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STRONG CONVERGENCE OF EXTENDED GENERAL VARIATIONAL INEQUALITIES AND NONEXPANSIVE MAPPINGS

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Abstract. In this paper, we suggest and analyze some three step iterative scheme for finding the common elements of the set of the solutions of the extended general variational inequalities involving three operators and the set of the fixed points of nonexpansive mappings. We also consider the convergence analysis of suggested iterative schemes under some mild conditions. Since the extended general variational inequalities include general variational inequalities and several other classes of variational inequalities as special cases, results obtained in this paper continue to hold for these problems. Results obtained in this paper may be viewed as a refinement and improvement of the previously known results.

1. Introduction

Throughout this paper we assume that H is a real Hilbert space, whose inner product and norm are denoted by $\langle\cdot,\cdot\rangle$ and $\|\cdot\|$ respectively. Let K be nonempty closed and convex set in H, and $T, g, h : H \to H$ be given nonlinear operators. We consider the problem of finding $u \in H, h(u) \in K$ such that

$$\langle Tu, g(v) - h(u)\rangle \geq 0, \qquad \forall v \in H, g(v) \in K. \tag{1.1}$$

An inequality of type (1.1) is called extended general variational inequality involving three operators, which was introduced and studied by Noor [2]. One can show that the extended general variational inequalities provide us a unified, simple, and natural framework in which to study a wide class of problems which arise in various areas of pure and applied sciences. Using a projection technique, Noor [2] established the equivalence between the extended general

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variational inequalities and the generalized nonlinear projection equation. Using this equivalent formulation, Noor discussed the existence of a solution of the extended general variational inequalities under suitable conditions.

We now list some special cases of the extended general variational inequalities. These also can be found in Noor [2].

I. If $g = h$, then problem (1.1) is equivalent to that of finding $u \in H : g(u) \in K$ such that

$$\langle T(u), g(v) - g(u) \rangle \geq 0, \qquad \forall v \in H : g(v) \in K, \tag{1.2}$$

which is known as general variational inequality, introduced in Noor [3].

II. For $g \equiv I$, the identity operator, the extended general variational inequality (1.1) is equivalent to finding: $u \in H : h(u) \in K$ such that

$$\langle T(u), v - h(u) \rangle \geq 0, \forall v \in K, \tag{1.3}$$

which also called the general variational inequality; see Noor [4].

III. For $h \equiv I$, the identity operator, the extended general variational inequality (1.1) is equivalent to finding: $u \in K$ such that

$$\langle T(u), g(v) - u \rangle \geq 0, \forall v \in H : g(v) \in K, \tag{1.4}$$

which also called the general variational inequality; see Noor [5].

IV. For $g = h = I$, the identity operator, the extended general variational inequality (1.1) is equivalent to finding: $u \in K$ such that

$$\langle T(u), v - u \rangle \geq 0, \forall v \in K, \tag{1.5}$$

which is known as the classical variational inequality and studied by Stampacchia [1] in 1964.

Noor [2] emphasizes that the problem (1.1) is equivalent to that of finding $u \in H : h(u) \in K$ such that

$$\langle Tu + h(u) - g(u), g(v) - h(u) \rangle \geq 0, \qquad \forall v \in H, g(v) \in K. \tag{1.6}$$

We now recall the following well-known results and concepts.

Lemma 1.1. *For given $z \in H, u \in K$ satisfies the inequality*

$$\langle u - z, v - u \rangle \geq 0, \forall v \in K, \tag{1.7}$$

if and only if

$$u = P_K(z)$$

where P_K is the projection of H onto K. Also the projection operator P_K is nonexpansive.

Using Lemma 1.1, we can show that the extended general variational inequality (1.6) is equivalent to the fixed point problem. This result is mainly due to Noor [2].

Lemma 1.2. *The function $u \in H : h(u) \in K$ is a solution of the extended general variational inequality (1.6) if and only if $u \in H : h(u) \in K$ satisfies the relation*

$$h(u) = P_K[g(u) - \rho Tu], \tag{1.8}$$

where P_K is the projection operator and $\rho > 0$ is a constant.

