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Minimization of Inspection Cost in an Inspection System Considering the Effect of Lot Formation on AOQ*

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ABSTRACT

In this paper, we readdress the optimization problem for minimizing the inspection cost in a back-light unit inspection system, which forms a network including a K-stage inspection system, a source inspection shop, and a re-inspection shop. In order to formulate our objective function when the system is in a steady state, assuming that the number of nonconforming items in a lot follows a binomial distribution when a lot is formed for inspection, we make a steady-state network flow analysis between shops, and derive the steady-state amount of flows between nodes and the steady-state fraction defectives by solving a nonlinear balance equation. Finally we provide some fundamental properties and an enumeration method for determining an optimal value of K which minimizes our objective function. In addition, we compare our results numerically with previous ones.

Keywords: Back-light Unit, Inspection Cost, K-stage Inspection System

1. Introduction

Ever since the pioneering work of Dodge and Romig [1], a variety of acceptance sampling schemes for inspection have rapidly gained wide application in industry and have been evolved by many researchers. Wetherill and Chiu [2] reviewed some major principles of acceptance sampling schemes with emphasis on the economic aspect. Since then a few papers on acceptance sampling schemes have been published.

A single acceptance sampling plan, usually designated as (n, c), specifies the sample size n that should be taken and the number of defective units c that cannot be

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exceeded without the lots's being rejected. In BLU (Back-Light Unit) industries, (n, 0) acceptance sampling plan for inspection is widely used. Rejected lots must be 100% inspected later, and if some nonconforming items are found, then they should be reworked and pooled together with recalled items. Hence newly produced items, those reworked items in rejected lots, and the reworked items recalled from consumers are pooled together and form lots for inspection in a storage area. Since those items with different fraction defectives would be circulated, even estimating the fraction defective of items in the storage area is not easy. Similarly, measuring the effect of (n, 0) plan on the prior and posterior processes throughout the factory is not easy even though results of (n, 0) plans are simple.

Considering a BLU factory as a network of processes, which includes a K stage inspection system, a source inspection shop where (n, 0) plan for inspection is performed, and a re-inspection shop, Yang and Kim [4] provided a balance nonlinear equation for estimating the fraction defectives in nodes and the sizes of flows between nodes assuming that the system is in steady state. They formulated the cost objective function including the number of items inspected and reworked, and provided some fundamental properties and an enumeration method for determining an optimal value of K which minimizes their objective function.

They assumed, however, that the number of nonconforming items in a lot, which is formed for inspection in storage area, was not a variable but a constant. This "constant" assumption might not be realistic since the fraction defectives of different lots will be different. In this paper, we assume that the number of nonconforming items in a lot follows a binomial distribution, and we will readdress the optimization problem and provide some properties in addition to comparing our results with theirs. In Section 2, for reader's convenience we will describe briefly the problem. In Section 3, we will make the flow and cost analysis in order to formulate our cost objective function, and provide an enumeration method for determining an optimal value of K which minimizes our cost objective function. Finally, in Section 4, using the previous case study, we will compare our results with theirs.

2. Problem Statement

Yang [3] suggested a K-stage inspection system consisting of K stages, each of which includes an inspection process and a rework process as shown in Figure 1. In the first stage, if an item coming off from production lines is classified as conforming, then it is sent to a storage area called as Node 2. Otherwise, it is sent to the first rework process. After reworked, it is sent to the second stage. If the reworked item is classified as conforming, then it is sent to Node 2. Otherwise, it is sent to the second rework process. At the last K-th stage, an item classified as conforming is sent to Node 2 and an item classified as nonconforming is reworked and sent immediately to Node 2 without inspection. Assuming that inspectors are perfect in the sense that both type I error and type II error are zeros and using his result, we can express the average fraction defective of items stored at Node 2 as

$$p_K = p_0 p_R^K \tag{1}$$

where p_0= the average fraction defective of items produced from production lines, p_R= the average fraction defective of items reworked. Throughout this paper, we assume that $0 < p_0,\ p_R < 1$. If follows that $0 < p_K < 1$.

