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Promising Advantages and Potential Pitfalls of Reliance on Technology in Learning Algebra

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In a rapidly changing and increasingly technological society, the use of technology should not be disregarded in issues of learning algebra. The use of technology in learning algebra raises many learning and pedagogical issues. In this article, previous research on the use of technology in learning algebra is synthesized on the basis of the four issues: conceptual understanding, skills, instrumental genesis, and transparency. Finally, suggestions for future research into technological pedagogical content knowledge (TPCK) are made.

Key Words: Technology, Algebra. Conceptual understanding, Skills, Instrumental genesis, Transparency, Technological pedagogical content knowledge (TPCK)

I. INTRODUCTION

The use of technology in mathematics education raises many learning and pedagogical issues. Effective use of technology in the classroom may have enormous benefits for student understanding. However, technologies have many characteristics that make them complex tools for teachers and students to use correctly, creatively and efficiently. The skills necessary for effective technology use not necessarily the same as those required for traditional paper-and-pencil work. The process of learning with the use of technology is a slow and difficult one. Underlying these issues is the complex relationship among the student, the technology, and the mathematics. In what follows, the first section describes how the effective use of technology enhances students' conceptual understanding in learning algebra while the other sections offer potential hazards that can prevent students accessing technology's effectiveness. The second section addresses skills that students need to develop for the effective use of technology. The third section illustrates the different levels of facility that students can have with

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technology based on the process of instrumental genesis. In the fourth section, I will discuss the issue of transparency in the use of technology. Finally, I will synthesize the ideas coming from promising advantages and potential pitfalls of the use of technology in learning algebra for future study.

II. ENHANCING CONCEPTUAL UNDERSTANDING

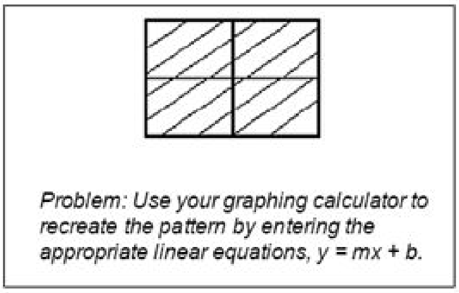
There are many learning issues related to technology. One issue is the changing student role. Another is a change in the nature of the mathematics students are taught. Under the changing student role, we can consider concept image, experimental aspects, dynamic aspects, modeling, student attitude, and reification. Also there are many features related to content and processes in mathematics for the nature of mathematics (Olds, Schwartz, & Willie, 1980). Since there are close relationships among all of these learning issues, the relationship between technology and conceptual understanding in relation to the function concept will be the focus of the discussion below.

1. Concept Image

First of all, the concept image that students can develop as a result of technology can improve students' conceptual understanding. Goldenberg (1988) posed the following problem: "Find the value of x in the equation $4x - 17 = 4x - 4$." As Goldenberg points out, many students added $-4x + 4$ to both sides of the equation and conclude, after getting $0 = -13$, that -13 is the only number left and then x equals -13. Students who have learned to visualize the graphic representations of each symbolic expression do not have this problem because they have a strong visual sense of two functions $f(x) = 4x - 17$ and $g(x) = 4x - 4$. Students using technology developed concept images of these functions as parallel and realized that therefore they do not intersect. Concept image can be well developed in technology and helps students have a deeper conceptual understanding of function. In general, manipulating multiple representations with the help of technology can become one of promising learning environments for students to study various algebra concepts.

2. Experimental Aspects

The experimental aspects of mathematics that can result from using dynamic representations provided by technology can improve students' conceptual understanding. In order to do the problem in figure 1, students need to know the graphic representations of linear equations with the same slope, the meaning of linear equations with the different y-intercepts and the relationships among these lines. In this problem, students can get a deeper conceptual understanding about the slope, y-intercept, and the concept of parallel of linear equations by trial and error with the help of technology. Such experiments, which are difficult without technology, can help students explore deeper concepts that are related to functions instead of memorizing equation forms and formulas. Therefore, the exploratory and inductive aspects of dynamic representations emphasize the heuristics involved in discovering results. In addition, the transition from an intuitive view to a theoretical one is greatly facilitated by the use of technology.



