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Understanding Statistical Terms: A Study with Secondary School and University Students[1]

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In this paper, we present an analysis of how students understand some statistical terms, mainly from inferential statistics, which are taught at the high school level. We focus our analysis on those terms that present more difficulties and are persistent in spite of having been studied until the college level. This analysis leads us to a hierarchical classification of responses at different levels of understanding using the SOLO theoretical framework.

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INTRODUCTION AND RESEARCH QUESTIONS

In recent years there has been an increasing amount of research on Statistics following the introduction of this subject into the Secondary Education curriculum in western countries. Many researchers in the field of Mathematics Education, especially in Statistical Education, have focused on understanding how Statistics is learnt and the difficulties the subject generates for students. Studies have been undertaken of random results, probabilistic experiments, averages by Strauss & Bichler (1988), Mokros & Russell (1995), García Cruz & Garrett (2007), of types of conceptualization of variability by Mokros & Russell (1995), Konold & Pollatsek (2002) and of the understanding of

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graphs by Friel, Curcio & Bright (2001), among others.

Shaughnessy (2007) states that aim of Statistical Education is to allow students of any age to read, analyze, criticize and make inferences regarding data distributions. Shaughnessy affirms that research needs to be done on classroom discourse when students are working with statistics. We should make students analyze data, criticize them, use multiple representations and communicate their results.

Niss (2003) describes what students learn in mathematics classes through what Niss defines as Mathematical Competence: the ability to understand, judge, do and use mathematics in a variety of situations and contexts both within mathematics itself or in situations outside mathematics but where mathematics plays or might play some role. This competence is shown by means of several mathematical capabilities which the student should possess and which Niss (2003) describes as follows:

A. Ability to make and solve questions on and with mathematics:

- Think mathematically
- Establish and solve mathematical problems
- Mathematical modelling
- Reason mathematically

B. Ability to develop and use mathematical language and tools:

- Represent mathematical entities
- Use symbols and formulae
- Communicate in, with and about mathematics
- Use aids and tools

Of the capabilities described by Niss (2003) we chose the ability to develop and use statistical language as the framework for our study, with specific focus on communication in statistics, with statistics and about statistics. We should take into account that mathematics also enhances linguistic competence, when this is understood as the competence that allows us to communicate and understand what is communicated to us. However, and more concretely, when we talk of linguistic competence with regard to statistics we refer to all the terms and modes of expression (graphic and analytic) used to communicate results or information.

In this work we focus on the understanding of statistical terms relating to Statistical Inference and which are used in secondary education. We understand language to be a basic element in the formation of statistical concepts, requiring clear and exact communication. On the other hand, the use of statistical terms allows their incorporation into everyday language and this is only possible if we promote students' classroom participation and their active explanation of those ideas that come up in class while statistical analysis is being worked on, and if we incorporate statistical meaning into everyday language.

The questions we ask in this work and for which we try to find answers are:

1. Does the working context determine the type of meaning the terms possess and, consequently, do students define them differently?
2. Do errors in understanding the terms disappear as students receive further training? In other words, do university students know and correctly apply statistical concepts?
3. Design a questionnaire to evaluate understanding.

The answers given by students are classified hierarchically to allow us to differentiate the types of answers and also compare the answers given by students at different educational levels. Using these categories we do not classify the students but rather their answers (Biggs & Collis, 1982) and thereby discover the quality of learning. In our bibliographical review we have not come across this type of analysis relating to the understanding of statistical terms and definitions and the evolution of this understanding over time and with greater training.

CONCEPTUAL FRAMEWORK

Shuard & Rothery (1984) show that students face difficulties understanding mathematical terms depending on the working context. More specifically, Shaurd & Rothery note that terms change in meaning according to the context where they are used (everyday context or mathematical context) and that it is important to know this in order to have the terms we use well defined and better communicate the concepts. Swan (2008) says that conceptual understanding is more efficient when collaboratively enriched tasks are set, mathematical language developed though communicative activities, the students' previous knowledge built upon, the student made to face difficulties rather than anticipate and avoid them, false conceptualizations brought up and discussed, higher order questions used, and interactive group/class teaching, individual work and collaborative work in small groups made suitable use of. Shuard & Rothery (1984) also point out that students do not know mathematics until they "speak them." Interpretation of concepts remains just "shadows" unless they are articulated through language (Vygostsky, 1996).

Orton (1990) also states that sometimes students face obstacles that have little to do with mathematics but are rather related to language. Orton adds that many aspects of language and mathematics can affect learning and make students interpret or change the real meaning for what they believe the teachers want them to say. Dickson, Brown & Gibson (1991) state that, as language development is dynamic in nature, it is essential that the student and teacher analyze the various meanings and interpretations of words and phrases so that they both clearly know what the other one understands and means when using certain linguistic forms.

Swan (2008) adds that students often find it difficult to generalize and transfer their knowledge to other subjects and contexts. Concepts that are related to one another therefore remain isolated. Swan calls teachers who can build bridges between ideas efficient teachers. Statistics is in this sense a valuable framework for making connections with elements outside the classroom and in different contexts. In order for students to make these connections they have to know statistical terms well.

García Alonso & García Cruz (2007) analyzed different secondary school textbooks and focused on statistical terms that are used when studying Statistical Inference. Their research shows that there are terms coming from everyday contexts that the student know but that in the classroom these terms take on a different meaning when the context changes. When this happens and the textbook fails to offer the new definition students might be led to understand the terms incorrectly. The authors also show that there are even some terms which might cause confusion as the meaning in the two contexts is contradictory. In this case errors may be exacerbated as students understand the exact opposite of what the term really means.

The aim of this work is to continue previous research and analyze how students understand terms relating to Statistical Inference, taking into account the classification of terms carried out by García Alonso & García Cruz (2007). The authors classify terms according to the meaning they possess in the context where they are used:

Same meaning in both contexts: Statistics, Population, Individual and Sample Size

Different meaning in both contexts: Sample, Estimation, Infer, Distribution, Probability, Representative, Risk and Significance.