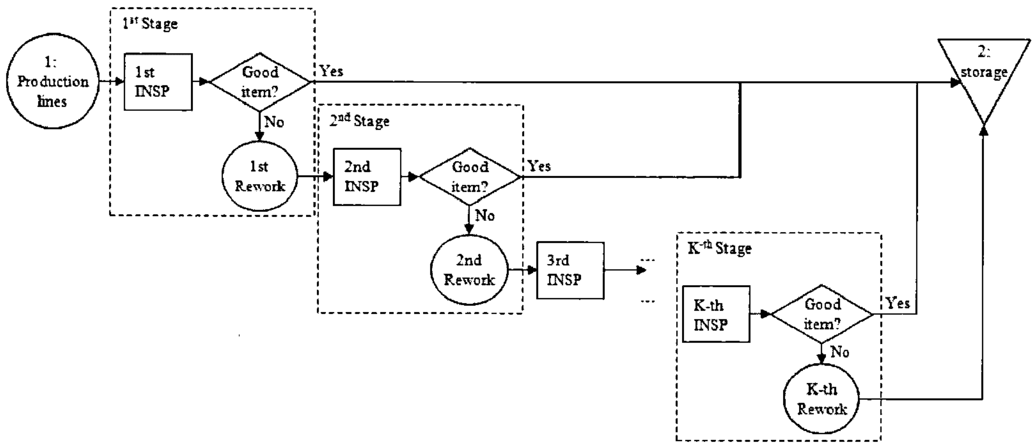

Figure 1. A Conceptual Process Diagram of the K-stage Inspection System

As shown in Figure 2, the inspection system consists of the production lines, the K-stage inspection system, the source inspection shop, and finally the re-inspection shop. Items stored in Node 2 in the K-stage inspection system are packed into lots and transferred to Node 3 in the source inspection shop, where they are stored. If demands arrive, the source inspector starts to inspect lots. If all the samples drawn from a lot are judged as conforming by him, the lot is accumulated and transported just in time to the consumer's production lines. Otherwise, lots are transferred to Node 5 in the re-inspection shop where all the items are re-inspected again. If those items in Node 5 are classified as conforming by inspectors, they are transferred to Node 3, the storage area in the source inspection shop. If not, they are sent to Node 6 and reworked in Node 7, located in the re-inspection shop. The nonconforming items returned from Node 9 are also reworked together with the items sent from Node 5.

We may consider various different types of costs. In this paper, we consider only the cost of inspecting items and the cost of reworking them. The ultimate loss from passing a defective is assumed to be equivalent to the cost of reworking. Now, define NIN(K) and NRW(K) to be the total number of items inspected at the K-stage inspection system, Node 4, and Node 5, and the total number of items reworked at the K-stage inspection system and Node 7 in the long run respectively. Assume that inspection costs per item occurring at different nodes are same and that rework costs per item occurring at different nodes are same too. Define κ to be the ratio of inspection cost per unit to rework cost per unit. Then given K stages, the total relevant inspection plus rework cost incurred throughout the factory, denoted by TC(K), can be expressed as NRW(K) + κ NIN(K). It may be conjectured that if K increases, the cost incurred at the K-stage inspection system increases while the cost incurred at Node 4, Node 5, and Node 7 decreases. Otherwise, reverse phenomenon will happen. Hence it can be expected that there exists an optimal value of K minimizing TC(K), and our problem can be stated as follows; Find the optimal value of K, denoted by K^*, so that we minimize TC(K).

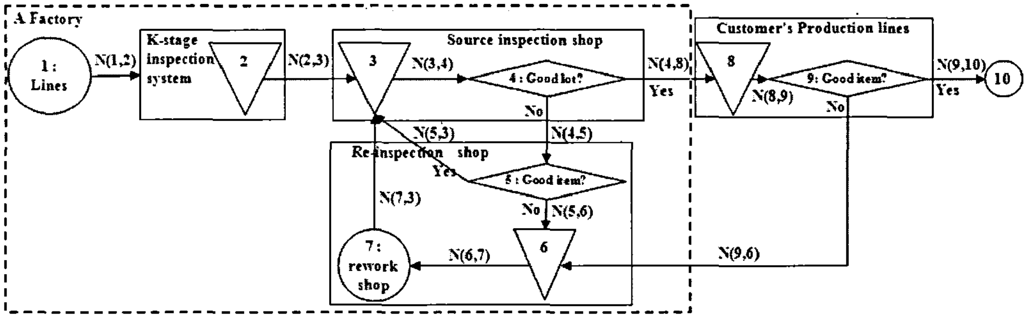

Figure 2. A Conceptual Process Diagram of an Inspection System

3. Flow and Cost Analysis

In this section, assuming that the number of nonconforming items in a lot follows a binomial distribution, we will make a steady-state flow analysis and derive a steady-state balance equation in order to formulate TC(K), and provide a procedure for determining an optimal value of K as well as fundamental properties.