[Figure 1] Sample graphing calculator exercise.
Adopted from activities published by Dan Ethier, Mounds Park Academy, St. Paul, MN.

3. Dynamic Aspects

Dynamic systems of technology also can improve students' conceptual understanding. As Forster and Taylor (2000) point out, when students looked at a function using technology, they saw it as part of a generalized equation, instead of evaluating only fixed expressions such as $f(4) = -1$. In addition, they saw a dynamic movement that is impossible without technology. For instance, instead of learning only how to produce the inverse to a function, students looked at the relationship between a function and its inverse through the use of dynamic properties of technology. They can see the relationship between coordinates of points on the function and on its inverse. Dynamic properties of technology can assist students to see a deeper relationship between two concepts and have a deeper conceptual understanding of the inverse function.

4. Modeling

Students can achieve a better conceptual understanding of functions through the processes of modeling, interpreting, and translating in relation to technology (O'Callaghan, 1998; 강윤수, 2005; 고상숙・고호경, 2007; 이헌수・박종률・정인철, 2009; 한혜숙・신현성, 2008). As Nemirovsky (1996) points out, a modeling perspective of algebra, we need to consider the modeling of physical and mathematical phenomena to make the meaning of algebra meaningful. Because of the availability of graphing calculators, a graphical-approach curriculum can include examples and problems for modeling real-world situations with functions that would be either too time-consuming or impractical without a graphing calculator (Hollar and Norwood, 1999). Students also can have both the ability to create equations, tables, and graphs quickly and the facility to move among the representations rapidly with the help of graphing calculators. There are mutually supportive relationships between modeling or interpreting in strategic competence and conceptual understanding. So, conceptual understanding of function can be improved by modeling a real-world situation using a function, interpreting a function in terms of a realistic situation, and translating among different representations of functions with the help of technology. Students can improve their conceptual understanding by modeling, interpreting, and translating with the help of technology. In addition, the modeling and interactive aspects of technology can help students develop together integration of multiple representations of the same concept.

5. Attitude

O'Callaghan (1998) also found that students in the Computer-Intensive Algebra (CIA) curriculum significantly improved their attitudes toward mathematics over the semester (see also 강윤수・이보라, 2004; 정인철・김승동・노영순・박달원, 2003; 황혜정・고유미, 2006). Positive attitudes toward mathematics led students to have more confidence in their knowledge and ability and a stronger belief that mathematics is understandable. In the long run, these attitudes can help students' conceptual understanding. As adding it up points out, most preschoolers enter school interested in mathematics and motivated to learn it. The challenge to educators is to help them maintain a productive disposition toward mathematics. The free environment that technology provides can also give powerful motivation for students, but the ability to use errors constructively needs to be reawakened in most school children. The use of technology in learning mathematics has the potential for changing students' attitudes toward how they learn mathematics and their behavior in classrooms.

6. Reification

Finally, technology may facilitate the reification of the function concept since it can help in making the transition from an operational to a structural understanding of a concept (Hollar and Norwood, 1999)[2]. As Kieran (1993) points out, a function can be thought of in two ways: operationally as a process and structurally as an object. Hollar and Norwood suggest that if students would have more opportunities with the graphing

2) Hollar and Norwood's particular claims about students' improvement in reification are based on questionable statistical assumptions. However, their general claims about the possible effects of graphing technology and, in particular, of the relative merits of handheld technology over computer-based implementations, are both plausible and intriguing.

calculator to explore functions and to examine abstract applications, they may demonstrate significantly better reification of function. Reification is a difficult process and involves high degree of abstraction. If technology can be used more often and appropriately, students can obtain a more complex and abstract conceptual understanding with the help of technology. Therefore, the graphical possibilities of technology can allow a reification of abstract algebraic concepts.

Technology can assist students in forming deeper conceptual understanding. The processes that are well developed with the help of technology can help students obtain better conceptual understanding. Concept image, experimental features, dynamic aspects, modeling, and reification, and students' positive attitude toward mathematics, can be a part of those contributions to student conceptual understanding.

