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Meromorphic Functions Sharing a Small Function with their Differential Polynomials

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Abstract. In this paper, we investigate uniqueness problems of meromorphic functions sharing a small function with their differential polynomials, and give some results which are related to a conjecture of R. Brück, and also improve several previous results.

1. Introduction

In what follows, a meromorphic (resp. entire) function always means a function which is meromorphic (resp. analytic) in the whole complex plane. We will use the standard notation in Nevanlinna's value distribution theory of meromorphic functions, see, e.g., [10,12,18]. As for the standard notation in the uniqueness theory of meromorphic functions, suppose that f, g are meromorphic and $a \in \hat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$, resp. a is a small meromorphic function in the usual Nevanlinna theory sense. Denoting by $E(a, f)$ the set of those points $z \in C$ where $f(z) = a$, resp. $f(z) = a(z)$, we say that f, g share a IM (ignoring multiplicities), if $E(a, f) = E(a, g)$. Provided that $E(a, f) = E(a, g)$ and the multiplicities of the zeros of $f(z) - a$ and $g(z) - a$ are the same at each $z \in C$, then f, g share a CM (counting multiplicities).

Meromorphic functions sharing values with their derivatives has become a subject of great interest in uniqueness theory recently. The paper [17] by Rubel and Yang is the starting point of this topic, along with the following.

Theorem A. *Let f be a nonconstant entire function. If f and f' share two distinct finite values CM, then $f = f'$.*

Examples of investigations in this field might be Mues and Steinmetz [16], Frank and Schwick [4], Yang [19], Gundersen [6–8]. In additional, we recall the following two representative results: Let k be a positive integer. If a meromorphic (resp. entire) function f shares two distinct finite values CM (resp. IM) with $f^{(k)}$, then $f = f^{(k)}$. For the proof, see [5] and [13].

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The following counterexample from [20] shows that the number 2 of shared values in the above results is necessary. Let k be a positive integer, and let $f = e^{bz} + a - 1$, where a and b are constants satisfying $b^k \neq 1$ and $a = b^k$. Clearly, f and $f^{(k)}$ share a CM, yet f and $f^{(k)}$ are not the same.

In order to get uniqueness theorems when a meromorphic function shares one finite value with its k-th derivative, some additional condition might be needed.

In 2003, Yu [23] considered the uniqueness problems with deficiency condition and obtained the following result.

Theorem B. *Let f be a nonconstant entire function, k be a positive integer, and let a be a small meromorphic function with respect to f such that $a(z) \not\equiv 0, \infty$. If $f - a$ and $f^{(k)} - a$ share the value 0 CM and $\delta(0, f) > \frac{3}{4}$, then $f = f^{(k)}$.*

For the other papers on this topic, the reader is invited to see the recent papers Lahiri [11], Zhang [24], Liu and Gu [14]. Theorem C below due to Lü and Zhang [15] is a closely related result involving linear differential polynomials. For shortness, we denote

$$L(f) = f^{(k)} + a_{k-1} f^{(k-1)} + \cdots + a_1 f', \tag{1.1}$$

where $a_j (j = 1, \ldots, k-1)$ are small meromorphic functions with respect to f.

Theorem C. *Let f be a nonconstant meromorphic function, n, k be positive integers and $a(z)$ be a small meromorphic function with respect to f such that $a(z) \not\equiv 0, \infty$. Let $L(f)$ be given by (1.1). Suppose that f^n and $L(f)$ share a IM (resp. CM) and $6\delta(0, f) + (2k + 6)\Theta(\infty, f) > 2k + 11$ (resp. $3\delta(0, f) + 3\Theta(\infty, f) > 5$), then $f^n = L(f)$.*

Recently, the present author and Yang [26] considered f^n sharing a small function with its k-th derivatives and got the following result.

Theorem D. *Let f be a nonconstant meromorphic function, n, k be positive integers and $a(z)$ be a small meromorphic function with respect to f such that $a(z) \not\equiv 0, \infty$. If $f^n - a$ and $(f^n)^{(k)} - a$ share the value 0 IM and*

$$n > 2k + 3 + \sqrt{(2k + 3)(k + 3)},$$

then $f^n = (f^n)^{(k)}$, and f assumes the form

$$f(z) = ce^{\frac{\lambda}{n} z}, \tag{1.2}$$

where c is a nonzero constant and $\lambda^k = 1$.