3.1 Flow Analysis

In order to facilitate our flow analysis, we define the following notations;

$NG(i, j)$ = the steady-state number of conforming items sent from Node i to Node j

$NB(i, j)$ = the steady-state number of nonconforming items sent from Node i to Node j

$N(i, j) = NG(i, j) + NB(i, j)$

$NG(i) = NG(i, i) =$ the steady-state number of conforming items in Node i

$NB(i) = NB(i, i) =$ the steady-state number of nonconforming items in Node i

$N(i) = N(i, i) = NG(i) + NB(i)$

$p(i)$ = the steady-state fraction defective of items in Node i

Consider Node 8. We assume that as soon as the items, the amount of which is N(4, 8), are moved to Node 8, they are temporarily stored in Node 8 and are inspected and loaded into a production line. In other words, we assume that there are actually no stored items in Node 8, and we represent the series of these activities as N(4, 8) = N(8) = N(8, 9). Assume that if items are classified as conforming, the inspectors in Node 9 send them to Node 10. Otherwise the inspectors send them to Node 6. Suppose that N(9, 10), the number of items required for the consumer's production lines, is Q items per day. Since the fraction defective of items stored in Node 8 is p(8), in the long run we have

$$N(4, 8) = N(8) = N(8, 9) = \frac{Q}{1-p(8)} \tag{2}$$

$$N(9, 6) = \frac{p(8)Q}{1-p(8)} \tag{3}$$

Consider Node 4. Assume that all the items available in Node 3 are packed into lots and are immediately sent to Node 4 for inspection, i.e., N(3, 4) = N(3). Suppose that a lot is composed of N items, each of which is taken randomly from the population of size N(3). Let X be the number of nonconforming items in a lot. Assume that the probability that a nonconforming item is taken from the population is a constant denoted by p(3) or p. That is, the number of nonconforming items in population is pN(3). Then the probability of making a lot with j nonconforming items can be approximated as

$$\Pr\{X=j\} = \pi_j = \binom{N}{j} p^j (1-p)^{N-j} \qquad \text{for } j = 0, 1, \cdots, N$$

where $\binom{m}{k} = \frac{m!}{(m-k)!k!}$ for nonnegative integers m and k. Suppose that a lot is accepted only if all the sampled n (≤ N) items per lot are judged as conforming by the source inspector. We assume that even if a rejected lot may have some conforming items, they are not partially accepted. Let s_j be the probability of accepting a lot with j nonconforming items. Then s_j can be expressed as

$$s_j = \binom{N-j}{n} \Big/ \binom{N}{n} = \frac{(N-n)!(N-j)!}{N!(N-n-j)!} \qquad \text{for } j = 0, 1, \cdots, (N-n)$$

$$0 \qquad \text{for } j = (N-n+1), (N-n+2), \cdots, N$$

Since the probability of accepting a lot given that it has been formed with j nonconforming items can be expressed as

$$\pi_j s_j = (1-p)^n \binom{N-n}{j} p^j (1-p)^{N-n-j} \qquad \text{for } j = 0, 1, \cdots, (N-n),$$

$$0 \qquad \text{for } j = (N-n+1), (N-n+2), \cdots, N,$$

the probability of accepting a lot given p, L(p), can be expressed as

$$L(p) = \sum_{j=0}^{N} \pi_j s_j = (1-p)^n \sum_{j=0}^{N-n} \binom{N-n}{j} p^j (1-p)^{N-n-j} = (1-p)^n$$

Let $\lfloor x \rfloor$ be the greatest integer less than or equal to x. Assume that N(3, 4) is very large enough to satisfy that $\lfloor N(3,4)/N \rfloor N \approx N(3,4)$. Since the average number of lots accepted by the source inspector given N(3, 4) will be $\lfloor N(3,4)/N \rfloor L(p)$, we have

$$N(4,8) = \left\lfloor \frac{N(3,4)}{N} \right\rfloor L(p)N \approx L(p)N(3,4)$$

$$N(3,4) = \frac{N(4,8)}{L(p)} \approx \frac{Q}{L(p)\{1-p(8)\}} \tag{4}$$

$$N(4,5) = \{1-L(p)\}N(3,4) \approx \frac{\{1-L(p)\}Q}{L(p)\{1-p(8)\}} \tag{5}$$

Since Q is sufficiently large, and the difference between two values computed by approximation and equal signs respectively is usually within an allowed tolerance, we will use an equal sign instead of an approximation sign from now on and we assume that the numbers used in this paper are real.

