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On Orthogonal Generalized (σ, τ)-Derivations of Semiprime Near-Rings

Shuliang Huang
Department of Mathematics, Chuzhou University, Chuzhou Anhui, 239012, P. R. China
e-mail : shulianghuang@sina.com

Abstract. In this paper, we present some results concerning orthogonal generalized (σ, τ)-derivations in semiprime near-rings. These results are a generalization of results of Bresar and Vukman, which are related to a theorem of Posner for the product of two derivations in prime rings.

1. Introduction

As is well known, the study of derivations of near-rings was initiated by Bell and Mason [3]. An additively written group $(N, +)$ (not necessary abelian) equipped with a binary operation $\cdot : N \times N \longrightarrow N, (x, y) \longrightarrow xy$, such that $(xy)z = x(yz)$ and $x(y + z) = xy + xz$ for all $x, y, z \in N$ is called a (left) near-ring. A near-ring N is said to be zero-symmetric if $0x = 0$ for all $x \in N$. Following example, due to Beidar et al.,[5] shows that such near-rings do exist. Let V be a linear space with a basis $e_1, e_2, \ldots, e_n$ over a field F of characteristic different from two. Define a multiplication $\cdot : V \times V \longrightarrow V$ by the rule $vw = 0$ for all $v, w \in V$ with $v \neq e_1$, $v \neq -e_1$ and $e_1 w = w$, $(-e_1)w = -w$. One can easily check that V is a left zero-symmetric near-ring with respect to this multiplication. In view of the above multiplication $e_1(e_2 + e_3) = e_2 + e_3$. On the other hand, neither $e_2 + e_3 = e_1$ nor $e_2 + e_3 = -e_1$ since $e_1, e_2, \ldots, e_n$ is linearly independent, and hence $(e_2 + e_3)e_1 = 0$. Obviously, V is not a ring since right distributive law fails. For more natural examples of left near-rings we refer the reader to [6]. In [4], Bresar and Vukman introduced the notion of orthogonality for two derivations in a semiprime ring and proved some results on the orthogonal derivations of semiprime rings which are related to Posner's First Theorem [9]. In [2], Argac et al., introduced the notion of orthogonality for a pair (D, d), (G, g) of generalized derivations on semiprime rings and gave several necessary and sufficient conditions for (D, d) and (G, g) to be orthogonal. Golbasi and Aydin [7] extended their results to orthogonal generalized (σ, τ)-derivations. In [8], Park and Jung proved some results on orthogonal generalized derivations in

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semiprime near-rings. Motivated by the above, our purpose is to present orthogonal generalized (σ,τ)-derivations in semiprime near-rings. In fact, our results extend and unify some results proved in [1], [2], [7] and [8].

Throughout this paper, N will denote a zero-symmetric left near-ring. We say that N is 2-torsion free if $2x = 0$, $x \in N$, implies that $x = 0$. Recall that a near-ring N is prime if $xNy = \{0\}$ implies $x = 0$ or $y = 0$, and N is semiprime if $xNx = \{0\}$ implies $x = 0$. An additive mapping $d : N \longrightarrow N$ is said to be a derivation on N if $d(xy) = d(x)y + xd(y)$ for all $x, y \in N$. An additive mapping $f : N \longrightarrow N$ is said to be a generalized derivation on N if there exists a derivation d on N such that $f(xy) = f(x)y + xd(y)$ for all $x, y \in N$, and denoted by (f, d). Let σ and τ be two near-ring endomorphisms of N. An additive mapping $d : N \longrightarrow N$ is called a (σ,τ)-derivation if $d(xy) = d(x)\sigma(y) + \tau(x)d(y)$ holds for all $x, y \in N$. An additive mapping $F : N \longrightarrow N$ is called a generalized (σ,τ)-derivation if there exists a (σ,τ)-derivation d such that $F(xy) = F(x)\sigma(y) + \tau(x)d(y)$ holds for all $x, y \in N$. A generalized (σ,τ)-derivation F associated with d will denote (F, d). Note that if $d = F$, then a generalized (σ,τ)-derivation F is just a (σ,τ)-derivation. If $\sigma = \tau = 1$, the identity map on N, then a generalized (σ,τ)-derivation F is simply a generalized derivation. If $\sigma = \tau = 1$ and $d = F$, then a generalized (σ,τ)-derivation F is a derivation. Hence the class of generalized (σ,τ)-derivations includes those of derivations, generalized derivations and (σ,τ)-derivations. Given an endomorphism α of N, an additive mapping $f : N \longrightarrow N$ is called a left (resp. right) α-centralizer of N if $f(xy) = f(x)\alpha(y)$ (resp. $f(xy) = \alpha(x)f(y)$) for all $x, y \in N$. Two additive mappings $d, g : N \longrightarrow N$ are called orthogonal if $d(x)Ng(y) = \{0\} = g(y)Nd(x)$ for all $x, y \in N$. It is obvious that a nonzero generalized (σ,τ)-derivation cannot be orthogonal to itself in semiprime near-rings.