It is clear from the Lemma 1.2 that the extended general variational inequality (1.6) and the fixed point problem (1.8) are equivalent. This alternative equivalent formulation has played a significant role in the studies of the variational inequalities and related optimization problems.

It is convenient to rewrite the relation (1.8) in the following form which is very useful in obtaining our results:

$$u = u - h(u) + P_K[g(u) - \rho Tu]. \tag{1.9}$$

Let $S : K \to K$ be a nonexpansive mapping, i.e., if $\|Sx - Sy\| \leq \|x - y\|, \forall x, y \in K$. We denote the set of the fixed points of S by $F(S)$ and the set of the solutions of the extended general variational inequalities (1.6) by $EGVI(K, T, g, h)$. If $u \in F(S) \cap EGVI(K, T, g, h)$, from the Lemma 1.2, it follows that

$$\begin{aligned} u &= Su = u - h(u) + P_K[g(u) - \rho Tu] \\ &= Su - h(u) + P_K[g(u) - \rho Tu], \end{aligned}$$

where $\rho > 0$ is a constant.

The fixed point formulation is used to suggest the following three-step iterative method for finding a common element of two different sets of the fixed points of the nonexpansive mappings and the extended general variational inequalities.

Algorithm 1.1. For a given $x_0 \in H$, compute the approximate solution x_n by the iterative schemes

$$z_n = (1 - \gamma_n)x_n + \gamma_n S\{x_n - h(x_n) + P_K[g(x_n) - \rho Tx_n]\}, \tag{1.10}$$

$$y_n = (1 - \beta_n)x_n + \beta_n S\{z_n - h(z_n) + P_K[g(z_n) - \rho Tz_n]\}, \tag{1.11}$$

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n S\{y_n - h(y_n) + P_K[g(y_n) - \rho Ty_n]\}, \tag{1.12}$$

where $\alpha_n, \beta_n, \gamma_n \in [0, 1]$ for all $n \geq 0$ and S is a nonexpansive mapping. Algorithm 1.1 is a three-step predictor-corrector method. For $S = I$ and $g = h$, Algorithm 1.1 is essentially due to Noor [6].

For $g = h$, Algorithm 1.1 reduces to the following method, which is studied by Noor [7].

Algorithm 1.2. For a given $x_0 \in H$, compute the approximate solution x_n by the iterative schemes

$$z_n = (1 - \gamma_n)x_n + \gamma_n S\{x_n - g(x_n) + P_K[g(x_n) - \rho Tx_n]\},$$

$$y_n = (1 - \beta_n)x_n + \beta_n S\{z_n - g(z_n) + P_K[g(z_n) - \rho Tz_n]\},$$

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n S\{y_n - g(y_n) + P_K[g(y_n) - \rho Ty_n]\},$$

where $\alpha_n, \beta_n, \gamma_n \in [0,1]$ for all $n \geq 0$ and S is a nonexpansive mapping.

For $g = h = I$, the identity operator, Algorithm 1.1 reduces to the following methods, which is basically Noor and Huang [8].

Algorithm 1.3. For a given $x_0 \in H$, compute the approximate solution x_n by the iterative schemes

$$z_n = (1-\gamma_n)x_n + \gamma_n SP_K[x_n - \rho T x_n],$$
$$y_n = (1-\beta_n)x_n + \beta_n SP_K[z_n - \rho T z_n],$$
$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n SP_K[y_n - \rho T y_n],$$

where $\alpha_n, \beta_n, \gamma_n \in [0,1]$ for all $n \geq 0$ and S is a nonexpansive mapping.
Note that for $\gamma_n = 0$, Algorithm 1.1 reduce to:

Algorithm 1.4. For a given $x_0 \in H$, compute the approximate solution x_n by the iterative schemes

$$y_n = (1-\beta_n)x_n + \beta_n S\{x_n - h(x_n) + P_K[g(x_n) - \rho T x_n]\},$$
$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n S\{y_n - h(y_n) + P_K[g(y_n) - \rho T y_n]\},$$

where $\alpha_n, \beta_n, \gamma_n \in [0,1]$. Algorithm 1.4 is also known as the two-step (Ishikawa iterations) iterative method.