Let $\lambda(0<\lambda<1)$ be n/N. Since the expected number of items accepted per lot is $NL(p)$ and the expected number of nonconforming items accepted per lot is $\sum_{j=0}^{N} j\pi_j s_j$, the AOQ of our inspection system, denoted by p(8), can be expressed as

$$p(8) = \frac{\sum_{j=0}^{N} j\pi_j s_j}{NL(p)} = \frac{(N-n)p(1-p)^n}{N(1-p)^n} = (1-\lambda)p$$

Consider Node 5. Assume that all the conforming items coming into Node 5 are sent to Node 3 and all the nonconforming items coming into Node 5 are sent to Node 6 by the inspectors in Node 5. The probability that a lot with j nonconforming items is rejected by a source inspector in Node4 is $(1-s_j)$ for j = 0, 1, $\cdots$, (N-n) and 1 for j = (N-n+1), (N-n+2), $\cdots$, N. Since the expected number of items rejected per lot is N{1-L(p)} and the expected number of nonconforming items rejected per lot is $\sum_{j=0}^{N} j\pi_j(1-s_j)$, p(5) can be expressed as

$$p(5) = \frac{\sum_{j=0}^{N-n} j\pi_j(1-s_j)}{N\{1-L(p)\}} = \frac{p\left[N\{1-(1-p)^n\}+n(1-p)^n\right]}{N\{1-L(p)\}} = (1+\delta)p$$

where $\delta = \frac{\lambda L(p)}{1-L(p)}$. Hence, using Eq. (5), we have,

$$N(5,3) = \{1-p(5)\}\, N(4,5) = \frac{\{1-L(p)\}Q - p[1-(1-\lambda)L(p)]Q}{L(p)\{1-(1-\lambda)p\}} \tag{6}$$

$$N(5,6) = p(5)N(4,5) = \frac{p\{1-(1-\lambda)L(p)\}Q}{L(p)\{1-(1-\lambda)p\}} \tag{7}$$

Consider Node 6. Since all the items corresponding to N(5, 6) and N(9, 6) are nonconforming and reworked in Node 7, using Eq. (3) and Eq. (7), we have

$$N(6,7) = N(5,6) + N(9,6) = \frac{pQ}{L(p)\{1-(1-\lambda)p\}} \tag{8}$$

$N(7,3) = N(7) = N(6,7)$, $NG(7,3) = (1-p_R)N(7,3)$ and $NB(7,3) = p_R N(7,3)$

Consider Node 3. Since the steady-state amount of the flow into Node 3 must be equal to the steady-state amount of the flow out of Node 3, we have,

$$N(3) = N(3,4) = \frac{Q}{L(p)\{1-(1-\lambda)p\}} \tag{9}$$

$$N(2,3) = N(3,4) - N(5,3) - N(7,3) = \frac{Q}{L(p)\{1-(1-\lambda)p\}} L(p)\{1-(1-\lambda)p\} = Q$$

$NG(2,3) = (1-p_K)Q$ and $NB(2,3) = p_K Q$

$$NB(3) = NB(2,3) + NB(7,3) = \frac{Q}{L(p)\{1-(1-\lambda)p\}}\left[p_K L(p)\{1-(1-\lambda)p\} + p_R p\right]$$

Suppose that we form lots in Node 3 after pooling and mixing those items transferred separately from Node 5 and Node 2. Now p(3) can be expressed as

$$p = \frac{NB(3)}{N(3)} = p_K L(p)\{1-(1-\lambda)p\} + p_R p$$

which can be further reduced to the steady-state equation for p(3),

$$p_K\{1-(1-\lambda)p\}(1-p)^n-(1-p_R)p=0 \tag{10}$$

Define p_E to be the steady-state fraction defective of items available in Node 3, which satisfies Eq. (10). Then, we have the following property for p_E, which says that Eq. (10) has only one root even though it is an (n+1)th-order polynomial equation, and that there exists only one steady-state value of N_E corresponding to N(3).

