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Bandwidth Allocation and Scheduling Algorithms for Ethernet Passive Optical Networks

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ABSTRACT

This paper considers bandwidth allocation and scheduling problems on Ethernet Passive Optical Networks (EPON). EPON is one of the good candidates for the optical access network. This paper formulates the bandwidth allocation problem as a nonlinear mathematical one and characterizes the optimal bandwidth allocation which maximizes weighted sum of throughput and fairness. Based upon the characterization, two heuristic algorithms are suggested with various numerical tests. The test results show that our algorithms can be used for efficient bandwidth allocation on the EPON. This paper also shows that the WSPT (Weighted Shortest Processing Time) rule is optimal for minimization the total delay time in transmitting the traffic of the given allocated bandwidth.

Keywords: Bandwidth Allocation, Ethernet Passive Optical Network, Nonlinear Mathematical Model, Lower Bound, Scheduling

1. Introduction

This paper considers a fiber network for an access area. For the fiber network, xDSL (x Digital Subscriber Line), HFC (Hybrid Fiber Coax), LAN (Local Area Network), and FTTH (Fiber-To-The-Home) can be considered, where one of the most promising candidates is FTTH due to its high capacity of transmission. However, FTTH requires a huge cost of fiber for connecting all the homes (access points). If there are many fibers and active components, cost for maintenance of the network also becomes large. To reduce the cost, we can consider using just one fiber and sharing its bandwidth among the access points. Such an access network is PON (Passive Optical Network),

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where there is one common fiber connecting several access points through an optical splitter as depicted in Figure 1. Since PON shares a common fiber to transmit traffics, it needs to solve collision resulted from the simultaneous transmission. The two major methods to solve the collision problem are TDMA (Time Division Multiple Access) and WDMA (Wavelength Division Multiple Access). Ethernet Passive Optical Network (EPON) uses the TDMA method. The EPON is a preferred candidate for an access network since Ethernet is a mature technology and most LANs are Ethernet. This paper considers bandwidth allocation and scheduling problems on the EPON. The EPON uses two types of passive components. The one is OLT(Optical Line Terminal), and the other is ONU(Optical Network Unit) in Figure 1. The OLT has a role of connection between the backbone and access network at a central office(CO), where the OLT allocates the bandwidth according to the requested demands from the ONUs connected to customers. The ONU has buffer memory for incoming traffic from customers and for outgoing traffic to the OLT, and arbitrates transmission priority of the packets waiting in the buffer. To transmit the traffic from the OLT to an ONU, the OLT broadcasts the traffic and then each ONU receives the packets if they are transmitted to it. However, it needs to classify channels for transmitting the packets from ONUs to the OLT since there are many ONUs and there occur conflicts when some ONUs try to transmit packets simultaneously. The bandwidth allocation problem is to find an algorithm for sharing the common bandwidth with respect to some criteria such as throughput and fairness.

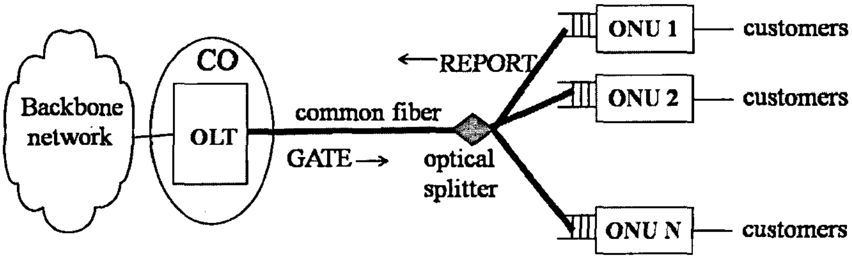


Figure 1. Access network architecture

The bandwidth allocation algorithms are performed by using two control messages, REPORT and GATE in Figure 1 to request and grant bandwidth, respectively. Since the bandwidth allocation algorithm for the EPON has declared to be out of scope of the standard[11], many bandwidth allocation mechanisms including 'polland-stop', 'interleaved polling', and 'interleaved polling with stop' algorithms are proposed in the literature. For example, Kramer *et al.* [9] developed the 'interleaved polling with stop' algorithms called IPACT (interleaved polling with adaptive cycle time, where a limited service discipline is proposed to predefine a maximal bandwidth for all the ONUs to reduce delay time of packet transmissions. However, the IPACT leads to a problem called light-penalty problem. That is, when the traffic load of an ONU is very light, the delay time of the transmission service for the ONU increases significantly. Kramer *et al.* [10] developed an allocation algorithm to reduce the light-penalty problem of the IPACT. Assi *et al.* [2] suggested using a predetermined minimum guaranteed bandwidth for the entire ONUs and classified some ONUs into the overloaded ones if they requested more bandwidth than the minimum guaranteed bandwidth, where the excessive bandwidth is fairly allocated among the overloaded ONUs to improve the bandwidth utilization. Joo and Kwon [7] evaluated a bandwidth allocation algorithm for an EPON system in Korea, where they used GATE and REPORT messages twice. The first messages are use to allocated static bandwidth and the final messages are used to distribute the excessive bandwidth fairly. As a meta-heuristic algorithm, Joo and Smith [8] suggested a Particle Swarm Optimization (PSO) algorithm for the allocation problem.

To incorporate the request on the quality of service (QoS), An *et al.* [1] introduced an algorithm using two parts of frame to reduce mean delay and variation of the service time for a higher priority traffic, where a frame (bandwidth) is divided into two parts of high-priority and low-priority parts. The length of the high-priority part is fixed and the part is dedicated to transmit the high-priority traffic. But, the low-priority part can be used for the high-priority traffic also. Yang *et al.* [14] used a scheme to allocate the surplus bandwidth for the higher priority class traffic, where the surplus bandwidth implies some extra bandwidth and the bandwidth is allocated for the higher priority traffic to improve QoS of the higher priority traffic. Bai *et al.* [3] considered QoS at the ONU level, where they suggested a bandwidth allocation algorithm to improve fairness among the overloaded ONUs. A good survey on the bandwidth allocation algorithm is given by [15].

This paper considers the QoS at the ONU level as same as that of Bai *et al.* Please notice that Bai *et al.* measured the fairness only for the overloaded ONUs and they showed their heuristic algorithm had good fairness for the highly loaded ONUs under an asymmetric traffic. But, we measure the fairness for the entire ONUs and develop an optimal bandwidth allocation algorithm under derived conditions.