Algorithm 1.5. For given $x_0 \in K$, the sequence $\{x_n\}$ is generated by the following scheme:

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n S\{x_n - h(x_n) + P_K[g(x_n) - \rho T x_n]\}.$$

In particular, three-step methods, Algorithm 1.1 is quite general and it includes several new and previously known algorithms for solving variational inequalities and nonexpansive mappings. It is well known fact three-step iterations are also called Noor iterations, which has stimulated recent research activities in the field of fixed point theory and related optimization problems. Clearly Noor iterations include Mann (one-step) and Ishikawa (two-step) iterations as special cases.

Definition 1. A mapping $T : K \to H$ is called *μ-Lipschitzian* if there exists a constant $\mu > 0$, such that

$$\|Tx - Ty\| \leq \mu \|x - y\|, \forall x, y \in K.$$

Definition 2. A mapping $T : K \to H$ is called *r-strongly monotonic* if there exists a constant $r > 0$, such that

$$\langle Tx - Ty, x - y \rangle \geq r\|x - y\|^2, \forall x, y \in K.$$

Definition 3. A mapping $T : K \to H$ is called *α-inverse strongly monotonic* if there exists a constant $\alpha > 0$, such that

$$\langle Tx - Ty, x - y \rangle \geq \alpha\|Tx - Ty\|^2, \forall x, y \in K.$$

Definition 4. A mapping $T : K \to H$ is called *relaxed* (γ, r)*-cocoercive* if there exist constants $\gamma > 0, r > 0$, such that

$$\langle Tx - Ty, x - y\rangle \geq -\gamma\|Tx - Ty\|^2 + r\|x - y\|^2, \forall x, y \in K.$$

2. Main results

In this section, we investigate the strong convergence of Algorithm 1.1 in finding the common element of two sets of the solutions of the variational inequalities $EGVI(K, T, g, h)$ and $F(S)$.

In order to prove our results we need the following Lemma:

Lemma 2.1. [10] *Suppose* $\{\delta_k\}_{k=0}^{\infty}$ *is a nonnegative sequence satisfying the following inequality:*

$$\delta_{k+1} \leq (1 - \lambda_k)\delta_n + \sigma_k, k \geq 0$$

with $\lambda_k \in [0, 1], \sum_{k=0}^{\infty} \lambda_k = \infty$, *and* $\sigma_k = o(\lambda_k)$. *Then* $\lim_{k\to\infty} \delta_k = 0$.

Theorem 2.2. *Let* K *be a closed convex subset of a real Hilbert space* H. *Let* T *be a relaxed* (γ, r) *cocoercive and* μ*-Lipschitzian mapping of* K *into* H. *Let* g *be a relaxed* (γ_1, r_1) *cocoercive and* μ_1*-Lipschitzian mapping of* K *into* H *and* h *be a relaxed* (γ_2, r_2) *cocoercive and* μ_2*-Lipschitzian mapping of* K *into* H. *Let* S *be a nonexpansive mapping of* K *into* K *such that* $F(S) \cap EGVI(K, T, g, h) \neq \emptyset$. *Let* x_n *be a sequence defined by algorithm 1.1, for any initial point* $x_0 \in K$, *with conditions*

$$|\rho - \frac{r - \gamma\mu^2}{\mu^2}| < \frac{\sqrt{((r - \gamma\mu^2)^2 - \mu^2(k_1 + k_2)[2 - (k_1 + k_2)]}}{\mu^2}, \tag{2.1}$$

where

$$k_1 = \sqrt{1 + 2\gamma_1\mu_1^2 - 2r_1 + \mu_1^2},$$

$$k_2 = \sqrt{1 + 2\gamma_2\mu_2^2 - 2r_2 + \mu_2^2},$$

and $k_1 + k_2 < 1$. $\alpha_n, \beta_n, \gamma_n \in [0, 1]$ *and* $\sum_{n=1}^{\infty} \alpha_n = \infty$, *then* x_n *obtained from Algorithm 1.1 converges strongly to* $x^* \in F(S) \cap EGVI(K, T, g, h)$.