The following example shows that orthogonal generalized (σ,τ)-derivations on semiprime near-rings do exist. Let N be any prime near-ring and d a (σ,τ)-derivation of N. Set $S = N \bigoplus N$, then S is semiprime near-ring. It is easy to see that $F : N \longrightarrow N$ defined by $F(xy) = a\sigma(xy) + d(xy)$ for some fixed $a \in N$, is a generalized (σ,τ)-derivation of N. Define $F_1, F_2 : S \longrightarrow S$ by $F_1((x, y)) = (F(x), 0)$ and $F_2((x, y)) = (0, F(y))$, then it is straightforward to check that F_1 and F_2 are orthogonal generalized (σ,τ)-derivations on S.

2. Preliminary results

We begin with the following lemmas which will be used in the sequel.

Lemma 2.1([8, Lemma 1])**.** *Let N be a 2-torsion free semiprime near-ring and $a, b \in N$. Then the following conditions are equivalent:*

(*i*) $axb = 0$ *for all* $x \in N$.

(*ii*) $bxa = 0$ *for all* $x \in N$.

(*iii*) $axb + bxa = 0$ *for all* $x \in N$.

If one of the three conditions is fulfilled, then $ab = ba = 0$.

Lemma 2.2. *Let (F, d) be a generalized (σ,τ)-derivation of near-ring N, where σ*

is an automorphism of N. Then the following hold:

(i) $(F(x)\sigma(y)+\tau(x)d(y))z = F(x)\sigma(y)z+\tau(x)d(y)z$ *for all* $x,y,z\in N$.

(ii) $(d(x)\sigma(y)+\tau(x)d(y))z = d(x)\sigma(y)z+\tau(x)d(y)z$ *for all* $x,y,z\in N$.

Proof. (i) For all $x,y,z\in N$, on the one hand,

$$F((xy)z) = F(xy)\sigma(z)+\tau(xy)d(z) = (F(x)\sigma(y)+\tau(x)d(y))\sigma(z)+\tau(x)\tau(y)d(z).$$

On the other hand,

$$F(x(yz)) = F(x)\sigma(yz)+\tau(x)d(yz) = F(x)\sigma(y)\sigma(z)+\tau(x)d(y)\sigma(z)+\tau(x)\tau(y)d(z).$$

Comparing these two expressions of $F(xyz)$, we have

$$(F(x)\sigma(y)+\tau(x)d(y))\sigma(z) = F(x)\sigma(y)\sigma(z)+\tau(x)d(y)\sigma(z)$$

for all $x,y,z\in N$. Since σ is an automorphism of N, and so

$$(F(x)\sigma(y)+\tau(x)d(y))z = F(x)\sigma(y)z+\tau(x)d(y)z$$

is fulfilled for all $x,y,z\in N$.