Property 1: If $0<p_R,\ p_K<1$ and $0<x<1$, then for positive integer n, there exists one and only one value p_E such that

(i) p_E is the solution of the following equation;

$f(x)=p_K\{1-(1-\lambda)x\}(1-x)^n-(1-p_R)x=0$, and

(ii) $N_E=\dfrac{Q}{\{1-(1-\lambda)p_E\}(1-p_E)^n}=\dfrac{p_KQ}{(1-p_R)p_E}$.

Proof: When n = 1, the first and second order derivatives of f(x) can be derived respectively as follows; $f'(x)=p_K\{2(1-\lambda)x+(\lambda-2)\}-(1-p_R)$ and $f''(x)=p_K2(1-\lambda)$. Since $f''(x)>0$ and $f'(1)=-\{\lambda p_K+(1-p_R)\}<0$, it follows that $f'(x)<0$ and $f(x)$ is a strictly decreasing convex function of x. Since $f(0)=p_K>0$ and $f(1)=-(1-p_R)<0$, there exists one and only one root, p_E, in (0, 1) such that $f(p_E)=0$. When $n\geq 2$, the first and second order derivatives of f(x) can be derived respectively as follows;

$$f'(x)=p_K\{(1-\lambda)(n+1)x-(n+1-\lambda)\}(1-x)^{n-1}-(1-p_R)$$
$$f''(x)=n(n+1)(1-\lambda)p_K(x_a-x)(1-x)^{n-2}$$

where $x_a=\dfrac{n-2\lambda+1}{(n+1)(1-\lambda)}$. It can be easily proved that $x_a>1$ since $0<\lambda\leq 1$ and $n\geq 2$. Since $f''(x)\geq 0$ and $f'(1)=-(1-p_R)<0$, it follows that $f'(x)<0$ and f(x) is strictly decreasing convex function of x. Since $f(0)=p_K>0$ and $f(1)=-(1-p_R)<0$, there exists one and only one root, p_E, in (0, 1) such that $f(p_E)=0$. Using Eq. (9) and Property 1-(i), Property 1-(ii) holds. □

From Property 1, we can compute the steady-state value of N(i, j) for all i and j and the steady-state value of the AOQ. In addition, it can be observed that p_E and N_E are a function of (N, n, p_0, p_R) and (Q, p_E) respectively. Now in order to represent explicitly that both N_E and p_E depend upon K, we change those notations to $N_E(K)$ and $p_E(K)$ respectively for a nonnegative integer K. We have the following basic properties.

Property 2: If $0 < p_R, p_K < 1$, then

(i) both $p_E(K)$ and $N_E(K)$ are strictly decreasing functions of K respectively, and

(ii) $\lim_{K\to\infty} p_E(K) = 0$,

(iii) $\lim_{K\to\infty} N_E(K) = Q$.

Proof: From Eq. (10), we have

$$\frac{\partial}{\partial K} p_E(K) = \frac{\{1-(1-\lambda)p_E(K)\}\{1-p_E(K)\}^n}{p_K\{1-p_E(K)\}^{n-1}\left[(1-\lambda)(1-p_E(K)+n\{1-(1-\lambda)p_E(K)\}\right]+(1-p_R)} \frac{\partial p_K}{\partial K} < 0 \ (\because \frac{\partial p_K}{\partial K} < 0)$$

Thus, $p_E(K)$ is a strictly decreasing function of K. Using Property 1-(ii), we have,

$$\frac{\partial}{\partial K} N_E(K) = \frac{g(p_E(K))\, Q}{\{1-p_E(K)\}^{n+1}\{1-(1-\lambda)p_E(K)\}} \frac{\partial p_E(K)}{\partial K}$$

where $g(p_E(K)) = (n-1)\{1-(1-\lambda)p_E(K)\} - \lambda$. Since $g(p_E(K)) > 0$, $N_E(K)$ is also a strictly decreasing function of K. Since p_K converges to zero as K goes to infinity, Property 2-(ii) holds from Eq. (10). It follows that Property 2-(iii) holds from Property 1-(ii) and Eq. (9). □

From Property 2, the shapes of $p_E(K)$ and $N_E(K)$ may be drawn as in Figure 3. Note that the first left parts of the shapes can be slightly different from the figures since the second derivatives of $p_E(K)$ and $N_E(K)$ can be either positive or negative respectively depending upon the input values.