Proof. Let $x^* \in F(S) \cap EGVI(K, T, g, h)$. Then

$$\begin{aligned} x^* &= (1 - \gamma_n)x^* + \gamma_n S\{x^* - h(x^*) + P_K[g(x^*) - \rho Tx^*]\} \\ &= (1 - \beta_n)x^* + \beta_n S\{x^* - h(x^*) + P_K[g(x^*) - \rho Tx^*]\} \\ &= (1 - \alpha_n)x^* + \alpha_n S\{x^* - h(x^*) + P_K[g(x^*) - \rho Tx^*]\}. \end{aligned}$$

From the nonexpansive property of the projection P_K and nonexpansive mapping S, we have

$$\begin{aligned}
\|x_{n+1}-x^*\| &= \|1-\alpha_n)x_n+\alpha_n S\{y_n-g(y_n)+P_K[g(y_n)-\rho Ty_n]\}- \\
&\quad (1-\alpha_n)x^*-\alpha_n S\{x^*-h(x^*)+P_K[g(x^*)-\rho Tx^*]\}\| \\
&\leq (1-\alpha_n)\|x_n-x^*\|+\alpha_n\|y_n-h(y_n)-x^*+h(x^*)\| \\
&\quad +\alpha_n\|g(y_n)-\rho Ty_n-g(x^*)+\rho Tx^*\| \\
&\leq (1-\alpha_n)\|x_n-x^*\|+\alpha_n\|y_n-h(y_n)-x^*+h(x^*)\| \\
&\quad +\alpha_n\|g(y_n)-g(x^*)-y_n+x^*\| \\
&\quad +\alpha_n\|y_n-x^*-\rho Ty_n+\rho Tx^*\|. \qquad (2.2)
\end{aligned}$$

From the relaxed (γ, r)-cocoercive and μ-Lipschitzian definition on T,

$$\begin{aligned}
&\quad \|y_n-x^*-\rho(Tx_n-Tx^*)\|^2 \\
&= \|y_n-x^*\|^2-2\rho\langle Ty_n-Tx^*, x_n-x^*\rangle+\rho\|Ty_n-Tx^*\|^2 \\
&\leq \|y_n-x^*\|^2-2\rho[-\gamma\|Ty_n-Tx^*\|^2+r\|y_n-x^*\|^2]+\rho^2\|Ty_n-Tx^*\| \\
&\leq [1+2\rho\gamma\mu^2-2\rho r+\rho^2\mu^2]\|y_n-x^*\|^2 \\
&= \theta_1^2\|y_n-x^*\|^2, \qquad (2.3)
\end{aligned}$$

where $\theta_1=\sqrt{1+2\rho\gamma\mu^2-2\rho r+\rho^2\mu^2}$.

In similar way, using the relaxed (γ_1, r_1)-cocoercivity and μ_1- lipschitzian of the operator g, and the relaxed (γ_2, r_2)-cocoercivity and μ_2- lipschitzian of the operator h, we have

$$\|y_n-x^*-[g(y_n)-g(x^*)]\| \leq \sqrt{1+2\gamma_1\mu_1^2-2r_1+\mu_1^2}\|y_n-x^*\|=k_1\|y_n-x^*\|. \qquad (2.4)$$

$$\|y_n-x^*-[h(y_n)-h(x^*)]\| \leq \sqrt{1+2\gamma_2\mu_2^2-2r_2+\mu_2^2}\|y_n-x^*\|=k_2\|y_n-x^*\|. \qquad (2.5)$$

From (2.2)-(2.5), we have

$$\|x_{n+1}-x^*\| \leq (1-\alpha_n)\|x_n-x^*\|+\alpha_n\theta\|y_n-x^*\|, \qquad (2.6)$$

where $\theta=k_1+k_2+\theta_1$. From (2.1), we have $\theta<1$.

