and mechanical tool. Although mathematics was not considered as language historically, the present view makes it consider as language.

Comparing mathematical ability with ability of English conversation, we can present levels by 4 stages as Table 1. Students evaluate their mathematical ability in accordance with mathematical level. Once specific mathematical level is determined, mathematics class is proceeded in accordance with the level as in English class. This way gives students higher compensation and achievement than traditional class.

3. Relation between mathematics and foreign language

Analyzing texts as foreign language not native language is a threatening work and it needs a strategy. Whether mathematics consists of numerical expressions or words, students are faced with the same challenge when they analyze the mathematics problem. To help students do such a work, we can apply outline on the problem-solving method Polya made.

Polya's problem-solving 4 stages are as follows.
1) Understanding the problem
2) Devising a plan
3) Carrying out the plan
4) Looking back

The stage of understanding the problem and the stage of looking back are particularly related to the assertion that mathematics is the language[5]. The following example shows how to relate mathematics with language by Polya's problem-solving four stages.

Example: Perimeter of a right triangle is 60cm and a perpendicular line perpendicular to hypotenuse is 12cm. Solve three lengths of given triangle.

Stage 1: Understanding the problem

To answer the problem not to be understood is foolish. Students should understand the problem. First of all, linguistic statements to explain problem should be understood. Teacher should check this fact to some degree. Teacher asks students to read the sentences by repetition and lets them state the problem fluently. Students should be able to point out main point of the problem, i.e. the unknown, data, and condition.

The beginning stage can be related to beginners to learn foreign language newly. Stage to know the vocabulary and structure of sentence is decisive to beginners. Thus beginning stage to read sentence is to understand the method to begin. Many mathematical statements take different meaning from daily language. An ability to interpret this kind of daily words mathematically is crucial to success. Next stage after students apply proper definition is to read the statement again and to recognize the subject-predicate structure to provide clue of context. This process is similar to interpretation of English.

When considering by subject-predicate approach to establish the equation and solve it, it gives help to students to learn mathematics. Solution of mathematical problems to all the students to interpret mathematical problem as language is the key of success. When students are immersed in the textbook and are connected to the aim teacher sets up, they can understand the contents. Mathematicians assert that practical life should make ideal environment to be connected mathematically.

By applying stage 1 to given example, we observe to understand the problem correctly. Through the conversation between teacher and students, they can point out the important part of the problem.

Stage 2: Devising a plan

If they know some calculations or some constructions or at least the outline to solve the unknown, then they draw up a plan from them. The way from understanding the problem to devising a plan can be long and complicated. In fact, if the thinking about a plan occurs, the solution of the problem is almost achieved. Such thinking can be occur gradually. Or after trial and error or hesitation, thinking as flash can suddenly occur. The best thing that teacher can do is to help students and let them be in this state. To understand students, teacher should think difficulty that he suffered from when he solved the problem and experience for the success.

To understand statements whether mathematics or English, students should interpret the textbook. When students can define their words and interpret the related information successfully, they can interpret as desired. Adams recognized that a student's ability to recognize and employ the formal definition is key to understanding and applying concepts when reading mathematical text[1].

The process to apply stage 2 to given example is as follows. Students understood somewhat the problems and showed the interest about the problem. Now they themselves can think and become to have the positivity.

To establish the equation, we should decide the hypotenuse. Let the hypotenuse be c and the other two sides of having right angle be a and b. Then two formulas are established as follows.

$$a + b + c = 60 \tag{1}$$

$$ab = 12c \tag{2}$$

Since given triangle is a right triangle, we can establish the following formula by Pythagorean theorem.

$$c^2 = a^2 + b^2 \tag{3}$$

Stage 3: Carrying out the plan

To draw up a plan i.e. to devise the solution is not easy. To achieve this, much thing is required. Previously acquired knowledge, great thinking habit, and concentration for the aim are demanded and the luck is needed. Compared with previous stage, to execute the plan is much easier. What is demanded in this step is the patience.

Since a plan presents a general outline, we should make sure that the details fit the outline perfectly. Until everything becomes very clear and an ambiguous place which error can be hidden does not remain, we have the patience and should check the details in order.

Once students look back the problem quickly and confront a linguistic challenge, we should find the solution of the problem summing all the informations. Students should execute the devised plan.