It is natural to ask whether n can be reduced in Theorem D. We give a result improving Theorem D in Section 2. In Section 3, we improve Theorem C by relaxing the deficiency condition. We offer some concluding remarks in the final Section 4.

2. Improvement of Theorem D

In order to get a general result, we consider f^n sharing a small meromorphic function with its differential polynomial $L(f^n)$, and obtain the following result.

Theorem 2.1. *Suppose that f is a meromorphic function, n and k are positive integers satisfying $n > 2k + 2$. Let $L(f)$ be given by* (1.1) *and $a(z)$ be a small meromorphic function with respect to f such that $a(z) \not\equiv 0, \infty$. If f^n and $L(f^n)$ sharing $a(z)$ IM, then $f^n = L(f^n)$.*

The following corollary that improves Theorem D comes from Theorem 2.1 immediately.

Corollary 2.2. *Let f be a nonconstant meromorphic function, n, k be positive integers and $a(z)$ be a small meromorphic function with respect to f such that $a(z) \not\equiv 0, \infty$. If f^n and $(f^n)^{(k)}$ share the value a IM and $n > 2k+2$, then $f^n = (f^n)^{(k)}$, and f assumes the form* (1.2).

Proof of Theorem 2.1. Denote

$$F = \frac{f^n}{a}, \quad G = \frac{L(f^n)}{a}.$$

Since f^n and $L(f^n)$ share $a(z)$ IM, then F and G share 1 IM except the zeros and poles of $a(z)$. Thus

$$\overline{N}\left(r, \frac{1}{F-1}\right) = \overline{N}\left(r, \frac{1}{G-1}\right) + S(r,f).$$

Suppose that $F \not\equiv G$. Noting the above equation and using logarithmic derivative theorem, we have

$$\begin{aligned}
\overline{N}\left(r, \frac{1}{F-1}\right) &\le \overline{N}\left(r, \frac{1}{G/F-1}\right) + S(r,f) \\
&\le T\left(r, G/F\right) + S(r,f) \\
&= N\left(r, L(f^n)/f^n\right) + m\left(r, L(f^n)/f^n\right) + S(r,f) \\
&\le k\overline{N}(r,f) + N_k\left(r, 1/f^n\right) + S(r,f) \\
&\le k\overline{N}(r,f) + k\overline{N}\left(r, 1/f\right) + S(r,f).
\end{aligned}$$

Substituting this into the second main theorem, we get

$$\begin{aligned}
T(r,f^n) &= T(r,F) + S(r,f) \\
&\le \overline{N}\left(r,F\right) + \overline{N}\left(r,1/F\right) + \overline{N}\left(r, 1/(F-1)\right) + S(r,F) \\
&\le (k+1)\overline{N}\left(r,f\right) + (k+1)\overline{N}\left(r,1/f\right) + S(r,f) \\
&\le (2k+2)T\left(r,f\right) + S(r,f),
\end{aligned}$$

which means $n \le 2k+2$, a contradiction. Then $F \equiv G$. The assertion follows. □

3. Improvement of Theorem C

In this section, we consider the case that f^n shares a small function with its differential polynomial $L(f)$, and get the following result.

Theorem 3.1. *Let $k(\geq 1)$, $n(\geq 2)$ be integers and f be a nonconstant meromorphic function, and let a be a small meromorphic function with respect to f such that $a(z) \not\equiv 0, \infty$. Let $L(f)$ be given by (1.1). Suppose that f^n and $L(f)$ share a IM and*

$$6\delta(0,f) + (2k+6)\Theta(\infty,f) > 2k+12-n, \tag{3.1}$$

or f^n and $L(f)$ share a CM and

$$3\delta(0,f) + (3+k)\Theta(\infty,f) > k+6-n, \tag{3.2}$$

then $f^n = L(f)$.