Even though Lannoo *et al.* [12] derived the packet delay time by using an analytical modeling, most of the previous studies including Kramer *et al.* [9] and Bai *et al.* [3] are focused on evaluation of their heuristic algorithms without explicit characterization the optimal solution. This paper considers a bandwidth allocation problem to maximize weighted sum of throughput and fairness. For the allocation problem, we formulate a nonlinear mathematical model and characterize the optimal allocation to develop two bandwidth allocation algorithms. We show our algorithms guarantee the optimal solution under derived conditions. Additionally we show the effectiveness and efficiency of our algorithms by comparison with Kramer *et al.* [9] and Bai *et al.* [3] when the derived conditions are not satisfied. Please notice the bandwidth allocation algorithm determines the time duration dedicated to each ONU for the traffic transmission. That's to say, the allocation algorithm does not provide how to sequence the transmission to avoid transmission conflict. Even though the sequencing has an effect on the delay time, most of previous papers use a trivial sequence, that's to say, transmit the granted packets according to the order of increasing index of the ONU. The sequencing problem also occurs within each ONU, where each ONU should determine transmission orders for the packets in its buffer. This paper characterizes the optimal sequencing which minimize total (mean) delay time for the given (gran-ted) bandwidth allocation.

This paper is composed as follows: Section 2 describes the bandwidth allocation and sequencing problems in detail, and formulates the bandwidth allocation problem. The bandwidth allocation problem is characterized, and two algorithms are suggested with a number of numerical tests at Section 3. Section 4 considers a sequencing problem for the given bandwidth allocation. Finally, Section 5 is added for concluding remarks.

2. Problem Formulation

This paper considers N ONUs (customers) to transmit their packets toward the OLT along a common fiber with capacity C bit/second. According to the protocol of EPON [11], each ONU i requests bandwidth assignment for transmission the packets r_i waiting in its buffer to the OLT by using a message called REPORT as depicted in Figure 2.

For the bandwidth requests $\{r_i\}$ from N ONUs, the OLT allocates bandwidth and determines the timeslot start time of the transmission. The allocation is performed with 'interleaved with stop' mechanism, that's to say, the OLT allocates bandwidth periodically at the initial time of each cycle, and the allocated bandwidth with its start time information is notified to the ONUs via a message denoted as GATE in Figure 2. Due to the capacity restriction C on the common link, only some portion of waiting packets will be serviced at the current cycle. Let X_i denote the proportion of allocated bandwidth to the requirement r_i from ONU i at the current cycle, $0 \leq X_i \leq 1$ for all i. If $X_i = 1$, it implies that 100% of its requirement is fulfilled. Otherwise, there is some portion of traffic not serviced in the current cycle and the traffic will be serviced at the succeeding cycles. Each ONU has one chance to transmit its packets at each cycle under the 'interleaved with stop' mechanism. Therefore, we can model the problem by considering just one cycle.

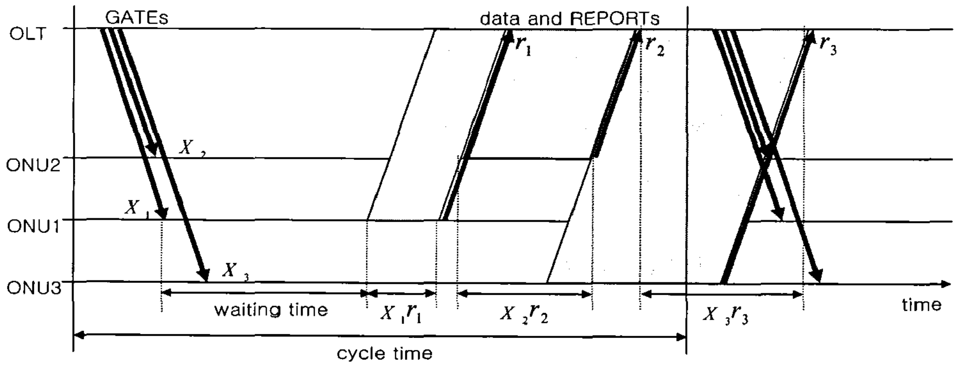


Figure 2. EPON bandwidth allocation model

Each ONU transmits the packets waiting in its buffer toward the OLT as the notified amount $X_i r_i$ at the timeslot start time along the common link by using the Ethernet frame. The timeslot start time of each ONU must be assigned so as not to overlap each other and the OLT needs to know round-trip time (RTT) to each ONU. The time duration to transmit unit packet depends on the distance between the OLT and each ONU. However, we can set the unit time as 1 for all ONUs for simplicity without loss of generality since our main concern is on the bandwidth allocation and the OLT can determine the timeslot start` time to start the transmission before RTT/2 to the target arrival time.

To transmit packets from N ONUs, a setup is required between each transmission for laser on/off and guarding its information, but the setup is independent on the ONU type and size of transmission. So, we can ignore the setup time for analysis. There are several classes of service (CoS) packets and the CoS can be applied to ONUs. That's to say, ONU i has its weight w_i representing its CoS determined by SLA (Service Level Agreement). This paper considers a bandwidth allocation problem to maximize utilization (throughput) and weighted fairness simultaneously. We suggest two algorithms for the bandwidth allocation. Furthermore, this paper finds the optimal schedule of packets for the given allocated bandwidth to minimize total delay time.

Firstly, we formulate the bandwidth allocation problem with a bicriteria objective function by using a throughput weight α, $0 \le \alpha \le 1$, as follows:

$$\text{Max.} \quad \alpha\,(\sum_{i=1}^{N} r_i X_i\,)/C + (1-\alpha\,)(\sum_{i=1}^{N} X_i \,/\, w_i)^2 \,/\, [N \cdot \sum_{i=1}^{N} (X_i \,/\, w_i)^2\,] \qquad (1)$$

$$\text{s.t} \quad \sum_{i=1}^{N} r_i X_i \le C \qquad (2)$$

$$0 \le X_i \le 1, \quad i = 1, 2, \cdots, N \qquad (3)$$

The first term of (1) denotes throughput, and the second term represents weighted fairness. Please notice that $\sum_{i=1}^{N} r_i X_i\,/C$ is a normalized form of the throughput which has 1 as its maximum value. We measure the fairness for all N ONUs, where we incorporate the ONU weight w_i instead of Jain's fairness measure [6]. Please notice that the value of (1) depends on the throughput weight α and the weight is determined by the decision maker (operator) of the EPON network. Equation (2) implies that the total amount of assigned bandwidth cannot be larger than capacity C. The decision variable X_i of (3) represents the proportion of demand allocated to the ONU i. We can obtain the allocated bandwidth $r_i X_i$ for the ONU i by solving the bandwidth allocation problem.

To transmit the packets from N ONUs, the OLT need to determine the transmission start time as well as the bandwidth allocation so as not to overlap each other in transmission the granted bandwidth $\{r_i X_i\}$. The OLT forms a GATE message for each ONU with its granted bandwidth and the start time. For the arrived GATE message, the ONU waits its transmission till its start time and then transmits its packets during the granted bandwidth as depicted in Figure 2, where the ONU also need to determine the transmission sequence for the packets within its buffer. At section 4, this paper considers both between in ONUs and within an ONU sequencing problems to minimize total delay time.