(ii) It is proved by the same arguments as (i). □

3. The main results

In all that follows, unless stated otherwise, we always assume that $F\sigma=\sigma F$, $F\tau=\tau F$, $d\sigma=\sigma d$, $d\tau=\tau d$ in the symbol (F,d), while σ and τ are automorphisms of N.

Theorem 3.1. *Let N be a 2-torsion free semiprime near-ring. Suppose that (F_1,d_1) (resp. (F_2,d_2))is a generalized (σ_1,τ_1)-derivation (resp. (σ_2,τ_2)-derivation) of N. If F_1 and F_2 are orthogonal, then the following conditions are true:*

(i) *d_1 and F_2 are orthogonal.*

(ii) *d_2 and F_1 are orthogonal.*

(iii) *d_1 and d_2 are orthogonal.*

(iv) $d_1F_2=F_2d_1=0$, $d_2F_1=F_1d_2=0$, $d_1d_2=d_2d_1=0$, $F_1F_2=F_2F_1=0$.

Proof. (i) By hypothesis,

$$F_1(x)zF_2(y)=0 \text{ for all } x,y,z\in N. \tag{1}$$

Application of Lemma 2.1 yields that

$$F_1(x)F_2(y)=0 \text{ for all } x,y\in N. \tag{2}$$

Replacing x by rx in (2) and using Lemma 2.2, we get

$$\begin{aligned} 0 &= F_1(rx)F_2(y) \\ &= (F_1(r)\sigma_1(x)+\tau_1(r)d_1(x))F_2(y) \\ &= F_1(r)\sigma_1(x)F_2(y)+\tau_1(r)d_1(x)F_2(y) \\ &= \tau_1(r)d_1(x)F_2(y) \end{aligned}$$

for all $x, y, r \in N$. Since τ_1 is an automorphism of N, we have $Nd_1(x)F_2(y) = \{0\}$ and hence

$$d_1(x)F_2(y) = 0 \tag{3}$$

by the semiprimeness of N. Replacing x by xr in (3) and using Lemma 2.2, we obtain

$$\begin{aligned} 0 &= d_1(xr)F_2(y) \\ &= (d_1(x)\sigma_1(r) + \tau_1(x)d_1(r))F_2(y) \\ &= d_1(x)\sigma_1(r)F_2(y) + \tau_1(x)d_1(r)F_2(y) \\ &= d_1(x)\sigma_1(r)F_2(y) \end{aligned}$$

for all $x, y, r \in N$. Since σ_1 is an automorphism of N, we have $d_1(x)rF_2(y) = 0$ for all $x, y, r \in N$, and so $F_2(y)rd_1(x) = 0$ by Lemma 2.1, which shows (i).

(ii) Using the same arguments in the proof of (i), we prove (ii).

(iii) Replacing x, y by xr, ys respectively in (2) and using Lemma 2.2, for all $x, y, r, s \in N$, we have

$$\begin{aligned} 0 &= F_1(xr)F_2(ys) \\ &= (F_1(x)\sigma_1(r) + \tau_1(x)d_1(r))(F_2(y)\sigma_2(s) + \tau_2(y)d_2(s)) \\ &= F_1(x)\sigma_1(r)(F_2(y)\sigma_2(s) + \tau_2(y)d_2(s)) + \tau_1(x)d_1(r)(F_2(y)\sigma_2(s) + \tau_2(y)d_2(s)) \\ &= \tau_1(x)d_1(r)\tau_2(y)d_2(s) \end{aligned}$$

where the last equation uses the orthogonality of F_1 and F_2, d_1 and F_2, d_2 and F_1.

Since τ_1 is an automorphism of N, the last relation gives $Nd_1(r)\tau_2(y)d_2(s) = \{0\}$ and hence $d_1(r)\tau_2(y)d_2(s) = 0$ by the semiprimeness of N. This implies that $d_1(r)td_2(s) = 0$ since τ_1 is also an automorphism of N. We have $d_2(s)td_1(r) = 0$ by Lemma 2.1 for all $r, s, t \in N$. Thus, d_1 and d_2 are orthogonal.