III. SKILLS

When incorporating technology in classrooms, teachers need to adapt to the changes caused by the technology. Teachers need to understand not only how to use technology but also how to change their practice in order to promote student learning in mathematics. Students also need to develop new skills to succeed in a classroom that employs technology. Many of the traditional skills are not important any longer. For instance, plotting points in order to graph functions is not a skill that students need to master because the graphing technology will plot functions for them. Instead, the student needs to develop new skills for dealing with graphs like estimation, interpreting information from a graph, and interpreting the output of technology. Recognizing that it is not possible to capture completely all aspects of skills that students need to develop in the use of technology, I have chosen three skills to capture what I believe is necessary for anyone to learn mathematics in the use of technology successfully: estimation skills, skills needed to interpret the output of technology, and skills needed to interpret the window of technology.

1. Estimation skills

Molyneax-Hodgson et al. (1999) did a study on spreadsheets and solution skills in Mexico and Britain. They found before the technology was introduced, a presentational teaching (Mexican) led to desire for precise answers by way of algorithmic manipulation of expressions while exploratory teaching (British) led to more estimation from a graph. Through the use of computer spreadsheets, students represented data using graphs, tables, and functions. After the study, British students were using expressions to find precise answers more than before. Likewise, more Mexican students were using graphs and giving estimates. The Mexican students saw that finding a precise answer was not always necessary, and that approximation was a useful skill to have. In this way using the spreadsheets helped students develop the need for estimation and approximation skills as well as improving those skills. In addition to approximating skills, students can also need to improve their ability to present exact calculation in Computer Algebra Systems (CAS) (see Artigue, 1999; Heid, 1999; Heid, 2001; and O'Callahan, 1998 for CAS research studies)

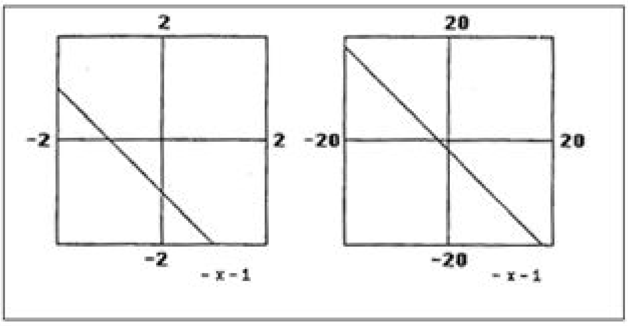
2. Skills needed to interpret the output of technology

Lindsay (1999) reported on a study where students used the CAS *Derive*. Only 18% of the students using the CAS were able to answer two questions about slope of the tangent and normal at a point on a cubic polynomial. The largest problem here was that the CAS had functions called "TANGENT" and "PERPENDICULAR" which outputted the tangent *line* and the normal *line*, not only the slopes of those lines. The most frequent response to the question was the output of the CAS: $9 \cdot x - 26$ and $\frac{12-x}{9}$. The students had to interpret the output of the CAS to find the slopes of each line, 9 and $-\frac{1}{9}$, respectively. In other words, students need to be able to use the output of the technology for solving problems and making informed decisions. This is an important skill for students to use a variety of technology outputs to communicate information and ideas with others.

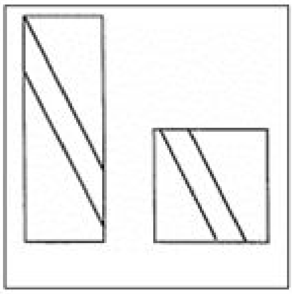
3. Skills needed to interpret the window of technology

Another kind of skill that is important is interpreting what has been graphed with respect to the viewing window. This is a skill that must be developed because students do not naturally interpret what is seen in a window. Goldenberg (1988) points this out when talking about looking through a window. If one sees a person outside a window, one knows that a roughly six foot tall person is immediately outside the window and it is not a 240 foot person forty feet away. In this manner students will not see that the lines in the two windows in figure 2 are the same line. The line on the right appears closer to the origin than is the line on the left, which tells students they have different y-intercepts. Because the students are not taking into account the dimensions of the window, they see two different lines.