3. Bandwidth Allocation

This section characterizes the optimal bandwidth allocation which maximizes the weighted sum of throughput and fairness. Based upon the characterization, two heuristic algorithms are developed.

3.1 Bandwidth Allocation Algorithms

This section characterizes the optimal allocation and develops two bandwidth allocation algorithms.

Proposition 1: If $w_i = w_0$ for all *i*, then the optimal allocation is $X_i = \text{Min}\{C/\sum_{i=1}^{N} r_i\ ,\ 1\}$ for all *i* and its optimal objective value is $\alpha\,\text{Min}\{1, \sum_{i=1}^{N} r_i\ /C\} + (1-\alpha)$.

Proof: If $w_i = w_0$ for all *i*, we can easily noticed that the throughput term of (1) is maximized when $X_i = 1$ for all *i* and the fairness term is maximized when $X_i = X_0$ for all *i*. That's to say, it need to maximize the value of X_0 in the equation of $X_i = X_0$ for all *i* for the optimal allocation. Since X_0 should satisfy (2) and (3), we can easily show the optimal value of X_0 is $\text{Min}\{C/\sum_{i=1}^{N} r_i,\ 1\}$, and its objective value is $\alpha\,\text{Min}\{C, \sum_{i=1}^{N} r_i\ \}/C + (1-\alpha)$. This completes the proof.

Proposition 1 gives the optimal allocation if all the weights of ONUs are identical.

However, if the weights are not identical, it is not easy to find the optimal solution. Now, we will consider only the general weight problem at the remaining part of the paper. For the general problem, let $Y_o = \text{Min}\{C/\sum_{i=1}^{N} w_i r_i,\ \sum_{i=1}^{N} r_i / \sum_{i=1}^{N} w_i r_i\}$. Proposition 2 finds an optimal allocation on a partial domain of Y_o.

Proposition 2: If $Y_o \leq \underset{1\leq i\leq N}{Min}\{1/w_i\}$, the optimal allocation is obtained as $X_i = w_i\, Y_o$ and it has the optimal objective value 1.

Proof: Fairness is trivially maximized because X_i/w_i equals the constant Y_o for all i. It is noticed that $Y_o \geq [\underset{1\leq i\leq N}{Min}\{1/w_i\}][\text{Min}\{C/\sum_{i=1}^{N} r_i,\ 1\}]$. Therefore, if $Y_o \leq \underset{1\leq i\leq N}{Min}\{1/w_i\}$, then it should be $C/\sum_{i=1}^{N} r_i \leq 1$. That is to say, we have $Y_o = C/\sum_{i=1}^{N} w_i r_i$ and $X_i = w_i\, Y_o$ = $w_i\, C/\sum_{i=1}^{N} w_i r_i$. Please notice that $\sum_{i=1}^{N} r_i X_i = C$ and X_i is feasible solution because $w_i\, C/\sum_{i=1}^{N} w_i r_i \leq w_i\, \underset{1\leq i\leq N}{Min}\{1/w_i\} \leq 1$. This completes the proof.

Even though Proposition 2 is useful for finding the optimal allocation, it does not guarantee an optimal solution if $Y_o > \underset{1\leq i\leq N}{Min}\{1/w_i\}$. Since Equation (1) is not concave function with respect to decision variable X_i, we try to find good heuristic algorithms for the case of $Y_o > \underset{1\leq i\leq N}{Min}\{1/w_i\}$. For the algorithms, we find two special solutions denoted as S1 and S2 when $\alpha = 0$ or $\alpha = 1$ in (1), respectively.

solution S1: $X_i = w_i\,(\text{Min}\{Y_o,\ \underset{1\leq i\leq N}{min}\{1/w_i\}\})$ (4)

solution S2: $X_i = \text{Min}\{C/\sum_{i=1}^{N} r_i,\ 1\}$ for all i (5)

Proposition 3: Both of the solutions S1 and S2 are feasible ones.

Proof. To show the feasibility of S1, we can classify the problem into two classes of $Y_o \leq \underset{1\leq i\leq N}{Min}\{1/w_i\}$ and $Y_o > \underset{1\leq i\leq N}{Min}\{1/w_i\}$. Firstly, if $Y_o \leq \underset{1\leq i\leq N}{Min}\{1/w_i\}$, S1 has the value of

$X_i = w_i Y_o$. The solution X_i satisfies that $0 \le X_i \le 1$ for all i and $\sum_{i=1}^{N} r_i X_i \le C$ since $Y_o = \mathrm{Min}\{C/\sum_{i=1}^{N} w_i r_i ,\ \sum_{i=1}^{N} r_i / \sum_{i=1}^{N} w_i r_i\}$ by definition. Secondly, if $Y_o > \underset{1 \le i \le N}{Min}\{1/w_i\}$, then the resulting solution becomes $X_i = w_i (\underset{1 \le i \le N}{Min}\{1/w_i\})$ which satisfies the relationship of $0 \le X_i \le 1$ for all i, and $\sum_{i=1}^{N} r_i X_i = \underset{1 \le i \le N}{Min}\{1/w_i\} \sum_{i=1}^{N} w_i r_i < Y_o \ (\sum_{i=1}^{N} w_i r_i) = \mathrm{Min}\{C,\ \sum_{i=1}^{N} r_i\} \le$ C. Similarly, we can show the feasibility of S2 by classifying the problem into two cases of $C/\sum_{i=1}^{N} r_i \le 1$ and $C/\sum_{i=1}^{N} r_i > 1$. If $C/\sum_{i=1}^{N} r_i \le 1$, the solution becomes $X_i = C/\sum_{i=1}^{N} r_i$ for all i which has the relationship of $0 \le X_i \le 1$ for all i, and $\sum_{i=1}^{N} r_i X_i = C$. Otherwise, $X_i = 1$ for all i, and the solution leads to the relationship of $\sum_{i=1}^{N} r_i X_i = \sum_{i=1}^{N} r_i \le C$ by the precondition of this case. This completes the proof.

It is noticed that solution S1 has maximum fairness, but may lose some amount of throughput with its objective value $\alpha (\sum_{i=1}^{N} w_i r_i)\ \mathrm{Min}\{Y_o,\ \underset{1 \le i \le N}{min}\{1/w_i\}\})/C + (1-\alpha)$. By the way, solution S2 has maximum throughput, but lose some amount of fairness with its objective value $\alpha + (1-\alpha)\mathrm{Min}\{C/\sum_{i=1}^{N} r_i ,\ 1\} (\sum_{i=1}^{N} 1/w_i)^2 / [N \cdot \sum_{i=1}^{N} (1/w_i)^2]$. Since both of S1 and S2 are feasible, we can find a good lower bound for the optimal bandwidth allocation if we select the better one between S1 and S2.