Specific meaning in mathematical context: Parameter, Statistic, Random Sampling, Sample Mean, Population Mean, Level of Confidence, Typical Deviation, Maximum Acceptable Error, Normal, Biased, Efficient, Sample Proportion, Contrast of Hypotheses.

In our bibliographical review we could not find any analysis of the understanding of the different statistical terms according to the working context or any study of how to evaluate students' understanding after they have received further training. We therefore designed a questionnaire to evaluate understanding by means of students' answers. To carry out this analysis we took as our basis the neo-Piagetian theory described by Biggs & Collis (1982; 1991). This theory constitutes a hierarchical model for the study of the development process in the teaching of students based on a series of stages for a given domain. The theoretical framework is composed of different stages.

The prestructural stage covers answers that reveal a low level of learning in relation to the level of abstraction required by the task, or which express subjective arguments and show that students are distracted by irrelevant aspects of the question. In some answers the students simply make an affirmation without any explanation, submit incomplete ideas or show that they have not made any real attempt to give a structured answer based on evidence.

The unistructural stage covers those answers based on imaginary ideas and/or ideas connected with students' everyday experiences. In some answers students focus on one aspect of the data and use this to justify their answer, or take a concrete attribute of their own experiences, and believe this suitable for the situation in question.

The multistructural stage is where students' answers are in line with the task and where they can demonstrate certain properties suitable for this task, but in which they are unable to incorporate correctly all the different elements, basically because they attribute meanings other than those required in the task. Students at this stage demonstrate isolated knowledge of definitions, algorithms and procedures. Finally, there is the relational stage where students make exact connections between the different elements. They integrate the various parts, definitions, properties, formulae, algorithms, procedures and application conditions into the process of doing the task and are then able to complete a coherent, meaningful structure.

Taking all of this into account we decided to work with original model set out by Biggs & Collis (1982; 1991) and which establishes five stages of understanding: *Prestructural*, *Unistructural*, *Multistructural*, *Transition stage between the Multistructural and the Relational*, and, finally, *Relational*. This adapted classification is based on the work by García Cruz & Garrett (2008). The authors describe the Transitional stage as that where students' answers show the use of suitable reasoning and which even manage to supply correct, coordinated paths to solution. The students recognize the elements required by the task and their answers progressively approach the desired structure, and might even be judged correct in some cases. However, the students' ideas need refining (finishing) as they are still unable to give a coherent, meaningful structure as required by the task.

METHODOLOGY

Sample

We carried out our study with 26 students. Of these 14 were in the final year in non-compulsory secondary schooling and were aged between 17 and 20 years old, while the other 12 were doing their final year in the Mathematics Faculty and were aged between

21 and 25 years old.

For one term in the final year of their secondary schooling students are introduced to Statistical Inference through estimation by means of confidence intervals and contrast of hypotheses. In both of these cases, students study mean and proportion in contexts where they can solve problems by using normal distribution. However, the students were handed the questionnaire prior to beginning their work on Statistical Inference, so the students in our study did not possess any specific training in these concepts.

The university students were finishing their course in Mathematical Sciences and had studied Statistical Inference for one term when they had worked on punctual estimation, confidence intervals and contrast of parametric hypotheses, among other elements. When asked to complete the questionnaire the students were not at that time working on Statistical Inference, so, although they had worked previously with the relevant terms, this had not been done so recently.

Data collection tools

Our aim is to see how students understand the various statistical terms that they come across when they start their course in the final years of secondary school and check whether the definitions supplied by the students are modified when working in the mathematics context. Accordingly, the questionnaire we designed was based on the work carried out on textbooks and the categorization of statistical terms made by García Alonso & García Cruz (2007). The questionnaire was reviewed by a number of teachers and the questions were modified on several occasions to adapt them to our aims. The questionnaire is heterogeneous: certain concepts that cause especial difficulty appear in different questions and are asked in various ways, allowing us to check whether the students are coherent in their answers and also permitting us to see to what degree terms can be transferred to the various contexts. According to Batanero, Díaz & Cobo (2003), this makes the evaluation of understanding of a mathematical concept more valid.

We designed the questionnaire to cover the categories described by García Alonso & García Cruz (2007). In Table A we establish a relationship between the categories, the term studied and, in brackets, the number of the question we use to analyze the term in question:

The final questionnaire is made up of sixteen questions, one of which contains four subsections, giving a total of nineteen items. The questionnaire is used to analyze students' knowledge of certain statistical terms, to study whether the context of the type of mathematics worked on modifies the definitions of some terms, and to find out if there are any underlying conceptual errors originating in the everyday context and which have not been overcome in the classroom. Some terms belonging to the mathematics context itself are also included in order to see how the students define them if the terms have not been previously used in class. For the contexts appearing in the questionnaire we have utilized those that the students are most familiar with, such as tossing a coin, selection of a sample of students from a school employing two different sets of criteria and drawing balls from a box.

Table A. A relationship between the categories

Category	Term studied (Number of the question)
Same meaning in both contexts	Population (Q2), Individual (Q3)
Different meaning in both contexts	Sample (Q4, Q5B, Q6), Infer (Q12), Significance (Q15), Distribution (Q13)
Specific meaning in mathematical context	Statistic (Q14), Random sampling (Q16)

One term which we pay special attention to is SAMPLE, a term analyzed in depth by García Alonso & García Cruz (2007). This term is a clear example of how the context affects the definition and hence understanding of the concept. Sample is defined in university textbooks as "*a subset of measurements selected from the population under study*" (Mendenhall, 1982) or "*the part of the population that we really examine in order to find information*" (Moore, 2005). On the other hand, the Spanish Academy Dictionary (Real Academia Española, 2001) defines sample as "*part or portion taken from a set by methods that permit it to be considered as* **representative** *of it*." Here we can see the difference between the definitions given in the two contexts (the everyday and the mathematical), there being a conceptual error in the everyday context when it says that the sample must be representative of the population from which it comes. Focusing on this term, our aim is to study how students react to these differences, find out if this error is already in evidence before students begin studying Statistical Inference and, on the other hand, whether university students manage to overcome the error completely, especially when we consider that these students will be responsible for working with these concepts with future secondary school students.