(iv) It follows from (iii) that d_1 and d_2 are orthogonal. Hence

$$0 = d_1(d_2(x)zd_1(y)) = d_1d_2(x)\sigma_1(z)\sigma_1d_1(y) + \tau_1d_2(x)d_1(zd_1(y))$$

for all $x, y, z \in N$. Using the facts that $d_1\sigma_1 = \sigma_1d_1$, $\tau_1d_2 = d_2\tau_1$ and the orthogonality of d_1 and d_2, the last relation reduces to $d_1d_2(x)\sigma_1(z)d_1\sigma_1(y) = 0$ and hence

$$d_1d_2(x)Nd_1(y) = \{0\} \tag{4}$$

since τ_1 is an automorphism of N. Replacing y by $d_2(x)$ in (4), we get $d_1d_2(x)Nd_1d_2(x) = \{0\}$ and hence $d_1d_2 = 0$ by the semiprimeness of N.

Similarly, since each of the equalities: $d_2(d_1(x)zd_2(y)) = 0$, $d_1(F_2(x)zd_1(y)) = 0$, $F_2(d_1(x)zF_2(y)) = 0$, $d_2(F_1(x)zd_2(y)) = 0$, $F_1(d_2(x)zF_1(y)) = 0$, $F_1(F_2(x)zF_1(y)) = 0$ and $F_2(F_1(x)zF_2(y)) = 0$ holds for all $x, y, z \in N$, we have $d_2d_1 = d_1F_2 = F_2d_1 = d_2F_1 = F_1d_2 = F_1F_2 = F_2F_1 = 0$, respectively. □

Theorem 3.2. *Let N be a 2-torsion free semiprime near-ring. Suppose that* (F_1, d_1)

(resp. (F_2, d_2)) is a generalized (σ_1, τ_1)-derivation (resp. (σ_2, τ_2)-derivation) of N. Then the following conditions are equivalent:

(i) F_1 and F_2 are orthogonal.

(ii) $F_1(x)F_2(y) = d_1(x)F_2(y) = 0$ for all $x, y \in N$.

(iii) $F_2(x)F_1(y) = d_2(x)F_1(y) = 0$ for all $x, y \in N$.

Proof. $(i) \Longrightarrow (ii)$ It is obvious by Theorem 3.1.

$(ii) \Longrightarrow (i)$ We are given that $F_1(x)F_2(y) = 0$ for all $x, y \in N$. Replacing x by xz in the above equation and using Lemma 2.2, we find that

$$\begin{aligned} 0 &= F_1(xz)F_2(y) \\ &= (F_1(x)\sigma_1(z) + \tau_1(x)d_1(z))F_2(y) \\ &= F_1(x)\sigma_1(z)F_2(y) + \tau_1(x)d_1(z)F_2(y) \\ &= F_1(x)\sigma_1(z)F_2(y), \end{aligned}$$

where the last equality uses the fact $d_1(x)F_2(y) = 0$ for all $x, y \in N$. Since σ_1 is an automorphism of N, we have $F_1(x)zF_2(y) = 0$ and hence $F_2(y)zF_1(x) = 0$ for all $x, y, z \in N$, by Lemma 2.1. Thus, F_1 and F_2 are orthogonal.

$(i) \Longleftrightarrow (iii)$ The proof is similar to $(i) \Longleftrightarrow (ii)$. □

When $\sigma_1 = \sigma_2 = \sigma$ and $\tau_1 = \tau_2 = \tau$, we can prove the following:

Theorem 3.3. *Let N be a 2-torsion free semiprime near-ring. Suppose that both (F_1, d_1) and (F_2, d_2) are generalized (σ, τ)-derivations of N. Then F_1 and F_2 are orthogonal if and only if (F_1F_2, d_1d_2) is a generalized (σ^2, τ^2)-derivation and $F_1(x)F_2(y) = 0$ for all $x, y \in N$.*