A further example of window interpretation comes when looking at lines. If the windows are not the same shape, lines will appear to have different slopes. In the pair of windows in figure 3, students typically viewed the lines on the left to have a smaller slope. There are two reasons for this. First, because the lines between the corners are longer on the left, the slope was seen to be smaller. Second, because of the rectangular shape of the window juxtaposed to the square window, a visual illusion is created which makes the lines on the left appear to have a smaller slope when, in fact, all four lines have the same slope.



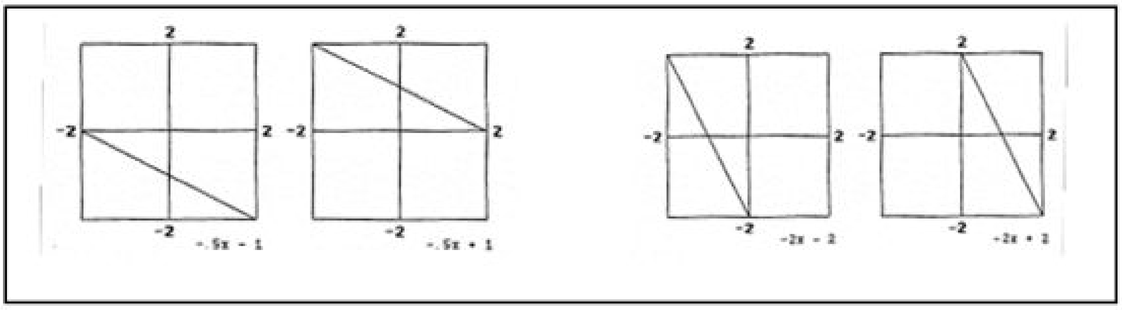
[Figure 2] The line y = −x−1 at two different zoom levels. Adapted from Goldenberg (1988), "Mathematics, metaphors, and human factors: Mathematical, technical, and pedagogical challenges in the educational use of graphical representation of functions," Journal of Mathematical Behavior.



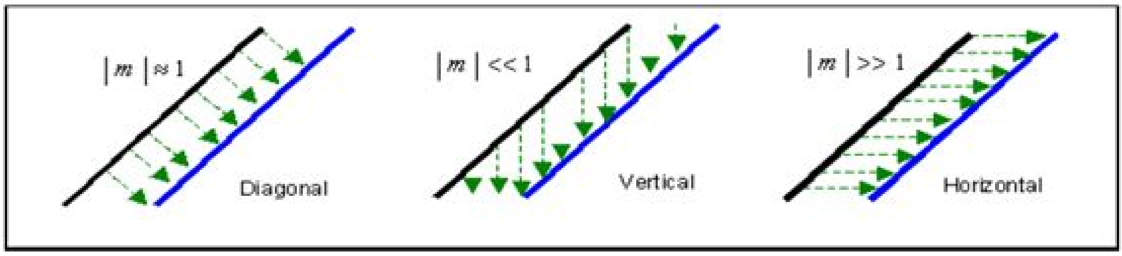
[Figure 3] Four parallel lines in two different-shaped windows. Adapted from Goldenberg (1988).

Another visual illusion that happens is in the translation of lines. Students typically see the pair of lines on the left of figure 4 as a vertical shift and the pair on the right as a horizontal shift. How students viewed the translation depended on the slope. Slopes with absolute value smaller than one appeared as vertical shifts. If the absolute value of the slope is larger than one the translation was thought to be horizontal. Lastly, if the absolute value is around one, then students perceived the shift to be diagonal (See figure 5).

There are a variety of skills that a student needs to develop in a technology-centered classroom. Some of the skills previously taught in school will no longer be a focal skill. Students need to demonstrate a sound understanding of the nature and operation of technology systems and they also need to be proficient in the use of technology in classrooms. The focus of classrooms that use technology should be on conjecturing and modeling, on approximation and estimation, and on interpretation.