Return to the given example.

$$(a+b)^2 = a^2 + b^2 + 2ab \tag{4}$$

From (1), we have

$$a + b = 60 - c \tag{5}$$

We substitute (5) for the left side of (4). Substituting (2),(3) for the right side of (4), we have the following

$$(60 - c)^2 = c^2 + 24c. \tag{6}$$

From this, we have

$$c = 25, \tag{7}$$

Substituting (7) for (1) and (2), we have

$$a + b = 35, ab = 12 \cdot 25 \tag{8}$$

Thus three lengths of the triangle are as follows.

$$a = 15, b = 20, c = 25 \tag{9}$$

or

$$a = 20, b = 15, c = 25. \tag{10}$$

Stage 4: Looking back

Now students executed the plan, checked the every stage, and described the solution. So there is enough reason to believe that their solutions are correct. Nevertheless errors can be always happened. Particularly, in case the process of proof or solution is long and complicated, more errors can be always happened. Thus to prove is desirable.

After solving all the problems, they should ask themselves what the figures mean. Unless students themselves don't ask, teacher should lead students to ask such questions. Looking back on a solved problem lets students encourage in discussions about the problem-solving process to further enhance their reasoning skills and abilities to explain and justify solutions[1].

In the given example, they finally got the solution. Three sides are $a = 15, b = 20, c = 25$ or $a = 20, b = 15, c = 25$. Teacher asks students to apply all the given conditions and make sure to satisfy all the conditions.

Students should be able to explain solution to friends and teacher. They give and take the mathematical technique with explanation. So they become to know the mathematical concepts and to understand the concepts completely. Writing is the good way to reflect the completed work. It supports learning because it requires students to organize, clarify and reflect on their ideas-all useful processes for making sense of mathematics[2].

To write down thinking and solution of every stage not only develops the future method which is the fittest to only learning style but also reflects students' own process. To apply Polya's problem-solving 4 stages is the efficient way to teach mathematics in the linguistic aspect. Each stage is logical and is familiar with mathematicians. Another efficient way can be deduced from the strategy used in linguistic education.

To teach mathematics in the aspect of language needs three important concepts: small class, connectedness, and immersion.

Small class is ideal to teach mathematics in the linguistic aspect. To teach mathematics with thinking the linguistic concept is as important as presenting the success of mathematics class in the classroom. But to enforce the process to learn mathematics as language affects students' abilities and will let them acquire the mathematical knowledge deeply.

If we give students small class, they can present their conclusion in the clear form. To work with friends relieves stresses to talk in front of all the students. Once students return to the whole class after finishing small class, they have chances to share thinking of the group as mathematical language.

The process to connect it to language was previously presented according to Polya's problem-solving 4 stages. To understand the world of number, students should connect thinking to material learned previously. Many students' parents did not have the mathematics-related job. Some students would be born at the culture which did not emphasize mathematics at all. Thus it is important for teacher to connect students' world to enormous world of mathematics. Students need to understand to connect mathematics not only to the classroom but also to all the area of the society. Although most mathematics teachers recognize the concept of connectedness, it casts a new light on the thinking to immerse students in the culture of mathematics.

To explain how to use mathematics in their work, to invite local students to represent various occupations can show students the various application of mathematics. Going back to the history of mathematics from Egyptian mathematician to Babylonian, Indian, Greek, Chinese, and mathematicians in today, there would be the method to connect each culture to their languages. Mathematicians say that objective of connectedness should develop the opportunity to enforce that students understand mathematical symbols and concepts.

Linguists say that immersion is one of the best way for students to learn foreign language. Although mathematical immersion does not mean to immerse completely as English, immersion in mathematics is the important element of learning.

Immersion is the word to express feeling that behaviors are accomplished naturally as the water flows in the moment life reaches the climax. When the clear objective is in the front of people, the possibility to immerse in it is high[3]. Immersion is the phenomenon which appears when pouring students' capability entirely in overcoming project which is not easy and not burdensome. When the harmony between behavior and opportunity is accomplished, we do the desirable experience. Project is so burdensome that students give up in fear of that. Project and capability are so low that they feel lukewarm enough no matter how often they experience. But to combine burdensome project with high level capability, the deep participation and immersion which are not experienced in daily life are occurred.

If objective is clear, result of activity appears soon, and project keeps the balance with students' capability, students can concentrate on study systematically. Since immersion demands the mental power wholly, students in immersion devote themselves to the study wholly. If difficulty of mathematics project have the harmony with the capability of students properly, we can raise up the quality of mathematics with tasting the immersion. Since the method of mathematics education in the linguistic aspect needs small class and immersion, we check to have the experience of immersion after trying small class.

Teachers will know that students who have the mathematics class through the immersion will understand mathematics concepts more deeply. Parents will encourage students to immerse in mathematics. These strategies attempt to aid students in acquiring a language through continuous immersion rather than periodic instruction[7]. If immersion is the great strategy to learn languages like English, Spanish, French, and German, it will be effective to teach mathematics in the same way as foreign language.