Below we present the questionnaire:

1. *What is statistics about?*
2. *What do you understand by population?*
3. *What do you understand by individual?*
4. *What is a sample?*

These four questions are designed to make the students give an explicit definition for each term.

Students often know the definition of the concepts; the difficulties and erroneous conceptualizations arise when applying the definitions. To study this phenomenon we include the following question where we ask them directly to choose the characteristics they attribute to the concept "sample."

5. *Say which statement or statements you think is or are correct and which one or ones not. Justify every answer you make.*

    (a) A sample is a quantity of elements we choose from a larger set.
    (b) A sample always has to represent the set from which it comes.
    (c) Taking all the white balls from a box which contains both black and white balls cannot be accepted as a sample.
    (d) A sample can be taken in any way.

While in the first four questions we wanted students to define the terms directly, in this question we modify the way of answering as we give them a statement which the students have to decide is true or false, following which they must justify their choice. In this way we aim to analyze the type of reasoning that leads students to make their decisions.

In Part (a) we wish to study if students believe that there are elements that cannot be present in the set of data.

In Part (b) we explicitly mention the term represent so that the students can express their ideas about this term when it comes to selecting samples.

In Part (c) we set a situation commonly used to study probabilities whereby an "extreme" sample is selected where all the balls drawn are of the same colour. We want to see whether students think that this type of sample is invalid and why they would reject it.

In Part (d) we set out the method of selection of the sample. When we ask the students if the sample can be selected in any way, we make the students ask themselves if there is any way of selecting the sample which they believe to be invalid and they are asked to explain why. Statistics is based on simple random sampling but this is seldom explained clearly in the classroom and the student does not even know how to do this.

6. *In order to study how much time students in a school devote to their studies, the teachers selected 100 students with aim of choosing "good", "average" and "weak" students. Is this a good way of selecting students? Why?*

This question presents a context close to students' own experiences: a school. Students are to be selected in a survey and the teachers classify them previously on the basis of their academic performance. The question aims to analyze if students are capable of understanding that this type of selection is not a product of simple random sampling, as the teachers' categorization is not a matter of chance. Moreover, there is an underlying idea among students that this type of categorization represents all the students in the school. We wish to study if the idea of representativity of the sample is revealed in this situation.

7. *In the same situation as in Question 6, the teachers decided to choose the first 100 students who get to school. Is this option better or worse than the first one? Why?*

As in the previous question students are given a situation close to their own experience where they have to analyze the way in which the sample is chosen, as this should be the only analysis that allows the sample to be validated. In contrast to the previous case there is no clear intention behind the selection process. Students should realize that coming to school first depends on many factors and if these are not analyzed prior to selection then the selection might not be valid (simple random sampling).

8. *What do you understand by sample size?*
9. *A coin is tossed 10 times. Which of the following results do you think the most likely?*

   a. HHHHHHHHHH
   b. HTHTHTHTHT
   c. HHHHHTTTTT

Explain your choice and if you do not agree with any of these options give a result that you think is more likely and say why you think this.

This question uses a context often used in the calculation of probabilities. We wish to study if this type of context is more familiar to students and why this is so. The students are given a series of results of the tossing of a coin that might be considered "extreme". In this way students will quickly come out in favour or against these results and will explain their reasoning, supplying, if they think it necessary, any other result they believe to be more probable.

10. *We wish to calculate the average of the following data: 7, 7.1, 7.5, 6.9, 6.5, 7.5, and 6.9. How would you do it?*
11. *What does the result from the previous question show?*

With these two questions analysis is made of the understanding of the arithmetic mean, but only algorithmically, and the students' interpretation of this idea.

12. *Which of the following statements do you believe to be the correct one? Justify your answer.*

    a. *From the data in a sample we can deduce the population mean.*
    b. *From the data in a sample we can infer the population mean.*

The terms inference and deduction are incorrectly defined in an everyday context, so we wish to know whether students understand their meaning and know how to differentiate them correctly.

13. *What do you understand by distribution? Give an example.*
14. *What is a statistic? Give an example.*

These two questions cover more specific terms. We note that students cannot know them as they have not yet been defined and this needs to be done. However, in the case of university students, if they do know the terms, we can check if they remember them properly.

15. *What do you understand statistically speaking by something meaningful?*

This is a term which changes meaning in the mathematical context. Moreover, the change in definition is subtle but significant and must therefore be taken into account when working with it.

16. *We are going to take a "random sampling." Say what we mean by this and how we can do it by giving a concrete example.*

Here we want students to explain the procedure by which simple random samples can be taken and why these can be considered as such. The aim is to study more deeply what the students understand by random

ANALYSIS AND CLASSIFICATION OF ANSWERS

From the questionnaire we have chosen some of the questions referring to the different types of terms in accordance with the classification made by García Alonso & García Cruz (2007). We have selected the terms and questions listed in Table B:

Table B. The terms and questions

Category	Term studied (Number of the question)
Same meaning in both contexts	Population (Q2), Individual (Q3)
Different meaning in both contexts	Sample (Q4, Q5B, Q6), Infer (Q12)
Specific meaning in mathematical context	Statistic (Q14), Random sampling (Q16)

Based on their answers we try to evaluate their understanding of the terms and concepts used. To this end, we classify the students' answers in accordance with the theoretical framework generated by the SOLO taxonomy.

Students' answers are classified by stages. The classification is made according to the theory described by Biggs & Collis (1982; 1991). The stages of understanding found for each question can be clearly seen in the examples of students' answers and which we show for each type. We have chosen the answers that we believe provide us with the greatest information with regard to the analysis we are making.

Question 2. What do you understand by population?

- *PRESTRUCTURAL ANSWER (P)*: Students answering at this stage are incoherent and write a definition in which they understand population as the inhabitants of a given place.