[Figure 4] Pairs of translated lines. Adapted from Goldenberg (1988).



[Figure 5] Translation of a line viewed as diagonal, vertical, and horizontal shift. Adapted from Goldenberg (1988).

IV. INSTRUMENTAL GENESIS

The different technologies that students are introduced to in mathematics classrooms are artifacts, in that they are human-made tools. An artifact does not automatically become an instrument, something that can be used to increase student understanding (in this case mathematical understanding), merely through exposure. Many researchers have mentioned a process called *instrumental genesis* which transforms an artifact into an instrument (see, e.g., Guin and Trouche, 1999; Artigue, 2001; 김부윤 · 이지성, 2008). Instrumental genesis is a process by which students' schemes for working with an artifact develop, adapt and assimilate. The development of instrumental genesis has both social and individual components. The individual component is a result of students' schemes being mental constructs of an individual. The social component considers the effect that teachers and other students have in this process. In particular, teachers play an important role and are faced with many complexities while trying to develop instrumental genesis in their students. Due to their high accessibility and current and increasing popularity in mathematical research and instruction, hand-held graphing technologies and CAS will be used to illustrate how a teacher can students move through this process.

The process of instrumental genesis has been described as having different phases.

Students may start first by using pre-existing schemes and then adapt their schemes depending on the task or the activity. Guin and Trouche (1999) saw instrumental genesis as two phases: the discovery phase and the organization phase. In the discovery phase, students explored the effects and organization of various commands. Students in the organization phase had more skepticism of calculator-produced answers and gave less authority to calculator. In relation to her research on CAS, Artigue (2001) mentions three phases. In the first phase, graphical representations are predominant in exploring and solving and the symbolic HOME screen rarely used. In second phase, the HOME screen is beginning to be used more and by the third phase, students use the symbolic HOME screen predominantly, while they interweave paper and pencil methods. In this third phase, the graphical applications are used mostly in problem solving to anticipate the analytical properties of functions. As students worked with the CAS, they came to value the CAS's capacity to perform analytic skills efficiently.

After conducting research involving the role teaching plays in how students use of technology, Lagrange (1999) commented that in order “for the support of the technology to be effective, teachers must control student's development of utilization schemes” (1999, p. 62). Lagrange's conclusion that teachers play a vital role in the instrumentalization process is shared by other researchers (see e.g., Artigue, 2001; Goos, Galbraith, Renshaw, & Geiger, 2000; Guin and Trouche, 1999). But how does a teacher go about fostering instrumentalization? Since schemes are internal constructs particular to individuals, they cannot be taught explicitly. Teachers can help students develop their schemes by introducing mathematical tasks that raise student awareness of the constraints and limitations involved in the technology and by teaching students techniques for using technology in mathematical problem solving. However it is important to pair rational reflection with student usage of these techniques, since this reflection is what can lead to students to acquire conceptual understanding as opposed to just manipulative skills (Lagrange, 1999, p. 63).

Mathematical tasks can be used by teachers to help students develop their instrumental schemes. Guin and Trouche (1999) noted that in France students had to learn the skills to obtain and read graphs on their own. If students are left to fend for themselves when technology is concerned, misconceptions may result. One misconception Guin and Trouche found was that students believed that the asymptotes depicted on the computer screen in a representation of the graphs $y=\tan(x)$ created additional intersection points with the graph of $y=x$. This task is an example of one way to talk to students about the calculator representations for asymptotes and maybe about continuity and discreteness issues. There are other tasks involving the discreteness issue of graphic displays. Artigue (2001) mentions one task where students are shown a function that appears monotonic in the display, however upon zooming-in many oscillations appear. Students are asked to find ways to reproduce this phenomenon. This task incorporates student understandings about function, about the characteristics of the technology they are using and about the discreteness issue. Another issue that teachers should address is that students who always study functions in a standard window may not notice the variation in the function. An example of a task that be used to give students an opportunity to adapt this scheme of graphing in the standard window is to ask students to graph the function $f(x) = x^3 - 12x^2 + 46 - 42$, which when graphed in the standard window appears to be a linear function. This task can illustrate the importance of examining functions analytically.