Remark 1. The deficiency condition (3.1) is weaker than $6\delta(0,f) + (2k+6)\Theta(\infty,f) > 2k+11$ when $n \geq 2$, and (3.2) is weaker than $3\delta(0,f) + 3\Theta(\infty,f) > 5$ when $n \geq 1 + \frac{k}{3}$. Therefore, Theorem 3.1 improves Theorem C when f^n and $L(f)$ share a IM. If $n \geq 1 + \frac{k}{3}$, Theorem 3.1 improves Theorem C when f^n and $L(f)$ share a CM.

In order to prove Theorem 2.1, we need the following lemmas. Firstly, we will give some notions.

Let p be a positive integer and $a \in \mathbb{C} \bigcup \{\infty\}$. We denote by $N_{p)}\left(r, \frac{1}{f-a}\right)$ the counting function of the zeros of $f-a$ with the multiplicities less than or equal to p, and by $N_{(p+1}\left(r, \frac{1}{f-a}\right)$ the counting function of the zeros of $f-a$ with the multiplicities larger than p; each point in these counting functions is counted only once. However, $N_p\left(r, \frac{1}{f-a}\right)$ denotes the counting function of the zeros of $f-a$ where m-fold zeros are counted m times if $m \leq p$ and p times if $m > p$. Obviously, $\overline{N}\left(r, \frac{1}{f-a}\right) = N_1\left(r, \frac{1}{f-a}\right)$.

Let F and G be two nonconstant meromorphic functions such that F and G share the value 1 IM. Let z_0 be a 1-point of F of order p, a 1-point of G of order q. We denote by $N_L(r, \frac{1}{F-1})$ the counting function of those 1-points of F where $p > q$; by $N_E^{1)}(r, \frac{1}{F-1})$ the counting function of those 1-points of F where $p = q = 1$; by $N_E^{(2}(r, \frac{1}{F-1})$ the counting function of those 1-points of F where $p = q \geq 2$; each point in these counting functions is counted only once. In the same way, we can define $N_L(r, \frac{1}{G-1})$, $N_E^{1)}(r, \frac{1}{G-1})$, and $N_E^{(2}(r, \frac{1}{G-1})$(see [22]). Particularly, if F and G share 1 CM, then

$$N_L\left(r, \frac{1}{F-1}\right) = N_L\left(r, \frac{1}{G-1}\right) = 0. \tag{3.3}$$

With these notations, if F and G share 1 IM, it is easy to see that

$$(3.4)\quad \overline{N}\left(r,\frac{1}{F-1}\right) = N_E^{1)}\left(r,\frac{1}{F-1}\right)+N_L\left(r,\frac{1}{F-1}\right) + N_L\left(r,\frac{1}{G-1}\right)+N_E^{(2}\left(r,\frac{1}{G-1}\right)=\overline{N}\left(r,\frac{1}{G-1}\right).$$

Lemma 3.2([21], Lemma 3)**.** *Let*

$$(3.5)\qquad H=\left(\frac{F''}{F'}-\frac{2F'}{F-1}\right)-\left(\frac{G''}{G'}-\frac{2G'}{G-1}\right),$$

where F and G are two nonconstant meromorphic functions. If $H\not\equiv 0$, then

$$(3.6)\qquad N_E^{1)}\left(r,\frac{1}{F-1}\right)\le N(r,H)+S(r,F)+S(r,G).$$

Lemma 3.3. *Suppose that two nonconstant meromorphic functions F and G share 1 and ∞ IM. Let H be given by* (3.5)*. If $H\not\equiv 0$, then*

$$(3.7)\quad T(r,F)+T(r,G)\le 3\overline{N}(r,F)+N_2\left(r,\frac{1}{F}\right)+N_2\left(r,\frac{1}{G}\right)+N_E^{1)}\left(r,\frac{1}{F-1}\right) + 2N_E^{(2}\left(r,\frac{1}{F-1}\right)+3N_L\left(r,\frac{1}{F-1}\right)+3N_L\left(r,\frac{1}{G-1}\right) + S(r,F)+S(r,G).$$