  "It's a set of individuals that can be classified by class or category according to the entity under study"

  "Group of persons"

  "Set of people living in one place"

  "Set of people in a town"

- *UNISTRUCTURAL ANSWER (U)*: At this stage we have chosen those answers where there are no determining elements as to why the set is chosen. So, definitions such as the following are given:

  "Set of personal or impersonal elements"

  "Set of elements, objects, persons, etc."

  "Set of individuals having some characteristic in common"

  "Set of persons, animals or even objects"

  "Group that is going to be researched" (includes spelling mistake)

  "Set of all the samples taken." Here there is an element that has attracted the student's attention, which is the sample.

  "Total number to be worked with," making population equivalent to the number of elements.

- *MULTISTRUCTURAL ANSWER (M)*: Here we include those answers that define population as the study target but fail to define it as a set.

  "What is going to be studied?"

  "The people who are under study"

- *TRANSITIONAL ANSWER (T)*: At this stage are those students' answers which provide a correct definition, speak of the set and relate this to statistical studies, but which fail to make it clear that it is the study target.

  "Set of elements which are put under statistical study" In order for this answer to pass onto the next stage we understand that the answer should be simplified.

"Total set of individuals to be studied"

"Set of elements of which a statistical study is made"

"Data base made up of all the individuals or measurements of which a statistical study is to be made"

"Set of people or things from which a sample can be taken to make a study"

"Set of all the individuals those are going to be studied." We place this answer at the transitional stage because of the definition that this student later gives for *individual* ("person, object or animal taken to carry out a concrete study").

- *RELATIONAL ANSWER (R)*: These are the correct answers for the question asked.

"Set of all the individuals those are going to be studied"

"Data base made up of all the individuals or measurements of which a statistical study is to be made"

"Set of elements of which a statistical study is made"

"Set of persons or things that we are going to study"

"Set of all the individuals those are going to be studied." We include this answer at this stage because of the definition the students later gives for the term individual ("each one of that population").

Question 3. What do you understand by individual?

- *PRESTRUCTURAL ANSWER (P)*: Students classified at this stage are unable or do not know how to give a suitable answer in the working context. We can find answers such as:

"Only one person"

"Well, a person chosen"

"Each of those persons whose identity is unknown"

"One person in particular"

"Unknown or alien person"

"A person"

- *UNISTRUCTURAL ANSWER (U)*: Students answering at this stage speak of individual within a sample or fail to say what type of work is to be carried out with them.

"Persons or objects that are going to form part of a sample"

"One of the parts of the sample"

"Persons, objects or animals that are chosen to make a specific study"

"A person or thing which we are going to work with"

"Only one person to be studied, that is, who is given a percentage of statistics"

- *MULTISTRUCTURAL ANSWER (M)*: At this stage we find one student who is aware that individuals form part of a larger whole which is the population; although in reality the student is defining a sample.

"Different subgroups of the population"

- *TRANSITIONAL ANSWER (T)*: At this stage we have included those students who appear to know the term but fail to express it correctly.

"One of the elements which make up the sample or population"

"Each one of said population"

- *RELATIONAL ANSWER (R)*: Correct answers for the question asked.

"Simplest element of the population"

"A single element of those that make up the sample or population"

"A particular element of the population"

"Each one of the population" As this student managed to define correctly the term population at this stage we can consider that this student has reached the relational stage.

Question 4. What is a sample?

- *PRESTRUCTURAL ANSWER (P)*: At this stage we have classified those students who give a poor definition by using the same term they are asked to define, or else they give a definition other than for sample.

"A sample is a sample of one of the elements from a larger set"

"It is to show what might happen as a result"

- *UNISTRUCTURAL ANSWER (U)*: At this stage we have included all those answers that in one way or another, either explicitly or implicitly, indicate that a sample must represent the set it comes from.

"Small set of individuals of the population that represents it"

"Representative quantity of the population"

"A determined quantity chosen to refer to all the population"

"Concrete study of a population used to represent all the others"

Another type of answer from secondary school students that can be found at this level is one where the sample is identified with other statistical elements:

"Number of data we require from a certain population." Those who define the term in this way are giving a definition of sample size.

"Survey carried out among a certain number." This student defines sample as a survey.

"What is chosen to carry out an experiment?" This student defines sample as an experiment.

- *MULTISTRUCTURAL ANSWER (M)*: At this stage we have included those answers that refer to a part, set or collection but do not refer to the population.

  "Part of the population to study the population in order to get results that are valid"

  "Selection of a number of individuals (at random or not) that might or might not represent the population" The student uses various concepts but does not present them in an integrated way and also expresses the idea of representativity.

- *TRANSITIONAL ANSWER (T)*: At this stage students give the correct definition but they add elements that might modify the meaning:

  "Subset of the population where we take measurements to extrapolate to a greater population"

  "Small part of a set (population) to make a study"

- *RELATIONAL ANSWER (R)*: At this stage we include those students' answers that match the definition given in university textbooks. Among the university students there appear, then:

  "Subset of a population"

  "Subset of a population of which the statistical study is to be carried out"

  "Collection of individuals taken from a population"

  There are no answers from secondary school students at this stage.

Question 5B. A sample always has to represent the set from which it comes. True or false? Justify your answer.

- *UNISTRUCTURAL ANSWER (U)*: At this stage we consider all those answers that focus on the representativity of the sample.

  "Representativity gives the sample sense"

  "True, because the sample is then made valid"

  "To be accepted as a sample"

  "By definition"

  "False, because, for example, if we wish to measure the mean economic level we cannot take a sample of the poorest families." For this student, it is important that the sample contain all types, that is, that it be representative.

- *MULTISTRUCTURAL ANSWER (M)*: At this stage we have included the answers from those students who speak of representativity to refer to whether the element belongs or not to the larger set, or those answers expressed in terms of homogeneity.

"It should, but if the sample is heterogeneous and if there is a large number of individuals it is highly probable that it will not represent the set"

"Correct, because in a sample there cannot be an element that is not in the population"

"If I speak of persons this represents persons"

"Yes, because it is homogeneous"

- *TRANSITIONAL ANSWER (T)*: At this stage we include those answers that affirm that the statement in question is false, but in the formulation of the answer we are unable to decide what the students understand by representativity.