Since there are many idiosyncrasies involved in the way simplification occurs in internally with CAS, tasks involving equivalent expression formulation and recognition are especially important. For example, the CAS will factor the two equivalent expressions $1-(1-x)(3+2x)-x$ and $1-x-(1-x)(3+2x)$ in two different ways $2x^2-2$ and $2(x-1)(x+1)$. Lagrange (1999) stressed this issue of students recognizing equivalent forms. On task he mentioned was to have student differentiate $3\cos(3x-\frac{\pi}{6})$ by hand and then check with the TI-92. The TI-92 produced the result of $3\cos(3x+\frac{\pi}{3})$. Only 8 out of 26 students recognized that they could use the property, $\cos(a+\frac{\pi}{2}) = -\sin(a+\frac{\pi}{6})$ to simplify their paper and pencil result of $-3\sin(3x-\frac{\pi}{6})$. A task like this can help guide students to make sense of calculator produced answers and to question why the answers are of a different form. Also, this task is an example of where pencil and paper can be used with instrumented techniques. Complementary tasks using paper and pencil techniques with instrumented techniques are encouraged (Artigue, 2001; Trouche and Guin, 1999). It is important to help students see the mathematics behind calculator-produced results. Artigue (2001) and Guin and Trouche (1999) mentioned organizational, internal and command constraints of CAS that teachers must be aware of and help students develop their own awareness. Through different experiences, such as those with the tasks mentioned above, teachers can explore these constraints with their students.

Guin and Trouche (1999) claimed, “It is only through a complex process that students will be able to combine different available sources of information (theoretical text, a calculator, calculation by hand) to construct their own mathematical understanding.” Students develop different relationships with graphing and symbolic calculators. As a result, teachers are working with a diverse group of students that vary in how competent they feel in mathematics and in using technology and vary in the strategies they use in problem solving. In order to look at the ways student behavior changed as a result of CAS usage, Guin and Trouche (1999) defined five student behavioral profiles based on accounts of their subjects previous graphing calculator experiences and their subjects main behavioral features: include random work, mechanical work, resourceful work, rational work and theoretical work. Guin and Trouche found that students in the rational work and theoretical work profiles chose more productive strategies and sought mathematical consistency between all information tools such as theoretical, paper and pencil and calculator.

Goos et. al. (2001) conducted a 3-year longitudinal study that investigated the role of graphing calculators and computers in supporting the mediation social interactions and focused on pedagogical issues. Varying degrees of sophistication with technology were categorized using 4 metaphors: master, servant, partner and extension of self. These are different from the profiles of behavior, in that they incorporate ways of talking about a person's individual interactions with technology, interactions among students using technology and interactions between teacher and student using technology. However both the profiles of behavior and the four metaphors illustrate the different levels of facility students can have with technology. These different levels create complexities for teachers in the coordination of activities and in the development of instrumental genesis in their students.

V. TRANSPARENCY

Any use of technology for learning necessarily will be implemented using a particular technological tool. Clearly not all tools are created equal, and the effectiveness of any technology-based intervention will be at least in part dependent on the specific tool or tools chosen. The "extent to which the technology being used highlights the mathematics that is being studied rather than obscures it" is called transparency (Heid, 1997, p. 7). A tool becomes transparent when it allows the user to forget its role and to believe he or she is interacting directly with the mathematics.

Heid's definition appears to position transparency as a feature of the technological tool itself－that a particular device is more or less transparent depending on its features. Meira (1998) cautions that this is not the case, but rather that transparency is a cultural product of the interaction of the tool with a particular individual in a specific social context. In other words, transparency is not about the tool per se, but about the relationship between the tool and the user at a particular time and in a particular context.