4. Method of study

4.1. The subject of study. The subjects of study are freshmen entered in 2010 in the department of computer science and engineering at my school. For continuous immersion, it is essential to organize small class. We organized small class and large class and compared achievement, presence, and attitude. Large class kept the traditional class but small class did the activities of language much for experiment about the relation between mathematics and language. Large class was organized into three classes which consisted of 58, 58, and 59 students. Small class was organized into one class which consisted of 5 students.

4.2. Process of study. Experiment 1: Comparison of achievement of large class and small class

For experiment, large class kept the traditional class but small class applied Polya's problem-solving 4 stages. At the end of semester, we compared the achievement of large class to that of small class. In small class, we asked all the students and called them individually to ask. These are not simple questions but language activities between teacher and students. This is Polya's problem-solving stage 1. This stage is the stage to check to understand problem correctly.

After students understand the problem, teacher draws up a plan of the problem with them. Teacher explains the concepts, looks for the principle, and makes the foundation of solving problem. This is Polya's problem-solving stage 2.

In small class teacher observes the students' expressions slowly and quickly looks for the part students don't understand. After finishing the fundamental explanation of the problem, teacher gives students time to solve the problem and then lets them solve it. This is the problem-solving stage 3. This stage is much easier than stage 1,2, but this stage needs the patience and fundamental knowledge. In case that they follow up to stage 1,2 but they don't solve the problem because there are no the patience or the fundamental knowledge, teacher helps the students individually.

After that, students to solve the problem come up and become to get the time to explain. This stage is Polya's problem-solving stage 4 that students to solve the problem look back the problem to solve by themselves. In this stage, meanwhile students explain and listen, they have the activities to check that the meaning is conveyed correctly and they know the learned contents correctly. They make the atmosphere to respect each other and meet the problem-solution to their belief. So the affirmative atmosphere of class is made.

To find achievements of the large class and small class, we compared GPA(grade point average) of mid-term exam and final exam.

Experiment 2: Comparison of rate of submission of reports and rate of presence

Experiment 2 compared the rate of submission of reports and rate of presence of large class and small class. Reports of large class and small class were handed in 4 times. We observed the rate of reports by class. By checking the attendance of each class, we compared the rate of attendance of large class and small class. Even if students in large class were present at first, some went out the classroom meanwhile the class and the rate of practical attendance made the difference. On the other hand, students in small class did not go out meanwhile the class and the rate of practical presence did not make the difference. But presence used in the experiment was the presence checked at first and presence of students who went out in the midway was not considered.

Experiment 3: Class attitudes and passions

Experiment 3 is the experiment that compares students' attitudes and passions. For this experiment, each time we compared students'

	Large class			Small class
class	A	B	C	
GPA	3.03	2.97	2.95	4.2

Table 2. GPA of large class and small class

attitudes and passions of large class and small class and checked this result through questionnaire.

5. Analysis of study result

The result of experiment 1:
Experiment 1 observed GPA of large class and small class. GPA of large class were 3.03, 2.97, and 2.95, by class, respectively and GPA of small class was 4.2. GPA of small class is much higher than that of large class. Students in small class were not good at the achievements at first and they were students inferior to students in large class as students taking a course again.

Since large class has many students and levels among students, it has the hard condition to teach with many linguistic activities. But since small class is possible to teach with linguistic activity, achievements of small class were better than those of large class. This says that students' understanding of small class is better than that of large class. Table 2 presents GPA of students taking discrete mathematics in large class and small class.

The result of experiment 2:
In large class the students' rates of submission of reports presented 83.6%, 82.6%, and 82.8% by class. In small class, the students' rates of submission of reports presented 95%. Table 3 presents the rate of submission of reports of large class and small class.

In large class, students' rates of presence presented 94.8%, 94.2%, and 93.6% by class. In small class, students' rates of presence presented 97.5%. Students in large class presented lower achievement than those in small class in the aspect of submission of reports and presence. Table 4 presents the rate of presence of large class and small class.