"False, not all samples are representative"

"It might represent the larger set or not"

- *RELATIONAL ANSWER (R)*: There are no answers at this level.

Question 6. In order to study how much time students in a school devote to their studies, the teachers selected 100 students with aim of choosing "good", "average" and "weak" students. Is this a good way of selecting students? Why?

- *UNISTRUCTURAL ANSWER (U)*: At this stage we account for those students who have focused on the representativity of the sample but who have not included any other type of element to find out if selection of the sample is suitable:

"If done randomly, yes"

"Yes. The students are diverse so the results from the study will give approximately the average time devoted to studies"

"This might represent the population, if this really provides us with what we want. To know this we need a statistical procedure that can tell us if this selection really represents the population"

"Yes it is, because it includes all the different types of students there can be. So in principle it's a representative sample"

"Yes, because the sample is representative, that is, it represents the reality of the population"

"Yes, because it represents all the likely cases of the population"

"Yes, because it tries to represent all the different types of student there are"

"Yes, because in this way we can find out the time devoted to studies according to students' abilities"

"Yes, because in this way we'll be able to see the different levels and results"

"It's a good way of selecting students because in this way there is variety in the study undertaken and the results might be more reliable"

"Yes, because you can research all types and not focus on just one"

> "Yes, because you'll be able to make a good selection by choosing the different types of level for each person"

> "No, because there would have to some other information in order to calculate this"

- *MULTISTRUCTURAL ANSWER (M)*: At this stage we select students' answers which accept as valid the selection proposed, but which take into account other aspects apart from representativity: proportionality with regard to the total and hourly dedication to studies. Or else, those answers where random chance appears in the type of selection and where students believe this might not favour selection.

> "It's good if students from each type are taken in proportion to the total; if not, the sample of 100 might contain more weaker students when in reality there are fewer such students"

> "Yes, provided the number of student representing their group (good, average and weak) is in proportion to the number of student there are in the school. (Supposing that there is some relation between the number of hours studied and the students' marks)"

> "Yes, because if we define intervals of time for each case, that is, if for the good students we consider that they devote 3 hours or more daily, for the average students 2 hours, and for the weak students less than two hours, we would be categorizing these 100 students in some of these groups"

> "Well, it isn't very good, because these 100 students might be weak and the rest who aren't selected might be the good students"

> "Yes, this will give mean values practically without error if the selection is good to begin with"

- *TRANSITIONAL ANSWER (T)*: Answers at this stage are expressed in accordance with the question, but the students have been unable to explain what the problem is and why the selection is not well made.

> "No, it isn't a good method as you're conditioning the types of people appearing in the sample and will fail to get a totally reliable result with regard to the population"

Question 9. A coin is tossed 10 times. Which of the following results do you think the most likely?

(a) HHHHHHHHHH
(b) HTHTHTHTHT
(c) HHHHHTTTTT

Explain your choice and if you do not agree with any of these options give a result that you think is more likely and say why you think this.

- *PRESTRUCTURAL ANSWER (P)*: There is one answer from a university student who marks (c) but fails to justify this option.

- *UNISTRUCTURAL ANSWER (U)*: Answers selected at this stage are based on the idea of representativity when choosing one or other option. The students' justifications are also based on the same idea.

  "The most likely thing is that one of each comes up during the 10 tosses of the coin as it is unusual for the same one to come up often. However, HHTHTTHTHT would be more likely"

  "Case (a) is the least likely (it only happens once). Case (b) is more probable than case (a)"

  "Answers (b) and (c) because the probability of coming up heads or tails is ½, but getting 5 heads and 5 tails is more likely than getting 10 heads"

  "(b), because there's a 50% chance it comes up heads and 50% tails"

  "I think Option (c) is more correct because there's the same probability that it will be heads as tails"

  Says (b): "Because with a coin you can only get heads or tails and then you can look at the probability"

  "My choice is (c) because the probability that it comes up heads is the same as tails"

  Says (b): "Because a coin has two sides (heads, tails) and the probability that it comes up one or the other is ½"

  Says (b): "Because coming up all heads is practically impossible. Half heads and half tails is too much of a coincidence. Neither do I agree with (b) but I think it might turn out like that as it is more disordered"

  Says (c): "Because we toss the coin 10 times"

  Says (c): "If we toss the coin ten times the probability is 5/10"

- *MULTISTRUCTURAL ANSWER (M)*: At this stage we include those answers that take into account other factors deemed relevant other than representativity. This leads the students to fail to express their choice correctly.

  "I think none of the results would turn out. They might do but we could not know the result until doing the task."

- *TRANSITIONAL ANSWER (T)*: At this stage we include answers from students who think any result possible, but cannot justify their answer correctly or base their answers on random chance.

  "I think when you toss a coin any result might come up. If the probabilities are calculated you'll get more or less what the probable result is"

  "I think any of the 3 possibilities might happen because when you toss a coin you don't know of it's going to come up heads or tails"

  "Any result might turn up because you can never know what exactly the result will be."

- *RELATIONAL ANSWER (R)*: Answers at this stage are correct and justifications adequate.

  "The three have the same probability of happening because each toss of the coin is independent and the probability of the three is $(1/2)^{10}$"

  "I think that the three, because, in the end, each side of the coin (heads or tails) has the same probability of coming up (1/2)"

  "All the answers have the same probability $(1/2)^{10}$ and any combination is as probable as any other. This can be explained by the property of the product of independent events and that the probability that it comes out heads or tails is ½"

  "The three results given (as well as any other) have the same probability of happening because each toss of the coin is an isolated event, that is, there's no relation with (or dependency on) the others"

  "None is more probable that any other because they are independent events"

  "The most probable thing is that there is a 50% probability that when you toss the coin you get heads and 50% tails, so any of the options could be valid"

  "They're all possible, because the probability when tossing a coin is ½ and is invariable, so anything might happen"

Question 12. Which of the following statements do you believe to be the correct one? Justify your answer.

(a) From the data in a sample we can deduce the population mean.
(b) From the data in a sample we can infer the population mean.