Proof. Suppose that (F_1F_2, d_1d_2) is a generalized (σ^2, τ^2)-derivation and $F_1(x)F_2(y) = 0$ for all $x, y \in N$. On the one hand,

$$F_1F_2(xy) = F_1F_2(x)\sigma^2(y) + \tau^2(x)d_1(x)d_2(y) \text{ for all } x, y \in N. \quad (5)$$

On the other hand, $F_1F_2(xy) = F_1(F_2(x)\sigma(y) + \tau(x)d_2(y))$, which implies that

$$F_1F_2(xy) = F_1F_2(x)\sigma^2(y) + \tau F_2(x)d_1\sigma(y) + F_1\tau(x)\sigma d_2(y) + \tau^2(x)d_1(x)d_2(y). \quad (6)$$

Comparing (5) with (6), we have $\tau F_2(x)d_1\sigma(y) + F_1\tau(x)\sigma d_2(y) = 0$ for all $x, y \in N$. Since $F_2\tau = \tau F_2$, $d_2\sigma = \sigma d_2$ and σ, τ are automorphisms of N, the above equation can be rewritten as

$$F_2(x)d_1(y) + F_1(x)d_2(y) = 0 \text{ for all } x, y \in N. \quad (7)$$

Recalling our hypothesis, $F_1(x)F_2(y) = 0$ for all $x, y \in N$, we have

$$0 = F_1(x)F_2(yz) = F_1(x)F_2(y)\sigma(z) + F_1(x)\tau(y)d_2(z) = F_1(x)\tau(y)d_2(z)$$

for all $x, y, z \in N$. Since τ is an automorphism of N, we have $F_1(x)yd_2(z) = 0$ for all $x, y, z \in N$. Making use of Lemma 2.1, we arrive at

$$F_1(x)d_2(y) = 0 \text{ for all } x, y \in N. \quad (8)$$

Comparing (7) with (8), we see that

$$F_2(x)d_1(y) = 0 \text{ for all } x, y \in N. \tag{9}$$

Replacing y by rs in (9), we have

$$0 = F_2(x)d_1(ys) = F_2(x)d_1(y)\sigma(s) + F_2(x)\tau(r)d_1(s) = F_2(x)\tau(r)d_1(s)$$

for all $x, y, r, s \in N$. Since τ is an automorphism of N, the last relation yields that $F_2(x)rd_1(s) = 0$ and hence $d_1(s)F_2(x) = 0$ by Lemma 2.1. Now we conclude that $F_1(x)F_2(y) = 0 = d_1(x)F_2(y)$ for all $x, y \in N$. Therefore, from Theorem 3.2, we obtain the result. Conversely, if F_1 and F_2 are orthogonal, then it follows from Theorem 3.1 that $F_1F_2 = d_1d_2 = 0$, as required. □

Theorem 3.4. *Let N be a 2-torsion free semiprime near-ring. Suppose that both (F_1, d_1) and (F_2, d_2) are generalized (σ, τ)-derivations of N. If both F_1, d_2 are orthogonal and F_2, d_1 are orthogonal, then the following holds:*
(i) $d_1d_2 = 0$ and F_1F_2 is a left $\sigma_1\sigma_2$-centralizer of N.
(ii) $d_2d_1 = 0$ and F_2F_1 is a left $\sigma_2\sigma_1$-centralizer of N.

Proof. (i) Since F_1 and d_2 are orthogonal, we have

$$F_1(x)yd_2(z) = 0 \text{ for all } x, y, z \in N. \tag{10}$$

Replacing x by rx in (10) and using Lemma 2.2, we get

$$0 = F_1(rx)yd_2(z) = F_1(r)\sigma_1(x)yd_2(z) + \tau_1(r)d_1(x)yd_2(z) = \tau_1(r)d_1(x)yd_2(z)$$