Heuristic algorithm H1: Select one solution with higher objective value between solutions S1 and S2.

H1 determines the allocation directly by using equations (4) and (5) without checking the total available bandwidth. So, it reduces computation time than improvement type algorithms such as Kramer *et al.* [9] and Bai *et al.* [3].

Proposition 4: H1 guarantees an optimal solution for the bandwidth allocation problem satisfying the relationship of $Y_o \le \underset{1 \le i \le N}{Min}\{1/w_i\}$.

Proof: If $Y_o \le \underset{1\le i\le N}{Min}\{1/w_i\}$, solution S1 of $X_i = w_i\, Y_o$ is the optimal one by Proposition 2. This completes the proof.

Proposition 4 implies that solution S1 is an optimal one if $Y_o \le \underset{1\le i\le N}{Min}\{1/w_i\}$. But, we cannot guarantee its optimality if $Y_o > \underset{1\le i\le N}{Min}\{1/w_i\}$. As an example of H1, consider two ONUs with $(r_1,\ r_2)$ = (100, 60) and $(w_1,\ w_2)$ = (1, 2) on a link with capacity C = 150. Suppose that $\alpha = 0.6$. Then, Y_o = Min{150/220, 160/220} $> \underset{1\le i\le N}{Min}\{1/w_i\}$ = 0.5, solution S1 is $(X_1\ X_2)$ = (0.5, 1) and has its objective value 66.4. The solution S2 leads to $X_i = \text{Min}\{C/\sum_{i=1}^{N} r_i\,,\ 1\}$ = 15/16 for all *i* and has objective value 90.1991. Therefore, the solution of H1 is S2 having larger objective value.

We can consider an improvement procedure for a given solution. For the procedure, let us use new notations *T* and *F* to denote the throughput and fairness for a given solution $\{X_i\}$, respectively: $T = \sum_{i=1}^{N} r_i X_i\,/C$, $F = (\sum_{i=1}^{N} X_i / w_i)^2 / [N \cdot \sum_{i=1}^{N} (X_i / w_i)^2]$.

Heuristic algorithm H2

Step 1: Find a solution (lower bound) by using H1. Let the solution as the current best one and denote the solution as S0. Let denote the throughput and fairness of the solution *S0* as *T0* and *F0*, respectively. Let $Z0 = \alpha\, T0 + (1-\alpha)F0$ and $Z^* = Z0$. For the solution $\{X_i^0\}$ of S0, let $X_i^T = X_i^0$ for all *i*.

Step 2: If *T0* = 1 or $X_i^0 = 1$ for all *i*, go to Step 3. Otherwise, find *i* = arg$\{\underset{1\le i\le N}{Min}\{X_i^0 / w_i\} \mid X_i^0 < 1\}$. Let $\delta_i = \text{Min}\{(1\text{-}T0)C/r_i,\ 1\text{-}X_i^0\}$ and $X_i^T = X_i^0 + \delta_i$. If $\delta_i > 0$, then calculate *F* for the $\{X_i^T\}$. If $\alpha\,(T0 + \delta_i\, r_i/C) + (1\text{-}\alpha)F > Z0$, let $X_i^0 = X_i^0 + \delta_i$, $T0 = T0 + \delta_i\, r_i/C$, $F0 = F$, and $Z0 = \alpha\, T0 + (1\text{-}\alpha)F0$.

Step 3: If F0 = 1, go to Step 4. Otherwise, go to Step 3.2 when $X_i^0 = 1$ for all *i*. If $X_i^0 \ne 1$ for any *i*, go to Step 3.1.

Step 3.1: Find *i* = arg$\{\underset{1\le i\le N}{Min}\{X_i^0 / w_i\} \mid X_i^0 < 1\}$. Let $\delta_i = \text{Min}\{1\text{-}X_i^0,\ w_i(\underset{1\le i\le N}{Max}\{X_i^0 / w_i\})\text{-}X_i^0\}$ and $X_i^T = X_i^0 + \delta_i$. If $\delta_i > 0$, then calculate *F* for the $\{X_i^T\}$. If $\alpha\,(T0$

$+\delta_i\, r_i/C$) + $(1-\alpha)F > Z0$, let $X_i^0 = X_i^0 + \delta_i$, $T0 = T0 + \delta_i\, r_i/C$, $F0 = F$, and $Z0 = \alpha\, T0 + (1-\alpha)F0$. Go to Step 4.

Step 3.2: Find $j = \arg \underset{1\le j\le N}{Max}\{X_j^0 / w_j\}$. Let $\delta_j = w_j\,(\underset{1\le j\le N}{Min}\{X_j^0 / w_j\}) - X_j^0$ and $X_j^T = X_j^0 - \delta_j$. If $\delta_j > 0$, then calculate F for the $\{X_j^T\}$. If $\alpha\,(T0 - \delta_j\, r_j/C) + (1-\alpha)F > Z0$, let $X_j^0 = X_j^0 - \delta_j$, $T0 = T0 - \delta_j\, r_j/C$, $F0 = F$, and $Z0 = \alpha\, T0 + (1-\alpha)F0$.

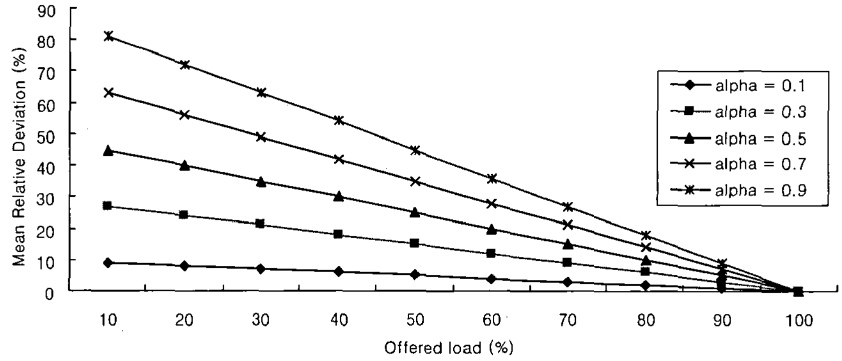
Step 4: If $Z^* < Z0$, let $Z^* = Z0$ and go to Step 2. Otherwise, stop the procedure and select the current solution $S0$ with $\{X_i^0\}$ and $Z0$ as the best one.

Algorithm H2 uses the solution of H1 at Step 1 as a lower bound, and Steps 2 and 3 try to improve the solution. Step 2 tries to increase throughput by addition the bandwidth $\delta_i r_i$ to the ONU i having the smallest ratio $\{X_i^0 / w_i\}$ for the fairness consideration. Additionally, Step 3 tries to improve the fairness by either addition or subtraction the bandwidth, where the throughput will be decreased if any bandwidth subtraction occurs. When the solution cannot be improved further, Step 4 selects the current solution as the best one. The major computational load of H2 comes from Step 3 and its worst case computational complexity is O(N). We could reduce the computational load of Steps 2 and 3 if we derived a simple equation for computation the value of F-$F0$. However, test results at Section 3.2 show that the CPU time is very small even though the current Steps 2 and 3 are employed as the improvement procedure.