- *PRESTRUCTURAL ANSWER (P)*: There are no answers at this level.
- *UNISTRUCTURAL ANSWER (U)*: At this stage we place those answers from students who do not know the terms or definitions and who thus take Answer (a) as valid.
- *MULTISTRUCTURAL ANSWER (M)*: There are no answers at this level.
- *TRANSITIONAL ANSWER (T)*: All those students correctly giving (b) as their solutions are placed at this stage, although the justification they give is not completely suitable:

  "We are supposing some degree of confidence"

  "With the data from a sample what we can do is estimate the value." This student does not mark any answer as correct.

- *RELATIONAL ANSWER (R)*: They say answer (b) is correct and give a suitable justification.

  "Because it is an approximate and representative quantity and we can't know exactly the population mean"

"With the data from the sample we can get an approximate idea of what the population mean is, but not deduce the mean"

"When you take a sample you do not take all the elements and you can only make an approximation"

Question 15. What is a statistic? Give an example.

- *PRESTRUCTURAL ANSWER (P)*: These are the answers given by students, in this case Secondary School students, from which we cannot conclude that they have understood the question or who fail to answer or do not argue the answer.

"The result of the population"

"A result of some type of statistical study"

"Necessary result when doing statistics"

- *UNISTRUCTURAL ANSWER (U)*: At this stage we find that students define the statistic as the person (statistician) who carries out statistics, as defined in Spanish dictionaries (for example, the DRAE).
- *MULTISTRUCTURAL ANSWER (M)*: A university student defines a statistic as "a property that we can analyze in a sample to extend this to the population." Here the student has confused statistic with the desired properties to be fulfilled by statistics.

- *TRANSITIONAL ANSWER (T)*: There are no answers at this level.

- *RELATIONAL ANSWERS (R)*: There is one answer at this level from a university student.

"It is a datum taken from the sample and which infers the population."

Question 16. We are going to take a "random sampling". Say what we mean by this and how we can do it by giving a concrete example.

- *PRESTRUCTURAL ANSWER (P)*: Here we include all those answers which cannot be understood or which fail to answer the question asked.

"It's a study of something specific"

"By sampling we can guess many of the probabilities that can arise"

"Calculating so many percent"

"A person is chosen at random and is asked questions"

"You can do it by choosing 100 people." The student does not mention random chance or how the people are chosen.

- *UNISTRUCTURAL ANSWER (U)*: Here we find those students who use the examples given in previous questions (Question 6 and Question 7) as examples of random sampling, but fail to justify their answer adequately.

> "You aim to analyze a population without taking into account any predetermined criteria, so you select a certain number of individuals at random, as in Question 7"

> "Getting a sample at random, for example as in Question 6"

> "Taking a certain number of individuals at random from a population"

> "We aim to find the statistical result of a sample that could be a box containing balls of different colours so that the probability would vary throughout the study." This example is not suitable as the results do not have the same probability of happening. Calculation is not made, then, with a simple random sample.

- *MULTISTRUCTURAL ANSWER (M)*: Students' answers at this stage include different statistical terms such as "simple random sampling," "representative," "taking at random," and "taking from a population" among other terms. Sometimes they use these terms well, but in none of the cases given below do they answer what is asked of them, which is to describe the steps they would follow to carry out random sampling.

> "By carrying out random sampling of a population we aim to take a representative sample which we can use as a reference to do a study. The choice is made at random to guarantee that the sample is representative of the total population."

> "Random sampling is the study made of a set. For example, in the case given in another question about the coin tossed 10 times we can get several options which we will call random variables and the study of these options is random sampling."

> "Random sampling is a sample that infers the total population. We might have said simple random sampling, that is, that the sampling is representative."

> "Taking a sample completely at random. For example, to do random sampling in Spain, instead of taking the same number of people from every autonomous region, it is done taking people completely at random."

> "We want the sample to represent the population as much as possible, taking measurements of individuals in a random manner. For example, if we want to measure people's eyesight in a population it is not random to take only data for people who wear glasses as this won't represent the total population."

> "Random sampling is taking in a random manner a sample of the population and studying it. From a town we take 500 people at random and make a study of how tall they are to see if taller people outnumber shorter people in this town."

> "Random sampling is choosing a population sample. The aim is for the statistical study to be as diverse as possible. For example, if we do a study of the height of 14-year-old to 16-year-old children in the Canary Isles we can make a random selection."

There are no answers at the transitional or relational stages as in no case were students able to describe what they were asked to do.

RESULTS AND DISCUSSIONS

The results are studied by analyzing the questions in accordance with the stage where we have placed each term. We give the absolute frequencies and the percentages in accordance with the stage of understanding and on the basis of the educational level (secondary or university). From the data collected we draw our conclusions based on students' actions.

Category 1. Same Meaning in Both Contexts

Question 2. What do you understand by population?

We begin by analyzing Question 2 where students were asked what they understood by "population". Using university manuals as a reference we can find population defined as "a complete group of individuals about which we want information" (Moore, 2005). This term is classified within the category of terms which have the same meaning in the two contexts. The secondary school students' answers are significant because, in spite of being in their second consecutive year studying statistics and being asked by their mathematics teachers what they understood by statistics when handing out the questionnaire, the answer most given (42%) is that the term refers to the inhabitants of a place. Among the university students a notable number of answers (38%) state that population is a set of persons, animals or things, but fail to indicate what the aim of this set is. Classification based on students' understanding is shown in Table 1.

Table 1. What do you understand by population?

Question 2		University students		Secondary sch. students	
Stage	P	2	16.7%	6	42.8%
	U	6	50%	2	14.2%
	M	0	0	2	14.2%
	T	1	8.3%	2	14.2%
	R	3	25%	2	14.2

All the students give an answer but the answers are distributed among all the answer categories. At the prestructural stage we have classified those students who fail to give the correct definition or who get distracted and enter into irrelevant aspects of the definition. Many secondary school students' answers (42.8%) are placed at this level. Most university students' answers (50%) are at this level, and here we have included those answers which fail to state the aim of the set or which simply speak of a varied set but fail to focus on the fact that the aim is to carry out a study or search for information.