Proof. Since F and G share ∞ IM, we deduce from (3.5) that

$$(3.8)\quad N(r,H) \le \overline{N}(r,F)+N_{(2}\left(r,\frac{1}{F}\right)+N_{(2}\left(r,\frac{1}{G}\right)+N_L\left(r,\frac{1}{F-1}\right) + N_L\left(r,\frac{1}{G-1}\right)+N_0\left(r,\frac{1}{F'}\right)+N_0\left(r,\frac{1}{G'}\right),$$

where $N_0(r,\frac{1}{F'})$ denotes the counting function corresponding to the zeros of F' which are not the zeros of F and $F-1$, $N_0(r,\frac{1}{G'})$ denotes the counting function corresponding to the zeros of G' which are not the zeros of G and $G-1$. The second main theorem yields

$$(3.9)\quad T(r,F) \le \overline{N}(r,F)+\overline{N}\left(r,\frac{1}{F}\right)+\overline{N}\left(r,\frac{1}{F-1}\right)-N_0\left(r,\frac{1}{F'}\right)+S(r,F),$$

$$(3.10)\quad T(r,G) \le \overline{N}(r,G)+\overline{N}\left(r,\frac{1}{G}\right)+\overline{N}\left(r,\frac{1}{G-1}\right)-N_0\left(r,\frac{1}{G'}\right)+S(r,G).$$

Noting that F and G share 1 IM, it is easy to get

$$\overline{N}\left(r,\frac{1}{F-1}\right)+\overline{N}\left(r,\frac{1}{G-1}\right) = 2N_E^{1)}\left(r,\frac{1}{F-1}\right)+2N_L\left(r,\frac{1}{F-1}\right) + 2N_L\left(r,\frac{1}{G-1}\right)+2N_E^{(2}\left(r,\frac{1}{G-1}\right).$$

Using Lemma 3.2 and substituting (3.8) into above equation, we obtain

$$\begin{aligned}(3.11)\quad \overline{N}\left(r,\frac{1}{F-1}\right)+\overline{N}\left(r,\frac{1}{G-1}\right)&\le\overline{N}(r,F)+N_E^{1)}\left(r,\frac{1}{F-1}\right)+3N_L\left(r,\frac{1}{F-1}\right)\\&+3N_L\left(r,\frac{1}{G-1}\right)+2N_E^{(2}\left(r,\frac{1}{G-1}\right)+N_{(2}\left(r,\frac{1}{F}\right)\\&+N_{(2}\left(r,\frac{1}{G}\right)+N_0\left(r,\frac{1}{F'}\right)+N_0\left(r,\frac{1}{G'}\right).\end{aligned}$$

The assertion follows by combining (3.9), (3.10) and (3.12). □

Lemma 3.4([25], Lemma 2.4). *Suppose that f is a nonconstant meromorphic function and k, p are positive integers. Let $L(f)$ be given by* (1.1). *Then*

$$N_p\left(r,1/L(f)\right)\le k\overline{N}(r,f)+N_{p+k}\left(r,1/f\right)+S(r,f).$$

Proof of Theorem 3.1. Denote

$$F=\frac{f^n}{a},\quad G=\frac{L(f)}{a}. \tag{3.12}$$

Let H be given by (3.5). Suppose that $H\not\equiv 0$. We discuss the following two cases.

Case 1. Suppose that f^n and $L(f)$ share a IM. Then F and G share $1,\infty$ IM except the zeros and poles of a. From Lemma 3.3, we have (3.7). Since

$$\begin{aligned}N_E^{1)}\left(r,\frac{1}{F-1}\right)+2N_E^{(2}\left(r,\frac{1}{F-1}\right)&+N_L\left(r,\frac{1}{F-1}\right)+2N_L\left(r,\frac{1}{G-1}\right)\\&\le N\left(r,\frac{1}{G-1}\right)\le T(r,G)+O(1),\end{aligned}$$

we get from (3.7) and (3.12) that

$$\begin{aligned}(3.13)\quad T(r,F)&\le 3\overline{N}(r,F)+N_2\left(r,\frac{1}{F}\right)+N_2\left(r,\frac{1}{G}\right)+2N_L\left(r,\frac{1}{F-1}\right)\\&+N_L\left(r,\frac{1}{G-1}\right)+S(r,F)+S(r,G)\\&\le 3\overline{N}(r,f)+2\overline{N}\left(r,\frac{1}{f}\right)+N_2\left(r,\frac{1}{L(f)}\right)+2N_L\left(r,\frac{1}{F-1}\right)\\&+N_L\left(r,\frac{1}{G-1}\right)+S(r,f).\end{aligned}$$