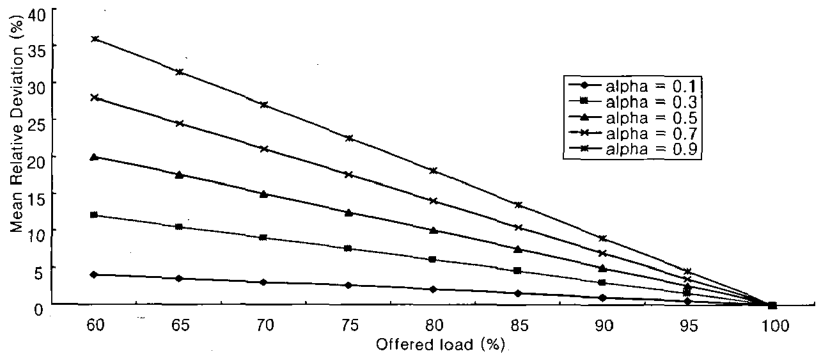
3.2 Performance Evaluation

For the performance evaluation of the bandwidth allocation algorithms H1 and H2, we consider an EPON system with 32 ONUs using a common fiber of 1000Mbps speed. We assume that the incoming traffic on the EPON system is composed of 20% of EF (Expedited Forwarding), 40% load of AF (Assured Forwarding), and 40% load of BE (Best Effort) traffic, where the size of EF traffic is 70bytes, and each size of AF and BE traffic is 64 through 1518 bytes [3]. If we suppose that the size of packet is distributed in uniform distribution and there are many packets, we can use the normal distribution with its standard deviation 0.0019Mbps in size as an approximated distribution of input traffic, where we consider two types of symmetric and asymmetric traffic. The symmetric traffic is consisted of generating the balanced traffic with each

ONU for the offered load of 10 through 100%. The asymmetric traffic is also balanced traffic except one ONU, where the ONU make the offered load of 60 through 100% and the other *N*-1 ONUs maintain the traffic of 60% load. Each ONU can have its class of service according to service level agreement (SLA). This paper considers six classes of service [5]. The classes are randomly generated from a discretely uniform distribution on the domain of [1, 6]. For each combination of parameters, we tested mean performance for the randomly generated 20 problems having $Y_0 > \underset{1\leq i\leq N}{Min}\{1/w_i\}$ by using Microsoft visual C++ on a PC (Intel Pentium 4 2.4GHz).



(a) Symmetric Traffic



(b) Asymmetric Traffic

Figure 3. Mean Relative Deviation of Lower Bound

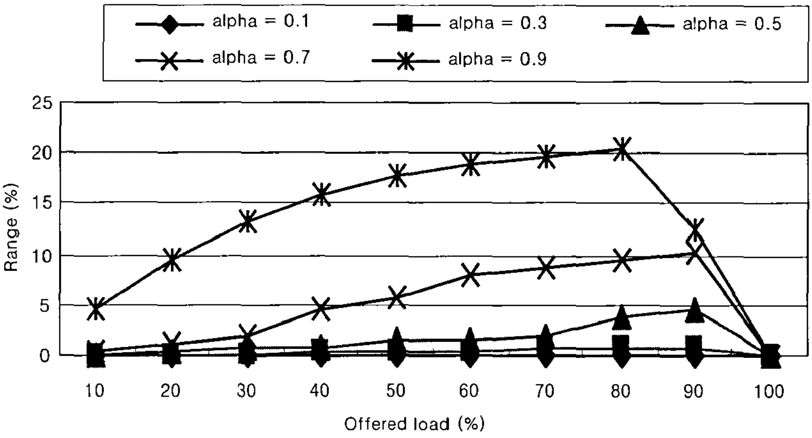
We use mean relative deviation to evaluate algorithms H1 and H2. It is very difficult to find the optimal solution, so we use the solution of algorithm H1 for comparison the performance. The first test is to know the performance behavior of H1. Let denote the solution of H1 as HS1. Please remind HS1 is a lower bound of the optimal objective value. We can presuppose that the lower bound HS1 increases as the offered load increases. Figure 3 shows the mean relative deviations of HS1 to upper bounds under the both of the symmetric and asymmetric traffic, where the mean relative deviations are calculated by the mean value of '100(1-HS1)/1' for randomly generated 20 problems since we can use 1 as the upper bound. Figure 3 represents that the mean relative deviation of HS1 decreases linearly as the offered load increases or α decreases. That's to say, the larger offered load or the smaller throughput weight α, the larger HS1. We can also observe that H1 derives almost optimal solutions under the offered load 100% in Figure 3, where the mean relative deviation is almost zero.

Secondly, Figures 4 through 6 test the performance of H2, Kramer *et al.* [9] and Bai *et al.* [3] to H1, where we use Kramer *et al.* [9] and Bai *et al.* [3] to compare their relative performance to H2, and denote IPACT and BAI for the Kramer's and Bai's solutions, respectively.

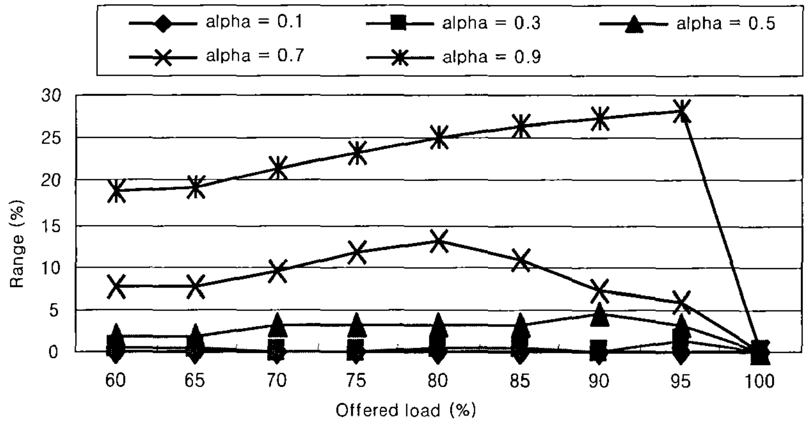
The mean relative deviation of H2 is measured as the average values of