for all $x, y, z, r \in N$. Since τ_1 is an automorphism of N, from the last relation $Nd_1(x)yd_2(z) = \{0\}$ and hence $d_1(x)yd_2(z) = 0$ for all $x, y, z \in N$, by the semiprimeness of N. Consequently, d_1 and d_2 are orthogonal and so $d_1d_2 = 0$ according to Theorem 3.1. On the other hand, by hypothesis, since F_1, d_2 are orthogonal and F_2, d_1 are orthogonal, we obtain that $F_1(x)d_2(y) = 0$ and $F_2(x)d_1(y) = 0$ for all $x, y \in N$. Noting that the fact $F_2\tau_1 = \tau_1F_2$ and $d_2\sigma_1 = \sigma_1d_2$, for all $x, y \in N$, we find that

$$\begin{aligned}
F_1F_2(xy) &= F_1(F_2(x)\sigma_2(y) + \tau_2(x)d_2(y)) \\
&= F_1F_2(x)\sigma_1\sigma_2(y) + \tau_1F_2(x)d_1\sigma_2(y) + F_1\tau_2(x)\sigma_1d_2(y) + \tau_1\tau_2(x)d_1d_2(y) \\
&= F_1F_2(x)\sigma_1\sigma_2(y) + F_2\tau_1(x)d_1\sigma_2(y) + F_1\tau_2(x)d_2\sigma_1(y) + \tau_1\tau_2(x)d_1d_2(y) \\
&= F_1F_2(x)\sigma_1\sigma_2(y)
\end{aligned}$$

(ii) It can be proved by using the same techniques. □

Corollary 3.1([6, Theorem 2])**.** *Let N be a 2-torsion free semiprime near-ring. Suppose that both (f, d) and (g, δ) are generalized derivations of N. If both f and δ are orthogonal and g and d are orthogonal, then we have*
(i) $d\delta = 0$ and fg is a left centralizer of N.

(ii) $\delta d = 0$ *and* gf *is a left centralizer of* N.

Proof. Take $\sigma_1 = \tau_1 = \sigma_2 = \tau_2 = 1$ in Theorem 3.4, where $1 : N \longrightarrow N$ is the identity map of N. □

The following result, without the assumption of $F\sigma = \sigma F$, $F\tau = \tau F$, $d\sigma = \sigma d$, $d\tau = \tau d$, is of independent interest.

Theorem 3.5. *Let* N *be a* 2*-torsion free semiprime near-ring. If* (F, d) *is a generalized* (σ, τ)*-derivation of* N *such that* $F(x)F(y) = 0$ *for all* $x, y \in N$*, then* $F = d = 0$.

Proof. We are given that $F(x)F(y) = 0$ for all $x, y \in N$. Writing yz for y in the above equation, we obtain

$$0 = F(x)F(yz) = F(x)F(y)\sigma(z) + F(x)\tau(y)d(z) = F(x)\tau(y)d(z)$$

for all $x, y, z \in N$. Since τ is an automorphism of N, we obtain $F(x)yd(z) = 0$ and so

$$d(z)F(x) = 0 \tag{11}$$

by Lemma 2.1. Replacing x by xz in (11), we get $0 = d(z)F(xz) = d(z)F(x)\sigma(z) + d(z)\tau(x)d(z) = d(z)\tau(x)d(z)$ for all $x, z \in N$. Since τ is an automorphism of N, we have $d(z)Nd(z)$=0 and so $d = 0$ by the semiprimeness of N. Now using hypothesis and Lemma 2.2, we get $0 = F(xz)F(y) = F(x)\sigma(z)F(y) + \tau(x)d(z)F(y) = F(x)\sigma(z)F(y)$ for all $x, y, z \in N$, and hence $F(x)NF(y) = \{0\}$, in particular, $F(x)NF(x) = \{0\}$ for all $x \in N$. The semiprimeness of N forces that $F = 0$, as required. □

Corollary 3.2([6, Theorem 3])**.** *Let* N *be a* 2*-torsion free semiprime near-ring. If* (f, d) *is a generalized derivation of* N *such that* $f(x)f(y) = 0$ *for all* $x, y \in N$*, then* $f = d = 0$.