$$\begin{aligned}
\|y_n-x^*\| &\leq (1-\beta_n)\|x_n-x^*\|+\beta_n\|S\{z_n-h(z_n)+P_K[g(z_n)-\rho Tz_n]\} \\
&\quad -S\{x^*-h(x^*)+P_K[g(x^*)-\rho Tx^*]\}\| \\
&\leq (1-\beta_n)\|x_n-x^*\|+\beta_n\|z_n-x^*-\rho(Tz_n-Tx^*)\| \\
&\quad +\beta_n\|z_n-h(z_n)-x^*+h(x^*)\|+\beta_n\|g(z_n)-g(x^*)-z_n+x^*\|,
\end{aligned}$$

from the relaxed (γ, r)-cocoercive and μ-Lipschitzian definition on T,

$$\begin{aligned}
& \|z_n - x^* - \rho(Tz_n - Tx^*)\|^2 \\
= \;& \|z_n - x^*\|^2 - 2\rho\langle Tz_n - Tx^*, z_n - x^*\rangle + \rho\|Tz_n - Tx^*\|^2 \\
\leq \;& \|z_n - x^*\|^2 - 2\rho[-\gamma\|Tz_n - Tx^*\|^2 + r\|z_n - x^*\|^2] + \rho^2\|Tz_n - Tx^*\| \\
\leq \;& [1 + 2\rho\gamma\mu^2 - 2\rho r + \rho^2\mu^2]\|z_n - x^*\|^2 \\
= \;& \theta_1^2\|z_n - x^*\|^2.
\end{aligned}$$

In similar way, using the relaxed (γ_1, r_1)-cocoercivity and μ_1-Lipschitzian of the operator g, and the relaxed (γ_2, r_2)-cocoercivity and μ_2-Lipschitzian of the operator h, we have

$$\|z_n - x^* - [g(z_n) - g(x^*)]\| \leq \sqrt{1 + 2\gamma_1\mu_1^2 - 2r_1 + \mu_1^2}\|z_n - x^*\| = k_1\|z_n - x^*\|.$$

$$\|z_n - x^* - [h(z_n) - h(x^*)]\| \leq \sqrt{1 + 2\gamma_2\mu_2^2 - 2r_2 + \mu_2^2}\|z_n - x^*\| = k_2\|z_n - x^*\|.$$

Therefore, we have

$$\|y_n - x^*\| \leq (1 - \beta_n)\|x_n - x^*\| + \beta_n\theta\|z_n - x^*\|, \tag{2.7}$$

$$\begin{aligned}
\|z_n - x^*\| &\leq (1 - \gamma_n)\|x_n - x^*\| + \gamma_n\theta\|x_n - x^*\| \\
&= (1 - \gamma_n(1 - \theta))\|x_n - x^*\| \\
&\leq \|x_n - x^*\|. && (2.8)
\end{aligned}$$

From (2,7),(2,8), we have

$$\|y_n - x^*\| \leq (1 - \beta_n(1 - \theta))\|x_n - x^*\| \leq \|x_n - x^*\|. \tag{2.9}$$

From(2.6),(2.9), we obtain that

$$\begin{aligned}
\|x_{n+1} - x^*\| &\leq (1 - \alpha_n)\|x_n - x^*\| + \alpha_n\theta\|y_n - x^*\| \\
&\leq (1 - \alpha_n)\|x_n - x^*\| + \alpha_n\theta\|x_n - x^*\| \\
&= (1 - \alpha_n(1 - \theta))\|x_n - x^*\|,
\end{aligned}$$

and hence by Lemma 2.1, $\lim_{n\to\infty}\|x_n - x^*\| = 0$, i.e., $x_n \to x^*$. □

Remark 1. For $g = h$, Theorem 2.2 reduce to Theorem 3.1 of Noor [7]; for $g = h = I$, the identity operator, Theorem 2.2 reduces to a result of Noor and Huang [8] for the variational inequalities and nonexpansive mappings.

Next we will prove the strongly convergence theorem of Algorithm 1.5 under the α-inverse strongly monotonicity (see[9]). With the following result, we extend Theorem 3.3 of [7] from the general variational inequality to the extended general variational inequality, while we also extend the result of [9].