$$100(\mathrm{HS2}-\mathrm{HS1})/\mathrm{HS1}, \tag{6}$$

where HS2 is the solution (objective value) resulted from algorithm H2. We have already known that HS2 is always better than or equal to HS1 and the mean value of (6) must be always non-negative since H2 tries to improve HS1. Please notice that the larger value of (6) implies the more improved solution than HS1, and the mean value of (6) will be zero if all the generated problems satisfy the relationship $Y_o \le \underset{1 \le i \le N}{Min}\{1/w_i\}$ since both H1 and H2 guarantee the optimal solution in the case. Even though we do not represent a figure on the mean analysis here, the test results show that the mean value of (6) increases as the offered load or the α increases with 2.02% and 2.30% in the overall average for symmetric and asymmetric traffic, respectively. Instead of presenting the mean value of (6), Figure 4 shows the ranges of (6) as a measure of dispersion of HS2, where the range is calculated as difference between the maximum and minimum values of (6) among 20 test problems. Figure 4 represents that the range increases as the throughput weight α or offered load increases. However, the average ranges are not big as 4% and 6.46% for the symmetric and asymmetric traffic, respectively. We tried to measure CPU time of H2 also. But, we cannot calculate the mean CPU times since the times are zero on the Pentium 4 PC, which implies that the mean CPU time will be less than 1 milisecond. Please remind the computational complexity of H2 is O(*N*). Based upon the test results, we can conclude H2 is valuable to find better solution robustly especially when the offered load or the throughput weight is large. By the way, we can use H1 to find a good solution more quickly especially when both of the offered load and throughput weight are small.



(a) Symmetric Traffic



(b) Asymmetric Traffic

Figure 4. Performance Range of H2

The mean relative deviations of BAI are calculated as the average values of 100(BAI-HS1)/HS1 for the randomly generated 20 problems in Figure 5. Figure 5 shows the mean deviations of BAI increases as α increases. But, the mean values are negative for all the problem cases. The negative value implies that HS1 is better than BAI on the average; in fact, we observe that HS1 is better than BAI for all the test problems. Even though Figure 5 shows the test result only for the symmetric traffic, we can observe the similar trends for the asymmetric traffic also.

Similarly, mean relative deviation of IPACT to HS1 is calculated as the average values of 100(IPACT-HS1)/HS1 for the 20 problems. As depicted in Figure 6, all the points are plotted under zero line.

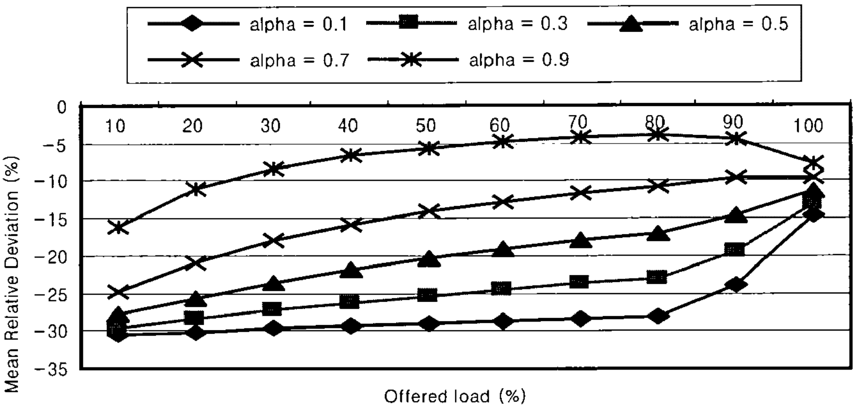


Figure 5. Mean relative deviation of BAI for symmetric traffic

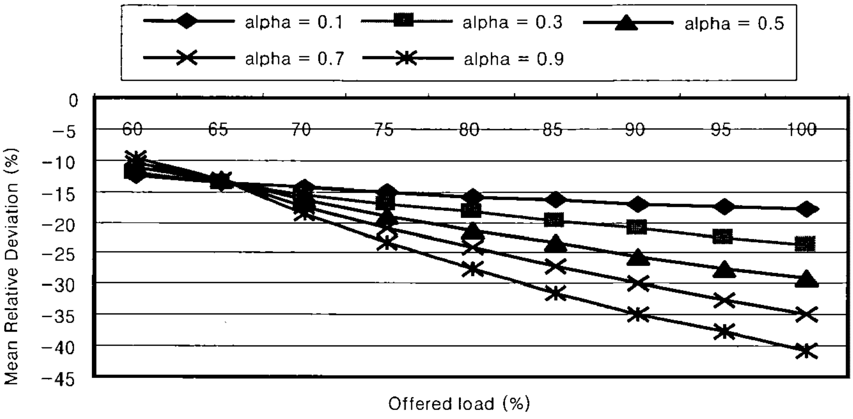
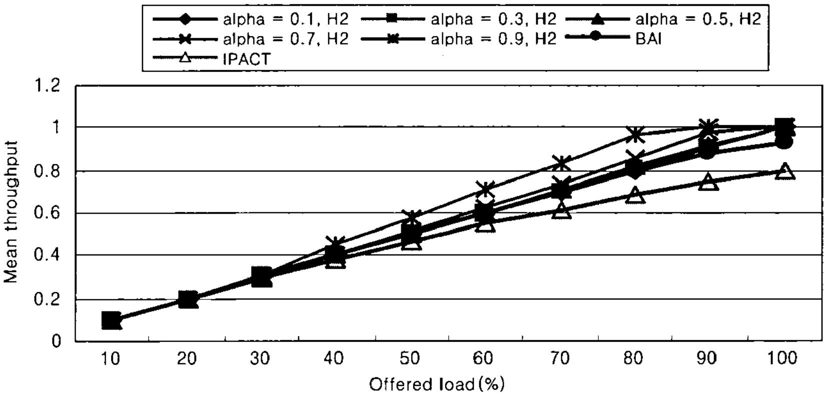
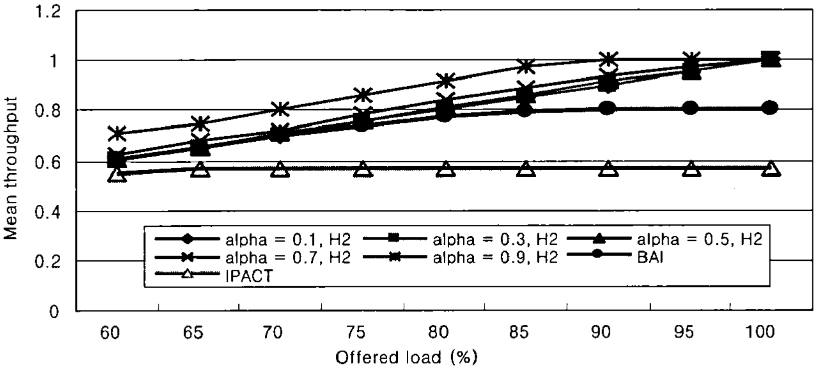


Figure 6. Mean relative deviation of IPACT for asymmetric traffic

Please notice that our algorithms use solutions S1 and S2 derived from our mathematical model, but BAI and IPACT use greedy solutions. Therefore, we can conclude that H1 and H2 have superior performance than BAI and IPACT. We can also observe such behaviors in Figures 7 and 8. Thirdly, we show the throughput (the first term of (1)) and fairness (the second term of (2)) in Figures 7 and 8, respectively.