Question 3. What do you understand by individual?

When we ask what students understand by individual we should take into account that according to the university manual used as a reference (Moore, 2005) this term is defined as "*the objects defined by a set of data. The individuals could be persons, but they could also be animals or things*."

In the classification of the answers given in Table 2, we can see that many secondary school students (50%) are unable to give a suitable definition.

Table 2. What do you understand by individual?

Question 3		University students		Secondary sch. students	
Stage	P	1	8.3%	7	50%
	U	1	8.3%	4	28.6%
	M	1	8.3%	1	7.1%
	T	3	25%	0	0
	R	6	50%	1	7.1%
NA		0	0	1	7.1%

Category 2. Different Meaning in Both Contexts

Question 4. What is a sample?

Table 3 shows the results of the answers to Question 4. The definition of this term can change depending on the context in which we are working.

Among the secondary school students we can find that when defining this term they use related synonyms which they use in class and which are not correct. For example, surveys, answers, results, approximations to the mean, data valid for the population and data representative of the population. University students generally define the term more specifically, the correct definition being given by 76% of the students taking part in the study.

There are no secondary school students' answers at the relational stage, showing that at this level of schooling all the elements are still not correctly integrated.

We have classified at the unistructural stage those answers that express the idea that the sample should represent the original set. We can see that a relatively high percentage of secondary school students base their justification of the definition of sample on the idea of representativity, but that this percentage falls for the university students. However, it is still significant that in spite of their mathematical and statistical training 25% of university students continue to believe that the sample should be representative.

The definitions classified at the multistructural stage can be considered incomplete as they fail to refer to the population. In other words, the definitions refer to part, set, collection but do not place these elements within the larger set of interest to us, which is the population under study. In this situation we find 25% of university students and 14.2% of secondary school students.

It is significant that 50% of university students manage to give a correct answer and are placed at the transitional and relation stages, while only 7.1% of secondary school students are so placed.

All the students attempted to give a definition and not one of them failed to provide some answer to this question. This shows that this term is often used in an everyday context, so any student might feel motivated to give the definition he or she understands by this.

Table 3. What is a sample?

Question 4		University students		Secondary sch. students	
Stage	P	0	0	2	14.2%
	U	3	25%	9	64.2%
	M	3	25%	2	14.2%
	T	1	8.3%	1	7.1%
	R	5	41.7%	0	0

Question 5 asks the students to decide whether the statement is true or false and then asks them to justify their answer. At this point some students are faced with a problem: secondary school students are not accustomed to having to explain the decisions they make and state that they feel incapable of giving a justification and do not know how to go about doing this. This is demonstrated by the number of subsections to the question where a high percentage of students do not answer. We focus here on one of the subsections of the question where the greatest number of students answered. We will analyze the justifications given for their choices.

Question 5B. A sample always has to represent the set from which it comes. True or false? Justify your answer.

Table 4 shows the answers given to this question. We find that both university and secondary school students mainly answer that the statement is true (85% and 46% for secondary and university students, respectively) and that they believe that this is a criterion that samples should fulfill.

When we study the categorization of the answers based on the justification students make of their choices we have included at the unistructural stage those answers where the justification focuses on the representativity of the sample. We can see that most university and secondary school students are placed at this level. Only three university students (25%) contradict the statement and state that not all samples are representative, or else that the representativity of the sample is based on the method of selection, that is, it is obtained when the sample is taken at random. There are no answers at the relational stage; we believe that answers at this stage should deny that the sample is necessarily representative and that the students should state that, owing to random chance, the sample might not contain some of the elements that we know the original population to have.

Table 4. A sample always has to represent the set from which comes.

Question 5B		University students		Secondary sch. students	
Stage	P	0	0	0	0
	U	5	41.7%	8	57.2%
	M	3	25%	4	28.6%
	T	3	25%	0	0
	R	0	0	0	0
NA		1	8.3%	2	14.2%

Question 6. In order to study how much time students in a school devote to their studies, the teachers selected 100 students with aim of choosing “good”, “average” and “weak” students. Is this a good way of selecting students? Why?

Students’ answers to Question 6 are displayed in Table 5.

Table 5. Students’ answers to Question 6

Question 6		University students		Secondary sch. students	
Stage	P	0	0	0	0
	U	7	58.3%	11	78.6%
	M	4	33.3%	2	14.3%
	T	1	8.3%	0	0
	R	0	0	0	0
NA		0	0	1	7.1%

Most secondary school students (71%) affirm that this is a good method of selection and put forward different justifications: the variety of data we are going to collect, that such variety makes it more reliable, and that there will be more probabilities. One student says there is some information missing but does not say what this is. Among those who state that this is not a good method some simply say that another option would be better but only give a very vague justification. Most university students (46%) accept this method of selection because the distribution made of the population makes the sample representative.

Analysis according to stages of understanding shows that most students remain anchored in the idea of representativity they attribute to the sample. Students at the multistructural or transitional stages do not focus on this aspect and in some cases consider the selection method to be unsuitable.

Question 9. A coin is tossed 10 times. Which of the following results do you think the most likely?

a. HHHHHHHHHH
b. HTHTHTHTHT
c. HHHHHTTTTT

Explain your choice and if you do not agree with any of these options give a result that you think is more likely and say why you think this.

In Question 9 we change the context. In this case the context given is more often used when working on the calculation of probabilities: tossing a coin. We attempt to study whether the context might condition the students' answers, in spite of being asked about the same term.

Table 6. A coin is tossed 10 times. Which of the following results do you think the most likely?

Question 9		University students		Secondary sch. students	
Stage	P	1	8.3%	0	0
	U	4	33.3%	8	57.1%
	M	0	0	1	7.1%
	T	0	0	3	21.4%
	R	6	50%	1	7.1%
NA		1	8.3%	1	7.1%

For this question we find that 28% of secondary school students state that the option chosen does not matter, while 53% of university students say the same. The number of students, including university students, choosing the second or third option as the most probable is significant.