Hancock (1995) extends the notion of transparency as a cultural product by suggesting the dialectic of transparency. This explication emerges from Hancock's examination of students' struggles with a particular database tool. Students began by viewing the program through the lens "naïve transparency," where the tool is assumed by the user to share his or her own meaning for the offered representations. In this case, given names (e.g., "John," "Mary") entered into a database were viewed by the students as gendered. When the students were unable to make the program distinguish between boys and girls based on the given name, they reached a state of "opacity," where their interpretation of the representation was brought into clear conflict with what the computer was able to do. As they came to some understanding of how the program treats character strings, thereby resolving the conflict, the students achieved "coordinated transparency," in which they were able to negotiate rather fluently between their own interpretation of the representation and the tool's.

In spite of this clearly individualized notion of transparency, it seems obvious that some technology is better suited to the promotion of transparency than others. Hancock acknowledges this, and proposes that any tool be treated as curriculum. Just as with technology, the interaction between student and curriculum (be it intended or enacted) is always necessarily individualized, but developers and implementers nevertheless place great stock in particular features of a particular curriculum to promote certain kinds of learning better than other possible features of other possible curricula.

Along these lines, Hancock offers three aspects of technology that should be considered both by developers and by implementers of technology. First, he suggests that like other curricula, a tool be judged "for its accessibility and interest to children, its cultural significance, [and] its contribution to intellectual and social empowerment" (p. 237). Second, he proposes that the tool be judged on the implicit mathematics it embodies, and that this be judged on form, content, coherence, and developmental appropriateness. Finally, he asserts that "where the mathematics of a … tool is judged worthwhile, it needs to be integrated into the accepted body of mathematics curriculum [which] may entail some reorganization of topics and branches of mathematics" (p. 239).

VI. CONCLUDING REMARKS

I tried to do a synthesis in order to put together the ideas coming from previous research on the use of technology in learning algebra on the basis of the four issues: conceptual understanding, skills, instrumental genesis, and transparency. Using technology in the classroom implies many things. In addition to the promising advantage of enhancing conceptual understanding, there are many learning issues related to technology such as changes in the nature of mathematics as well as in the roles in a learning context. The mathematics that can be covered is centered in real-world examples and more opportunities for deeper student understanding. The role of the student changes towards experimenter. The types of skills that the students learn need to change to put less focus on functions that the technology can easily perform and incorporate new skills to deal with what the technology cannot tell the student. However, the development of instrumental genesis from an artifact is complicated. In addition, technology should be transparent in order to be integrated into the mathematics that students interact with. To make the appropriate changes teachers need to consider the interactions between the students with the technology, and also the curriculum with the technology. These are some of the important issues to look at when implementing the use of technology in the classroom.

In a rapidly changing and increasingly technological society, the use of technology should not be disregarded in issues of learning algebra in particular and mathematics in general. It is important to dig further into the complex field of the use of technology in mathematics education and seek for practical and heuristic connections among the categories, as discussed above. However, mathematical learning cannot be fully understood without contemplation of the contribution made by a teacher, students, mathematical content, and their interaction within environments. Thus, the approach to teaching students algebra through technology should be developed and characterized on the basis of solid theories of mathematical learning. However, the amount of previous research on teaching algebra through technology is minimal. Future research into the strengths and weaknesses of technology implementation in learning algebra (and mathematics) and their relationships with teaching is critical for being able to better student understanding of the nature of mathematics, increase reasoning abilities, and improve dispositions toward the subject. In particular, technological pedagogical content knowledge (TPCK) is possibly one of the areas in need of research attention in learning algebra through technology.

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대수학습에서 테크놀로지 사용의 긍적적인 요소와 잠정적인 문제점

김동중[3)]

초록

급변하는 테크놀로지 사회에서 대수학습에서 테크놀로지 사용은 중시하지 않을 수 없다. 대수학습에서 테크놀로지 사용은 많은 학습적 그리고 교육학적 논점을 제기한다. 본 연구에서 이전 연구들을 네 가지 중요한 논점을 중심으로 종합적으로 요약한다: 개념적 이해, 기능, 도구적 형성, 그리고 투명성. 마지막으로 대수 학습에서 테크놀로지 교수학적 내용 지식에 대한 필요성을 제시한다.

주요용어 : 테크놀로지, 대수, 개념적 이해, 기능, 도구적 형성, 투명성, 테크놀로지 교수학적 내용 지식

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