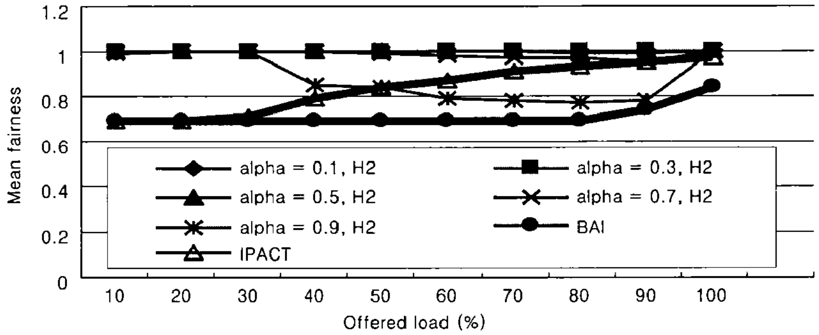
(a) Symmetric Traffic



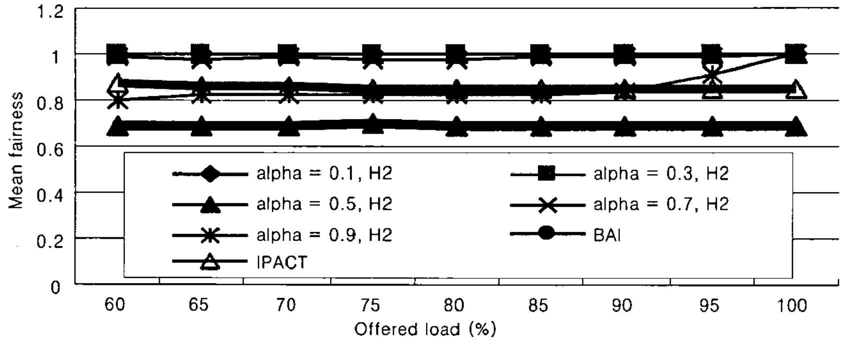
(b) Asymmetric Traffic

Figure 7. Mean throughput

Figure 7 shows the mean throughputs of H2, BAI and IPACT increase as the offered loads increase, where the throughput of IPACT is the worst especially when the non-uniform (asymmetric) traffic of Figure 7(b). Figure 7 implies that our algorithm H2 leads to higher utilization (throughput) than BAI and IPACT. For the fairness measure, Figure 8 represents that the fairness of H2 depends on value of α and is not worse than IPACT and BAI, where we can observe that BAI has robust fairness in the asymmetric traffic (see Figure 8(b)), but the fairness is the smallest. Therefore, we can conclude that our algorithms H1 and H2 outperforms than [3] and [9] with respect to the mean value of (1).



(a) Symmetric Traffic



(b) Asymmetrci Traffic

Figure 8. Mean Fairness

Based upon the test results, we recommend using H1 for a very fast good solution. We can also use H2 for the improved solution especially when the throughput weight α is large, where added computational load is very small negligibly.

Both algorithms H1 and H2 find the bandwidth allocation for each ONU. Please notice the bandwidth allocation algorithm does not provide how to sequence the transmission of the granted bandwidth. Since there are many ONUs and packets within an ONU, there occur conflicts when some ONUs or packets try to transmit packets simultaneously. The following section considers the sequencing problem to avoid the transmission conflict and to minimize mean delay time.

4. Optimal Sequencing

We use some additional notations for the sequencing problem. Let G_i denote the allocated bandwidth to the ONU *i*, $G_i = r_i X_i$. Suppose that the granted bandwidth G_i is composed of n_i packets waiting in the buffer of ONU *i*. The size of the granted packet *j* in ONU *i* is denoted as p_{ij}, $i = 1, 2, \cdots, N; j = 1, 2, \cdots, n_i$. For the packet $\{p_{ij}\}$, let denote the total size of packets in ONU *i* as T_i, $T_i = \sum_{j=1}^{n_i} p_{ij}$. If the weight of packet *j* in ONU *i* is q_{ij}, the total weight of packets in ONU *i* is denoted as Q_i, $Q_i = \sum_{j=1}^{n_i} q_{ij}$. Our objective is to find the optimal sequence which minimizes total(mean) delay time for the given allocated bandwidth $\{G_i\}$, where the delay time of ONU *i* is a completion time of the entire transmission of packets composing the allocated bandwidth G_i. That's to say, the transmission is performed as the batch availability [13]. Since the transmission is accomplished along a common fiber, we can consider the transmission speed is 1 without loss of generality. The sequencing problem occurs within an ONU and between in ONUs. The OLT determines and notifies the timeslot start time and duration of each transmission to each ONU, and then each ONU transmits the packets during the allocated time duration. For the transmission, each ONU need to determine the sequence of transmission the waiting packets within the buffer of the ONU. Proposition 5 characterizes an optimal sequencing rule for the granted packets.

Proposition 5: For a given bandwidth $\{G_i\}$, the optimal sequence of transmission is determined as nondecreasing order of $\{T_i / Q_i\}$ and the optimal sequence of packets within ONU *i* is nondecreasing order of $\{p_{ij} / q_{ij}\}$.

Proof: We can regard the scheduling problem as the basic single machine scheduling one. So, it is obvious since the WSPT (Weighted Shortest Processing Time) rule is the optimal one for minimization the total weighted completion time on the basic single machine ([4], [13]).

Please notice that the computational load of Proposition 5 is small since computational complexity of WSPT sequencing is O(N log N). As an example for the optimal sequencing, consider an EPON system with three ONUs. Suppose that the granted bandwidth is determined as $(G_1, G_2, G_3) = (7, 60, 80)$ by algorithm H1 or H2 at Section 3. Assume that the packets composing the granted bandwidth are given follows: for ONU 1, $n_1 = 2$ with $(p_{11}, p_{12}) = (2, 5)$ and $(q_{11}, q_{12}) = (1, 1)$; for ONU 2, $n_2 = 5$ with $(p_{21}, p_{22}, p_{23}, p_{24}, p_{25}) = (10, 20, 5, 15, 10)$ and $(q_{21}, q_{22}, q_{23}, q_{24}, q_{25}) = (1, 1, 2, 2, 3)$; for ONU 3, $n_3 = 3$ with $(p_{31}, p_{32}, p_{33}) = (20, 30, 30)$ and $(q_{31}, q_{32}, q_{33}) = (1, 2, 2)$.