The result of experiment 3:
Since mathematics is strict and logical and concentrates the process

	Large class			Small class
class	A	B	C	
%	83.6	82.6	82.8	95

Table 3. The rate of submission of reports of large class and small class

	Large class			Small class
class	A	B	C	
%	94.8	94.2	93.6	97.5

Table 4. The rate of presence of large class and small class

of thinking highly and represents it economically, high concentration is needed to learning. So in the large class only a few good students showed reaction to class with the teacher. As we should memorize English by mouth continuously, we should solve mathematics by hand continuously. But large class is hard to give and take lessons with students and it is hard for students to solve many problems. On the other hand, in small class teachers are able to make eye contact with all the students and to ask students step by step when implementing Polya's problem-solving stage 4. Teachers let students attend the class and can teach them individually. When they study mathematics, it is not enough for them to listen to teacher's explanation and to just look at the solution. Learning how to analyze the problem by themselves and how to write the solution practically and to look back is demanded.

In small class, teacher can ask to individual and the chances to solve the problem practically are much given. In large class, students felt nervous about solving the problem wrong in front of many students. But in small class, students did not feel nervous even solving hard problems and they experienced to take the pleasure of success and discovery.

In fact, according to the result of questionnaire, most students in small class answered that small class made them concentrate well, understand the contents of mathematics easily, have the interest, and not feel nervous about solving the problem in front of many students. Students in small class immersed in mathematics and had the experience to challenge the hard problems. When teacher teaches mathematics in aspect of linguistic concepts, small class is ideal. This represents that the number of students affects the related class with mathematics and language.

Experiment 3 is the experiment about how the language relatedness in small class and large class affects the class through attitudes and passions. In small class, even students to take courses again were full of passions and earnest. Because the class related with language was proceeded, the understanding of class and students' correspondence were high. Because time to solve the problem in front of many students was given much, students didn't look at teacher's explanation with just eyes and solved the problem directly. Whenever solution of the problem was wrong, students communicated with teachers.

Small class did not mean cooperative class but all the students who participated in small class studied hard for the common objective which would be for them to have the good grades. Their learning attitudes were improved, too. They cooperated each other, achieved the assignments, liked each other, and became to form the friendly and affirmative bond. Students were able to have the affirmative self concept through the class.

Since teacher proceeded the class which fitted some students' level in large class, students in mid-low level did not follow the class. Teacher played the assistant role for promoting students' education. But except a few excellent students, students in large class felt isolated and passive and a desirable class did not get accomplished.

6. Conclusion

To answer 'How should we teach mathematics to students with stressing in the aspect of understanding', we observed the data about the relation between mathematics and foreign language. To teach mathematics in the linguistic aspect, three important concepts, which are small class, connectedness, and immersion, are needed. Among them small class is the ideal and important element to teach mathematics in the linguistic aspect.

This study observed the relation between mathematics and language, centered on small class which is the most important element of immersion. Small class and large class were organized and we observed how achievement, the rate of attendance, rate of submission of reports, and attitudes and passions were presented in small and large class. As a result, we concluded as the following.

First, the achievement in small class which has many language activities between teacher and students was higher than that in large class and so understanding of students in small class was higher than that of students in large class.

Second, students in small class showed higher accomplishment than students in large class in the aspect of attendance and submission of reports. This represents that students in small class were more faithful than students in large class.

Third, in the experiment of class attitudes and passions, even if students took the course again in small class, they were full of passions and faithful. Since the class related with language was proceeded, understanding of class and correspondence with teacher were high. All the students who participated in small class studied hard for the common objective which would be for them to have good grades and their learning attitudes were improved. They cooperated each other, achieved the assignments, became to like each other, and became to form the friendly and affirmative bond. Students were able to have the affirmative self concept through the class. However, in large class, one-sided class rather than two-sided class was proceeded. Since activities of language in large class were not much, the evaluation in qualitative aspect of the understanding of mathematics concept, correspondence with teacher, and achievements was negative. Except a few excellent students, students in mid-low level felt isolated and passive about the class and so they were short of confidence and pride.

Above conclusions tell us that it was not until small class is supposed that there is good achievement when teacher teaches mathematics in the linguistic aspect. In this study, small class presented the mathematics class which had language activity vividly and large class presented the mathematics class which did not have language activity vividly.

The way to give effective mathematics class in most classroom which have carried out large class is through the method of language education. In fact, large class did not have many language activities as small class. If large class has many language activities as small class, then large class will have good achievements. The method to do language activity much should be considered.

To think mathematics through the method of language education will give the insight in the defect appeared in the traditional mathematics instruction. Now non-traditional opinion to use the common access to linguistic education provides suggestion to educational innovation which have the potential to prospective result. Doing language activity much with considering the relation with mathematics and language in math class presents good result.

We add language activity and reading activity much to classroom to learn mathematics. By doing so, education method related with language becomes lubricating oil and will anticipate good achievement in mathematics learning which uses symbols much and thought process is concentrated highly.

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