Theorem 2.3. *Let K be a closed convex subset of a real Hilbert space H, and $\alpha > 0$ and $\alpha_1 > 0, \alpha_2 > 0$. Let T be an α-inverse strongly monotone mapping of K into H. Let g be an α_1-inverse strongly monotone mapping of K into*

H and h be an α_2-inverse strongly monotone mapping of K into H, S be a nonexpansive mapping of K into K such that $F(S) \cap GVI(K,T,g,h) \neq \emptyset$. If

$$|1-\alpha_1| + |1-\alpha_2| < \alpha, \tag{2.9}$$

$$|\rho - \alpha| \leq |\alpha[1-(v_1+v_2)], \tag{2.10}$$

where

$$v_1 = \frac{|1-\alpha_1|}{\alpha_1}, v_2 = \frac{|1-\alpha_2|}{\alpha_2},$$

then the approximation solution obtained from Algorithm 1.5 converges strongly to $x^* \in F(S) \cap GVI(K,T,g,h)$.

Proof. It is well known that if T is α-inverse strongly monotonic with the constant α, then T is $\frac{1}{\alpha}$-Lipschitzian continuous (see [9]). For $x^* \in F(S) \cap GVI(K,T,g,h)$, we have

$$\begin{aligned}
& \|x_n - x^* - \rho(Tx_n - Tx^*)\|^2 \\
= \;& \|x_n - x^*\|^2 + \rho^2\|Tx_n - Tx^*\| - 2\rho\langle Tx_n - Tx^*, x_n - x^*\rangle \\
\leq \;& \|x_n - x^*\|^2 + \rho^2\|Tx_n - Tx^*\| - 2\rho\|Tx_n - Tx^*\|^2 \\
\leq \;& \left(1 + \frac{\rho^2 - 2\rho\alpha}{\alpha^2}\right)\|x_n - x^*\|^2.
\end{aligned}$$

So we have

$$\|x_n - x^* - \rho(Tx_n - Tx^*)\| \leq \frac{|\rho - \alpha|}{\alpha}\|x_n - x^*\|. \tag{2.11}$$

In similar way, using the α_1-inverse strongly monotonicity of g and the α_2-inverse strongly monotonicity of h, we have

$$\|x_n - x^* - g(x_n) + g(x^*)\| \leq v_1\|x_n - x^*\|, \tag{2.12}$$

$$\|x_n - x^* - h(x_n) + h(x^*)\| \leq v_2\|x_n - x^*\|, \tag{2.13}$$

where $v_1 = \frac{|1-\alpha_1|}{\alpha_1}, v_2 = \frac{|1-\alpha_2|}{\alpha_2}$. From Algorithm 1.5, (2.11), (2.12) and (2.13), we have

$$\begin{aligned}
\|x_{n+1} - x^*\| \leq \;& (1-\alpha_n)\|x_n - x^*\| + \alpha_n\|S\{x_n - h(x_n) + P_K(g(x_n) - \rho Tx_n)\} \\
& -S\{x^* - h(x^*) + P_K(g(x^*) - \rho Tx^*)\}\| \\
\leq \;& (1-\alpha_n)\|x_n - x^*\| + \alpha_n\|x_n - x^* - \rho(Tx_n - Tx^*)\| \\
& +\alpha_n\|x_n - x^* - g(x_n) + g(x^*)\| \\
& +\alpha_n\|x_n - x^* - h(x_n) + h(x^*)\| \\
\leq \;& (1-\alpha_n)\|x_n - x^*\| + \alpha_n\left(\frac{|\rho-\alpha|}{\alpha} + v_1 + v_2\right)\|x_n - x^*\| \\
= \;& \left\{1 - \alpha_n\left[1 - \left(\frac{|\rho-\alpha|}{\alpha} + v_1 + v_2\right)\right]\right\}\|x_n - x^*\| \\
= \;& [1 - \alpha_n(1-v)]\|x_n - x^*\|,
\end{aligned}$$

where $v = 1 - \left(\frac{|\rho-\alpha|}{\alpha} + v_1 + v_2\right)$, from (2.9)and (2.10), it follows that $\theta < 1$. Using Lemma 2.1, we have $\lim_{n\to\infty} \|x_n - x^*\| = 0$, i.e., $x_n \to x^*$. $\square$

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