Firstly, the optimal transmission sequence between in ONUs is determined by Proposition 5 as the nondecreasing order of $\{T_i / Q_i\}$: ONU1→ONU2→ONU3 since $T_1 / Q_1 = (2+5)/(1+1) = 3.5 < T_2 / Q_2 = 6.66 < T_3 / Q_3 = 16$. That's to say, the OLT can determine the transmission starting time to be sequence of ONU1→ONU2→ONU3 for the minimal total delay time.

Secondly, the optimal sequences of the packets within each ONU are derived as followings: for ONU 1, $p_{11} \rightarrow p_{12}$ since $p_{11} / q_{11} = 2 < p_{12} / q_{12} = 5$; for ONU 2, $p_{23} \rightarrow p_{25} \rightarrow p_{24} \rightarrow p_{21} \rightarrow p_{22}$ since $p_{23} / q_{23} = 2.5 < p_{25} / q_{25} = 10/3 < p_{24} / q_{24} = 7.5 < p_{21} / q_{21} = 10 < p_{22} / q_{22} = 20$; for ONU 3, $p_{32} \rightarrow p_{33} \rightarrow p_{31}$ similarly. Therefore, overall sequence is determined as $p_{11} \rightarrow p_{12} \rightarrow p_{23} \rightarrow p_{25} \rightarrow p_{24} \rightarrow p_{21} \rightarrow p_{22} \rightarrow p_{32} \rightarrow p_{33} \rightarrow p_{31}$.

As a special case, if $q_{ij} = q_0$ for all i and j, then the optimal sequence of transmission is determined as nondecreasing order of $\{T_i / n_i\}$ and the optimal sequence of packets within ONU i is nondecreasing order of $\{p_{ij}\}$. For the between in ONUs scheduling of Proposition 5, the OLT needs to know the values of T_i and Q_i. The OLT can use G_i for T_i to implement the sequencing procedure. But, ONU i should send the additional information Q_i to the OLT, where Q_i of ONU i is the total weight of packets composing r_i. Even though REPORT frame format is standardized by [11], the way to use each field of the REPORT message has left the choice to equipment vendors. Therefore, we can set two bits in position 1 and 2 of the 'Report bitmap' as 1 to use 'Queue #1 report' and 'Queue #2 report' fields in the REPORT message [11] for r_i and Q_i, respectively. For the received information r_i and Q_i in REPORT message, the OLT determines the value of X_i and can set the values of T_i and Q_i as $T_i = r_iX_i$ and $Q_i = Q_iX_i$ for the optimal sequencing. By the way, each ONU can transmit packets to the OLT according to WSPT rule of Proposition 5 without any additional information.

5. Conclusion

This paper considers bandwidth allocation and sequencing problems in EPONs. For the bandwidth allocation problem, we formulate a nonlinear programming to maximize weighted sum of throughput and fairness. The optimal allocation is characterized, and two algorithms are suggested based upon the characterization. The first algorithm is a very fast heuristic one, and the second one tries to improve solution of the first one with additional improvement procedure. These algorithms guarantee the optimal solution in the case of $Y_0 \leq \underset{1 \leq i \leq N}{Min}\{1/w_i\}$ problems. To evaluate their performance for the other problem case, various numerical problems are tested. The test results show that the heuristic algorithms are good for the allocation problem. For the determined amount of allocated bandwidth, this paper also formulates a transmission sequencing problem as a basic single machine problem. This paper shows that the WSPT(Weighted Shortest Processing Time) order is the optimal sequencing which minimizes the mean delay time in transmission all the packets.

Even though this paper considers a bandwidth allocation problem under the 'interleaved with stop' mechanism, a stochastic model on the whole service periods need to be analyzed if we want to allocate the bandwidth without the cycle-based allocation. As another further study, bandwidth allocation problems for the other measures need to be analyzed.

References

[1] An, F-T., H. Bae, Y-L. Hsueh, K. S. Kim, M. S. Rogge, and L. G. Kazovsky, "A New Media Access Control Protocol Guaranteeing Fairness among Users in Ethernet-based Passive Optical Networks," *IEEE Optical Fiber Communication*

conference 11 (2003), 134-136.

[2] Assi, C. M., Y. Ye, S. Dixit, and M. A. Ali, "Dynamic Bandwidth Allocation for Quality-of-Service over Ethernet PONs," *IEEE Journal on Selected Areas in Communications* 21 (2003), 1467-1477.

[3] Bai, X., A. Shami, and C. Assi, "On the Fairness of Dynamic Bandwidth Allocation Schemes in Ethernet Passive Optical Networks," *Computer Communications* 29 (2006), 2123-2135.

[4] Baker, K. R., *Introduction to Sequencing and Scheduling*, John Wiley and Sons, 1974.

[5] ITU-T Y.1541, *Network Performance Objectives for IP-based Services* 2002.

[6] Jain, R., A. Durresi, and G. Babic, "Throughput Fairness Index: An Explanation, ATM Forum," http//www.cse.ohio-state.edu/~jain/atmf/a99-0045.html, 1999, accessed 29 February 2008.

[7] Joo, U. G. and Y. Kwon, "Performance Evaluation of a Bandwidth Allocation of E-PON," *IE Interfaces* 20 (2007), 427-437.

[8] Joo, U. G. and A. E. Smith, "Bandwidth Allocation with a Particle Swarm Metaheuristic for Ethernet Passive Optical Network," to be appeared in *Computer Communications*.

[9] Kramer, G., B. Mukherjee, and G. Pesavento, "IPACT: A Dynamic Protocol for an Ethernet PON (E-PON)," *IEEE Communications Magazine* (2002), 74-80.

[10] Kramer, G., B. Mukherjee, S. Dixit, Y. Ye, and R. Hirth, "Supporting Differentiated Classes of Service in Ethernet Passive Optical Networks," *Journal of Optical Networking* l1 (2002), 280-298.

[11] Kramer, G., *Ethernet Passive Optical Networks*, McGraw-Hill, 2005.

[12] Lannoo, B., L. Verslegers, D. Colle, M. Pickavet, M. Gagnaire, and P. Demeester, "Analytical Model for the IPACT Dynamic Bandwidth Allocation Algorithm for EPONs," *Journal of Optical Networking* 6 (2007), 677-688.

[13] Potts, C. N. and M. Y. Kovalyov, "Scheduling with Batching: A Review," *European Journal of Operational Research* 120 (2000), 228-249.

[14] Yang, Y-M., J-M. Nho, N. P. Mahalik, K. Kim, and B-H. Ahn, "A Traffic-class Burst-polling based Delta DBA Scheme for QoS in Distributed EPONs," *Computer Standards and Interfaces* 28 (2006), 721-736.

[15] Zheng, J. and H. T. Mouftah, "Media Access Control for Ethernet Passive Optical Networks: An Overview," *IEEE Communications Magazine* (2005), 145-150.