By Lemma 3.4 and (3.12), we obtain

$$\begin{aligned}
N_2\left(r, \frac{1}{L(f)}\right) &\leq k\overline{N}(r,f) + N_{2+k}\left(r, 1/f\right) + S(r,f) \\
&\leq k\overline{N}(r,f) + N\left(r, 1/f\right) + S(r,f), \\
N_L\left(r, \frac{1}{F-1}\right) &\leq N\left(r, \frac{F}{F'}\right) \leq N\left(r, \frac{F'}{F}\right) + S(r,f) \\
&\leq \overline{N}\left(r, F\right) + \overline{N}\left(r, \frac{1}{F}\right) + S(r,f) \\
&\leq \overline{N}\left(r, f\right) + \overline{N}\left(r, \frac{1}{f}\right) + S(r,f), \\
N_L\left(r, \frac{1}{G-1}\right) &\leq N\left(r, \frac{G}{G'}\right) \leq N\left(r, \frac{G'}{G}\right) + S(r,f) \\
&\leq \overline{N}\left(r, G\right) + \overline{N}\left(r, \frac{1}{G}\right) + S(r,f) \\
&\leq (k+1)\overline{N}\left(r, f\right) + N_{k+1}\left(r, 1/f\right) + S(r,f) \\
&\leq (k+1)\overline{N}\left(r, f\right) + N\left(r, 1/f\right) + S(r,f).
\end{aligned}$$

Substituting the above three inequalities into (3.13) yields

$$T(r,F) \leq (2k+6)\overline{N}\left(r,f\right) + 6N\left(r, 1/f\right) + S(r,f).$$

Noting that $T(r,F) = nT(r,f) + S(r,f)$, we get

$$nT(r,f) \leq (2k+6)\overline{N}\left(r,f\right) + 6N\left(r, 1/f\right) + S(r,f), \tag{3.14}$$

which contradicts with (3.1).

Case 2. Suppose that f^n and $L(f)$ share a CM. Then F and G share 1 CM, ∞ IM except the zeros and poles of a. By the same reasoning discussed in Case 1, we obtain (3.13). Since now (3.3) holds, we have

$$T(r,F) \leq 3\overline{N}(r,f) + 2\overline{N}\left(r, \frac{1}{f}\right) + N_2\left(r, \frac{1}{L(f)}\right) + S(r,f).$$

Thus

$$\begin{aligned}
nT(r,f) &\leq 3\overline{N}(r,f) + 2\overline{N}\left(r, 1/f\right) + k\overline{N}(r,f) + N_{2+k}\left(r, 1/f\right) + S(r,f) \\
&\leq (k+3)\overline{N}(r,f) + 3N\left(r, 1/f\right) + S(r,f),
\end{aligned}$$

which contradicts with (3.2). Therefore, $H = 0$. By integration, we get from (3.5) that

$$\frac{1}{F-1} = \frac{A}{G-1} + B, \tag{3.15}$$

where $A(\neq 0)$ and B are constants. From (3.15) we have

$$G = \frac{(B-A)F + (A-B-1)}{BF - (B+1)}. \tag{3.16}$$

We discuss the following three cases.

Case I. Suppose that $B \neq 0, -1$. From (3.16) we have $\overline{N}\left(r, 1/\left(F - \frac{B+1}{B}\right)\right) = \overline{N}(r, G)$. From the second fundamental theorem, we have

$$\begin{aligned} nT(r,f) &\leq T(r,F) + S(r,f) \\ &\leq \overline{N}(r,F) + \overline{N}(r,1/F) + \overline{N}\left(r, \frac{1}{F - \frac{B+1}{B}}\right) + S(r,f) \\ &\leq \overline{N}(r,1/f) + \overline{N}(r,F) + \overline{N}(r,G) + S(r,f) \\ &\leq \overline{N}(r,1/f) + 2\overline{N}(r,f) + S(r,f), \end{aligned}$$

which contradicts with (3.1) and (3.2).