When we categorize the justifications given for their selections we find that a high percentage of students (57.1% secondary, 33.3% university) base their justifications on the fact that there are results which are more representative than others when it comes to tossing a coin. We believe that it is this idea that leads them to choose one option rather than another when they analyze which result they expect to turn out with greater probability. The answers classified at the transitional stage are those where students, on the basis of the knowledge they have, realize that all events are equally probable, but where their intuition or experience leads them to doubt this fact. These doubts can be seen in their answers.

Question 12. Which of the following statements do you believe to be the correct one? Justify your answer.

a. From the data in a sample we can deduce the population mean.
b. From the data in a sample we can infer the population mean.

Moore (2005) defines the term infer as "methods that allow us to draw conclusions from a population based on the data from a sample." With this question we try to see if the students can correctly differentiate the terms infer and deduce.

There are many answers regarding this term, especially from secondary school students, where students choose an option but fail to explain their choice. We believe that these students do not have any clear idea of the difference between infer and deduce and that choosing one or other of the options is a matter of indifference. All of these students choose Option A as valid. One possible interpretation is that the term "infer" has not yet been used in the classroom.

Table 7. Differentiate the terms infer and deduce

Question 12		University students		Secondary sch. students	
Stage	P	0	0	0	0
	U	3	25%	12	85.7%
	M	0	0	0	0
	T	5	41.7%	0	0
	R	3	25%	0	0
NA		0	0	2	14.2%

Most university students choose Option B (53%). A significant number (23%) state that with the data from the sample we can only approximate to the real mean value, as the sample is representative. In these cases, Option B was chosen.

Category 3. Specific Meaning In Mathematical Context

Question 14. What is a statistic? Give an example.

University manuals define this term as "*a number that can be calculated from the data of the sample without using any unknown parameter*."

42% of secondary school students answering this question (only half the students in our study do so) state that a statistic is the person (statistician) who carries out statistics.

We classify at the prestructural stage those answers that give an incoherent definition or which fail to give a conclusion in one sense or another. At the unistructural stage are those replies that define the term by relating it to statistics in some way but where the definition has nothing to do with the statistical meaning of the term. Most of the secondary school students' answers are found at this stage while none of the answers from these students can be placed at a higher stage. On the other hand, there are university students who give a definition approximating to the correct one.

Table 8. What is a statistic?

Question 14		University students		Secondary sch. students	
Stage	P	0	0	4	28.6%
	U	0	0	5	35.7%
	M	1	8.3%	0	0
	T	0	0	0	0
	R	1	8.3%	0	0
NA		9	75%	4	28.6%

Question 16. We are going to take a "random sampling." Say what we mean by this and how we can do it giving a concrete example.

This question allows students to show their knowledge of the process of random sampling and the steps that should be taken. We can see that at secondary school level no student was able to answer this question. However, what is noteworthy is that the university students at most manage to state that it is a matter of selecting data in a random fashion. None of the students in our study is able to describe in detail a procedure to obtain random samplings.

In the classification of the stages for the answers obtained, Table 9, we can see that secondary school students' answers do not go beyond the unistructural stage; at this stage we include all those answers that do not even manage to name the term random chance or some other synonym. University students who reach the multistructural stage merely describe random sampling as the chance selection of data. There are no answers at the higher levels because students do not respond to what they are asked; that is, there is no answer that describes in detail the way in which the sampling should be carried out and why this should be called random in contrast to other types of sampling.

Table 9. A random sampling

Question 6		University students		Secondary sch. students	
Stage	P	0	0	5	35.7%
	U	3	25%	2	14.3%
	M	9	75%	0	0
	T	0	0	0	0
	R	0	0	0	0
NA		0	0	7	50%

CONCLUSIONS

This research work has allowed us to document some of the difficulties faced by students when trying to understand some important statistical terms mainly related to statistical inference. The results obtained show that students make conceptual errors regarding the terms analyzed and that these errors persist even after several years of higher mathematical education. Also, our research has permitted us to document students' understanding through their answers to the questionnaire designed to this end. We can see here that there are no answers at the transitional and relational stages for terms in Category 3. This shows that the terms included here are specifically terms from statistical language. Furthermore, if they are not properly defined by teachers, the students never manage to know them.

The difficulties found in answers regarding terms in Category 1 should be underlined. These terms are defined in the same way in the everyday context as in the mathematical context. Very few students were able to give an answer that we could classify at the relational stage. Students did not supply the correct acceptance of the terms for the working context. It would be hard for them, then, to understand correctly the explanations or situations in which these terms are used, thus giving rise to an obstacle based on language (Orton, 1990). Teachers might be totally unaware of this type of obstacle.

Of the terms from Category 2 (terms with different meaning in the everyday and mathematical contexts) we selected the term sample to analyze students' understanding of this term in more detail. We were able to see that most university students are able to define the term properly, their answers being classified at the relational or transitional stages. This suggests that greater training allows students to be more precise in their definition of terms. However, when students have to use the definition to answer questions regarding samples, most of the answers are given at the unistructural or multistructural stages. This demonstrates that students believe that the samples should resemble the original set (representativeness heuristic) (Kahnemann, Slovic & Tversky, 1982). In this type of question there are no major differences between the answers from university and secondary school students. Obviously, we expected university students to demonstrate a greater degree of development as we supposed that greater training, maturity and experience would lead to better understanding of the term sample in the various situations set. The importance of this point must be underlined: there will be unfortunate future consequences as in years to come some of these university students will be teaching these terms in schools.

In our opinion, difficulties with statistical terms arise because students are not familiar with the use of the terms in different situations and contexts, and are thus unable to distinguish properly the different conceptual aspects.

In conclusion, the working context does not determine the type of meaning these terms have for students and, consequently, the students fail to adapt the meaning to the working context. Moreover, errors in understanding do not disappear with great training: students do not correctly apply the statistical concepts they learn and conceptual errors can continue throughout students' training. Taking into account these results we believe that we have to study what teaching strategies are most suitable to define correctly statistical terms and, at the same time, diversify the situations in which they are used in order to enhance all their conceptual aspects.

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