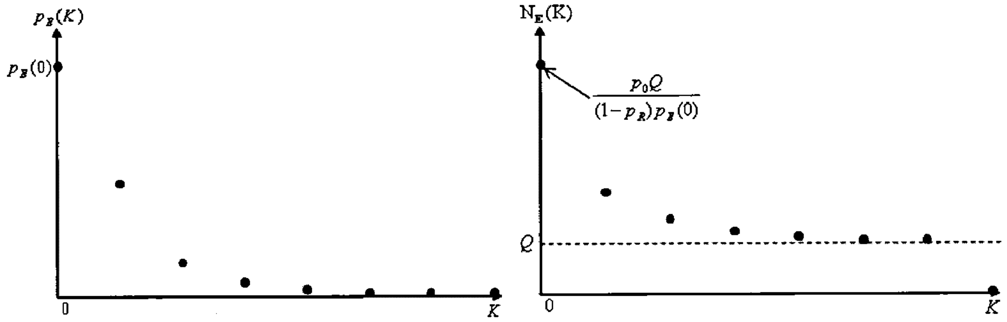

Figure 3. The shapes of $p_E(K)$ and $N_E(K)$

From Property 1 and 2, it is not clear whether $p_E(K)$ is always less than p_K or not. If our inspection system gives $p_E(K)$ greater than p_K, then there is no reason for the rework shop corresponding Node 7 to exist. In fact, the following property gives the necessary and sufficient condition for its existence. Hence we assume that $p_R < 1-\{1-(1-\lambda)p_K\}(1-p_K)^n$ holds.

Property 3: $p_E(K) < p_K$ if and only if $p_R < p_{RUB}(K)$ and $0 < p_R, p_K < 1$, where $p_{RUB}(K) = 1-\{1-(1-\lambda)p_K\}(1-p_K)^n$.

Proof: Since $f(x)$ is a strictly decreasing convex function of x and $f(p_E(K)) = 0$, $f(p_K) < 0$ if and only if $p_E(K) < p_K$. Thus solving $f(p_K) < 0$ gives the necessary and sufficient condition. □

3.2 Cost Analysis

Consider the total number of items reworked. Define $NRW_1(K)$ and $NRW_2(K)$ to be the number of items reworked at the K-stage inspection system and the number of items reworked at the re-inspection shop respectively. Define NRW(K) to be the sum of $NRW_1(K)$ and $NRW_2(K)$. Then the following property indicates that NRW(K) is invariant irrespective of the value of K. This important result is explicitly the same as that of Yang [3] even though we relax the "constant" assumption.

Property 4. $NRW(K) = \frac{p_0 Q}{1-p_R}$.

Proof: We can express $NRW_1(K)$ and $NRW(2)$ respectively as

$$NRW_1(K) = \frac{(1-p_R^{\ K})p_0 Q}{1-p_R} \qquad \text{(from Yang [3])}$$

$$NRW_2(K) = N(6,7) = \frac{p_K Q}{(1-p_R)} \qquad \text{(from Eq. (8) and Eq. (10))}$$

Thus $NRW(K) = \dfrac{p_0 Q}{1-p_R}$. □

Since NRW(K) is constant irrespective of the value of K, we can exclude the rework cost and our total inspection plus rework cost, TC(K), is now redefined as the only inspection costs incurred at three shops; the K-stage inspection system, the source inspection shop, and the re-inspection shop. Utilizing the results of Yang [3] again, the number of items inspected at the K-stage inspection system, denoted by $NIN_1(K)$, can be expressed as

$$NIN_1(K) = 0 \qquad \text{if } K = 0$$
$$= \left\{1 + \frac{(1-p_R^{\ K-1})p_0}{1-p_R}\right\} Q \qquad \text{if } K \geq 1 \qquad (11)$$

Assume that the source inspector must examine all of the n samples per lot even though he may happen to find a defective item and reject the lot without inspecting the remaining samples. Then, using Eq. (4) and Eq. (10), the number of items inspected in Node 4 , denoted by $NIN_2(K)$, can be expressed as

$$NIN_2(K) = n\left\lfloor \frac{N(3,4)}{N} \right\rfloor = \lambda N_E(K) \qquad (12)$$

From Eq. (4) and Eq. (7), the number of items inspected in Node 5, denoted by $NIN_3(K)$, can be expressed as,

$$NIN_3(K) = N(4,5) = \left[\,1 - \{1 - p_E(K)\}^n\,\right] N_E(K) \qquad (13)$$

Hence, we can express the total relevant inspection cost as TC(K) $= \sum_{i=1}^{3} NIN_i(K)$. It is not easy to make a graph of TC(K). However, the following property might be useful to sketch and explain an approximated shape of TC(K).