Case II. Suppose that $B = 0$. From (3.16) we have

$$G = AF - (A-1). \tag{3.17}$$

If $A \neq 1$, from (3.17) we obtain $\overline{N}\left(r, 1/\left(F - \frac{A-1}{A}\right)\right) = \overline{N}(r, 1/G)$. By Lemma 3.4 and the second fundamental theorem, we have

$$\begin{aligned} nT(r,f) &\leq T(r,F) + S(r,f) \\ &\leq \overline{N}(r,F) + \overline{N}(r,1/F) + \overline{N}\left(r, \frac{1}{F - \frac{A-1}{A}}\right) + S(r,f) \\ &= \overline{N}(r,f) + \overline{N}(r,1/f) + N_1(r,1/G) + S(r,f) \\ &\leq (k+1)\overline{N}(r,f) + 2N(r,1/f) + S(r,f), \end{aligned}$$

which contradicts with (3.1) and (3.2). Thus $A = 1$. From (3.17) we have $F = G$. Then $f^n = L(f)$.

Case III. Suppose that $B = -1$. From (3.16) we have

$$G = \frac{(A+1)F - A}{F}. \tag{3.18}$$

If $A \neq -1$, we obtain from (3.18) that $\overline{N}\left(r, 1/\left(F - \frac{A}{A+1}\right)\right) = \overline{N}(r, 1/G)$. By the same reasoning discussed in Case II, we obtain a contradiction. Hence $A = -1$. From (3.18), we get $F \cdot G = 1$, that is

$$f^n \cdot L(f) = a^2,$$

and

$$N(r, f) = S(r, f), \quad N(r, 1/f) = S(r, f).$$

From the last three equations, we have

$$T\left(r, \frac{f^{n+1}}{a^2}\right) = T\left(r, \frac{a^2}{f^{n+1}}\right) + O(1) = T\left(r, \frac{L(f)}{f}\right) + O(1) = S(r, f).$$

So $T(r, f) = S(r, f)$, which is impossible. This completes the proof of Theorem 3.1.

Theorem 3.5. *Let k, n be positive integers and f be a nonconstant meromorphic function, and let $L(f)$ be given by (1.1). If $n > 2k + 12$ (resp. $n > k + 6$), then there does not exist a small function $a(z)(\not\equiv 0, \infty)$ with respect to f such that f^n and $L(f)$ share a IM (resp. CM).*

Proof. Suppose that there exists a small function $a(z)$ satisfying the condition of the Theorem 3.5. Then we obtain $f^n = L(f)$ by Theorem 3.1.

Suppose that z_0 is a pole of f with the multiplicity p. Then z_0 is a pole of f^n and $L(f)$ with the multiplicity np and $k + p$ respectively. Thus $np = k + p$ and $k = (n - 1)p \geq (n - 1)$, which is a contradiction. So, f is an entire function. Then

$$(n-1)T(r, f) = T\left(r, f^{n-1}\right) = m\left(r, f^{n-1}\right) = m\left(r, \frac{L(f)}{f}\right) = S(r, f),$$

which is impossible since $n > 1$. □

Remark 2. From the proof of Theorem 3.5. We know that Theorem 3.1 is valid when $n \leq k + 1$.

4. Concluding remarks

As for an entire function sharing a finite value with its derivative, the following conjecture proposed by Brück [2] is widely studied:

Conjecture. *Let f be a nonconstant entire function. Suppose that the hyper-order of f,*

$$\rho_2(f) := \limsup_{r \to \infty} \frac{\log \log T(r, f)}{\log r},$$

is not a positive integer or infinite. If f and f' share one finite value a CM, then

$$\frac{f' - a}{f - a} = c$$

for some non-zero constant c.

The conjecture has been verified in special cases only: (1) $\rho_2(f) < \frac{1}{2}$, see [3]; (2) $a = 0$, see [2]; (3) $N(r, 1/f') = S(r, f)$, see [2]. However, the corresponding conjecture for meromorphic functions fails in general, as shown by Gundersen and Yang [9], while it remains true in the case of $N(r, 1/f') = S(r, f)$, see Al-Khaladi [1].

Theorem 2.1 shows that the conjecture holds if a meromorphic function f^n shares 1 IM with $(f^n)'$, where $n > 4$ is an integer. A natural question is:

Question 4.1. *Can n in Theorem 2.1 be reduced?*

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