Proof. Setting $\sigma = \tau = 1$ in Theorem 3.5, we obtain the result of the corollary. □

The following example shows that the hypothesis of semiprimeness is essential in Theorem 3.1(*iv*),Theorems 3.4-3.5 and Corollaries 3.1-3.2.

Example 3.1. Let S be any near-ring and $N = \left\{\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} \mid a, b \in S\right\}$. We define maps $d_1, d_2 : N \to N$ as follows: $d_1\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix}$ and $d_2\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -b \\ 0 & 0 \end{pmatrix}$. Then it is easy to see that d_1 and d_2 are nonzero orthogonal derivations of N satisfying $d_1(x)d_2(y) = 0$ for all $x, y \in N$. We know that a derivation is a special type of generalized (σ, τ)-derivation, namely, $\sigma = \tau = 1$ and $d = F$ in the symbol (F, d). However, it is straightforward to check that neither $d_1d_2 = 0$ nor $d_2d_1 = 0$.

The following example demonstrates that Theorem 3.2 fails if we omit the semiprimeness of N.

Example 3.2. Let S be any near-ring and $N = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \mid a, b, c \in S \right\}$. Define maps $d_1, d_2 : N \to N$ as follows: $d_1 \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} -a & b \\ 0 & 0 \end{pmatrix}$ and $d_2 \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} 0 & a-c \\ 0 & 0 \end{pmatrix}$. It is easy to check that d_1 and d_2 are nonzero derivations of N such that $d_2(x)d_1(y) = 0$, however $d_1(x)d_2(y) \neq 0$ for all $x, y \in N$.

The following example shows that the hypothesis of semiprimeness is crucial in Theorem 3.3.

Example 3.3. Let S be any near-ring and $N = \left\{ \begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix} | a, b, c \in S \right\}$. Define maps $F,\ d : R \longrightarrow R$ and $\sigma,\ \tau : R \longrightarrow R$ as follows:

$$F \begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & b \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ d \begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & a & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\sigma \begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -a & b \\ 0 & 0 & -c \\ 0 & 0 & 0 \end{pmatrix}\ \tau \begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -a & -b \\ 0 & 0 & -c \\ 0 & 0 & 0 \end{pmatrix}.$$

One can verify that $(F,\ d)$ is a generalized (σ, τ)-derivation of N which is orthogonal to itself, but (F^2, d^2) is not a generalized (σ^2, τ^2)-derivation of N.

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References

[1] M. Ashraf, A. Ali and S. Ali, (σ, τ)*-derivations on prime near-rings*, Arch. Math., (Brno), **40**(2004), 281–286.

[2] N. Argac, A. Nakajima and E. Albas, *On orthogonal generalized derivations of semiprime rings*, Turk J. Math., **28**(2004), 185-194.

[3] H. E. Bell and G. Mason, On derivations in near-rings, Near-rings and Near-fields, 1987, 31-35.

[4] M. Bresar and J. Vukman, *Orthogonal derivations and an extension of a theorem of Posner*, Rad. Math., **5**(1989), 237-246.

[5] K. I. Beidar, Y. Fong, W. F. Ke and S. Y. Liang, *Near-ring multiplication on groups*, Comm. Algebra, **23**(1995), 999-1015.

[6] J. R. Clay, Nearrings geneses and applications, Oxford University Press, New York, 1992.

[7] O. Golbasi and N. Aydin, *Orthogonal generalized* (σ, τ)*-derivations of semiprime rings*, Siberian Mathematical Journal, **48**(2007), 979-983.

[8] K. H. Park and Y. S. Jung, Semiprime near-rings with orthogonal derivations, J. Korea Soc. Math. Edu. Ser.B: Pure Appl. Math., **13**(2006), 303-310.

[9] E. C. Posner, Derivations in prime rings, Proc. Amer. Math. Soc., **8**(1957), 1093-1100.