Property 5: If $0 < p_0,\ p_R < 1$, then we have,

(i) $NIN_1(K)$ is a strictly increasing concave function of K.

(ii) $\lim_{K\to\infty} NIN_1(K) = \left(1 + \frac{p_0}{1-p_R}\right)Q.$

(iii) $NIN_2(K)$ is a strictly decreasing function of K.

(iv) $\lim_{K\to\infty} NIN_2(K) = \lambda Q.$

(v) $NIN_3(K)$ is a strictly decreasing function of K.

(vi) $\lim_{K\to\infty} NIN_3(K) = 0.$

Proof: For K = 0 and 1, Property 5-(i) is clear since. $NIN_1(0) = 0$ and $NIN_1(1) = Q$ from Eq. (11). Since $\ln p_R < 0$, we have,

$$\frac{\partial}{\partial K} NIN_1(K) = -\frac{p_0 p_R^{K-1}(\ln p_R)Q}{1-p_R} > 0, \text{ and } \frac{\partial^2}{\partial K^2} NIN_1(K) = -\frac{p_0 p_R^{K-1}(\ln p_R)^2 Q}{1-p_R} < 0.$$

Hence, $NIN_1(K)$ is a strictly increasing concave function of K. Since $\lim_{K\to\infty} p_R^{K-1} = 0$, Property 5-(ii) holds. Taking the first derivative of $NIN_2(K)$, we have

$$\frac{\partial}{\partial K} NIN_2(K) = \lambda \frac{\partial N_E(K)}{\partial K} < 0$$

Hence, $NIN_2(K)$ is a strictly decreasing function of K. From Property 2-(ii), Property 5-(iv) holds. Taking the first order derivative of $NIN_3(K)$, we have

$$\frac{\partial}{\partial K} NIN_3(K) = n\{1-p_E(K)\}^{n-1} N_E(K) \frac{\partial p_E(K)}{\partial K} + \left[1 - \{1-p_E(K)\}^n\right] \frac{\partial N_E(K)}{\partial K} < 0$$

It follows that $NIN_3(K)$ is a strictly decreasing function of K. From Property 2-(ii), Property 5-(vi) holds. □

From Property 5, the shape of $NIN_i(K)$ may be drawn as in Figure 4. Note that the first left parts of the shapes of $NIN_2(K)$ and $NIN_3(K)$ can be slightly different since the second derivatives of $NIN_2(K)$ and $NIN_3(K)$ can be either positive or negative respectively depending upon the input values.

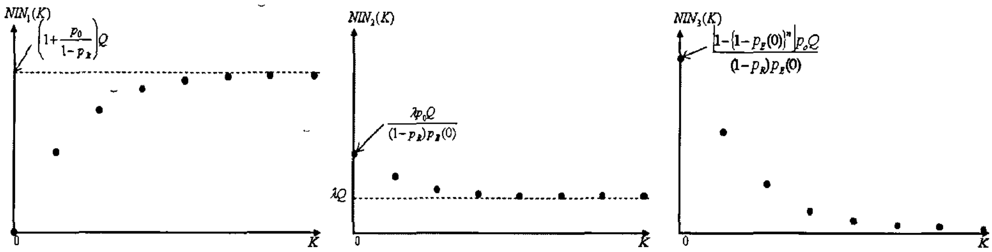

Figure 4. The shapes of $NIN_1(K)$, $NIN_2(K)$, and $NIN_3(K)$

3.3 A Procedure for determining K^*

Since the candidate value of K is very limited, an enumeration method for determining K may work well. Hence we suggest the following procedure; For an appropriate value of K_{MAX},

Step 1: For K = 0 to K_{MAX}

Begin

Find a solution of the equation in Eq. (10) and let $p_E(K)$ be the solution.

Compute $N_E(K) = \dfrac{p_K Q}{(1-p_R)p_E(K)}$.

Compute TC(K) using $p_E(K)$ and $N_E(K)$.

End

Step 2: Find K^* which minimizes TC(K).

4. Comparison with previous results

Using the same input values of (Q, p_0, p_R, N, n) estimated by Yang and Kim [4] as

(4,800 units/day, 16.1%, 5.0%, 240 units, 16 units), the values of TC(K) for $0 \le K \le 6$ are computed sequentially as 9443, 5779, 5928, 5933, and so on as shown in Table 1. It can be observed that as K increases, p_K decreases very rapidly up to zero, and that both $N_E(K)$ and $p_E(K)$ also decrease and converge to zero and 4,800 units respectively as proved in Property 2. $NIN_1(K)$ increases up to 5,613 units. $NIN_2(K)$ and $NIN_3(K)$ decrease up to 320 units and zero respectively as proved in Property 5.

Table 1. Computational results of TC(K) and AOQ(K)

K	0	1	2	3	4	5	6
p_K (PPM)	16.1000%	0.8050%	0.0403%	0.0020%	0.0001%	0.0000%	0.0000%
$N_E(K)$	13,619	5,449	4,834	4,802	4,800	4,800	4,800
$p_E(K)$ (PPM)	5.9733%	0.7464%	0.0421%	0.0021%	0.0001%	0.0000%	0.0000%
$NIN_1(K)$	0	4,800	5,573	5,611	5,613	5,613	5,613
$NIN_2(K)$	908	363	322	320	320	320	320
$NIN_3(K)$	8,535	616	32	2	0	0	0
TC(K)	9,443	5,779	5,928	5,933	5,933	5,933	5,933
AOQ(K)	5.5750%	0.6966%	0.0393%	0.0020%	0.0001%	0.0000%	0.0000%

Those values of $(K^*, TC(K^*), N_E(K^*), p_E(K^*), AOQ(K^*))$ related with the optimal solution turn out to be (1, 5778.9 units, 5449 units, 0.7464%, 0.6966%) instead of those previous values, (1, 5779.1 units, 5452 units, 0.7461%, 0.7461%). Assuming that current output is closer to true value, we define the percentage error of previous output as the ratio of previous output minus current output to current output. The percentage errors of previous output are computed as shown in Table 2. The percentage errors of $(TC(K^*), N_E(K^*), p_E(K^*), AOQ(K^*))$ turn out to be (0.0031%, 0.0444%, -0.0455%, 7.0953%). It can be observed in this case study that the current results are very similar to the previous results except AOQ(K) and that the values of AOQ's are overestimated more than 5%. Our strong conjecture is that the values of AOQ(K) derived by Yang and Kim [4] are always overestimated. However, we failed to prove this observation mathematically. Note that NRW(K) is computed as 813 units, which is invariant irrespective of both K and the "variable" assumption.

Table 2. Percentage errors of previous output

K	0	1	2	3	4	5	6
$N_E(K)$	0.2031%	0.0444%	0.0028%	0.0001%	0.0000%	0.0000%	0.0000%
$p_E(K)$ (PPM)	-0.2035%	-0.0445%	-0.0028%	-0.0001%	-0.0000%	-0.0000%	-0.0000%
$NIN_1(K)$	n.a.	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%
$NIN_2(K)$	0.2031%	0.0444%	0.0028%	0.0001%	0.0000%	0.0000%	0.0000%
$NIN_3(K)$	0.0801%	0.0025%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%
TC(K)	0.0920%	0.0031%	0.0002%	0.0000%	0.0000%	0.0000%	0.0000%
AOQ(K)	6.9253%	7.0953%	7.1399%	7.1427%	7.1428%	7.1429%	7.1429%

5. Concluding Remarks

In this paper, assuming that the number of nonconforming items in a lot follows a binomial distribution, we readdressed the optimization problem for minimizing our objective function, TC(K). Since flows between nodes were interrelated, we made a network flow analysis and derived a steady-state balance equation for solving the fraction defectives such as $p_E(K)$ and AOQ(K), and provided a formula for $N_E(K)$. Based on the values of $(p_E(K), N_E(K))$, our objective function could be obtained recursively. Since we found that NRW(K) was constant irrespective of K, we redefined TC(K) as NIN(K), and provided an enumeration method for determining an optimal value of K which minimized TC(K). In addition, we compared our results with previous ones, and found in this case study that the current results were very similar to the previous ones except AOQ (K) and that the previous values of AOQ's were overestimated more than 5%.

Further research may be concentrated on the problems maximizing the combination of different benefits, or on the problem finding some conditions under which the number of items reworked throughout a factory does not change. It may be an interesting topic to prove our strong conjecture mathematically. In addition, since one of our assumptions is that inspectors are perfect in the sense that both type I error and type II error are zeros, this assumption may be relaxed and very complicated results could be derived in the future.

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